Parton Model from field theory via Light-front Current Algebra: The good, the bad, and the terrible

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Abstract

The emergence of parton model from field theory in the context of light-front current algebra and naive canonical manipulations is reviewed. Shortcomings of the naive canonical picture, especially concerning renormalization issues are discusssed. In order to illustrate the novel aspects of the renormalization problem in light-front dynamics, the scaling behaviour of different components of currents under the dual power counting is stressed. It is noted that the application of dual power counting to deep inelastic phenomena may provide a simple intuitive understanding of twist.

1. Introduction

The analysis and the resolution of renormalization problems associated with Hamiltonians is an important area of study in light-front field theory. Just like the Hamiltonian, matrix elements of currents or products of currents are also directly related to observables. Thus along with the study of renormalization problems associated with light-front Hamiltonians we need the study of renormalization problems associated with light-front currents.

Before we embark on the issues of renormalization of light-front current matrix elements, however, it may be helpful to recall the canonical structure of light-front currents, their relevance to observables and the inferences from canonical theory[1]. It is useful to recognise from a physical point of view the shortcomings of this picture so that we may be guided in studying the problems of renormalization.

It is worthwhile to remember that one of the motivations for proposing Quantum Chromodynamics (QCD) as the underlying theory of strong interactions was indeed the structure of light-front current algebra[2].

In the following we review[1,2] the emergence of parton model from canonical manipulations via light-front current algebra, mention its shortcomings from renormalization point of view, and briefly indicate how the dual power counting on the light-front may be beneficial in addressing various issues.

2. Fermionic Currents and Canonical Light-Front Commutators

Borrowing from the Lagrangian formalism we may define the vector current $J^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$ where $\psi(x)$ is the four-component Dirac field. (We will ignore the internal flavor symmetry in this discussion).

In light-front variables, not all the four components of ψ are dynamical. It is customary to define projection operators $\Lambda^\pm = \frac{1}{4}\gamma^\mp\gamma^\pm$ where $\gamma^\pm = \gamma^0 \pm \gamma^3$. Define $\psi^\pm = \Lambda^\pm\psi$. In gauge theory (QED or QCD), with the choice $A^+ = 0$, it follows from the equation of motion that ψ^- is constrained. Explicitly,

$$\psi^-(x^-, x^\perp) = \frac{-i}{4}\int dy^- \epsilon(x^- - y^-)[i\alpha^\perp.\partial^\perp - g\alpha^\perp.A^\perp + \gamma^0 m]\psi^+(y^-, x^\perp).$$

Thus the relation between ψ^- and ψ^+ is nonlocal and as we shall see the nonlocality has far reaching consequences.

From the definition of current, we have,

$$J^+ = 2(\psi^+)^\dagger\psi^+,\ \ J^\perp = (\psi^+)^\dagger\alpha^\perp\psi^- + (\psi^-)^\dagger\alpha^\perp\psi^+,\ \ J^- = 2(\psi^-)^\dagger\psi^-.$$

Using the canonical commutation relation, $\{\psi^+(x), (\psi^+)^\dagger(y)\}_{x^+=y^+} = \Lambda^+\delta^3(x-y)$, we get $[J^+(x), J^+(y)]_{x^+=y^+} = 0$. To compute $[J^+(x), J^-(y)]$ we need the equation of constraint and hence the equation of motion. We have

$$[J^+(x), J^-(y)]_{x^+=y^+} = \partial_x^+\Big\{-\frac{1}{2}\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)\bar{\psi}(x)\gamma^-\psi(y)\Big\}$$

$$+\partial_x^i\Big\{\frac{1}{2}\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)[\bar{\psi}(x)\gamma^i\psi(y) + i\epsilon^{ij}\bar{\psi}(x)\gamma^j\gamma^5\psi(y)]\Big\}.$$

Thus bilocal vector and axial vector currents emerge canonically. It is important to note that the nonlocality is only in the longitudinal (x^-) direction.

For future use define

$$\sqsubseteq^\mu(x|y) = \frac{1}{2}(J^\mu(x|y) + J^\mu(y|x)),\ \ \bar{\sqsubseteq}^\mu(x|y) = \frac{1}{2i}(J^\mu(x|y) - J^\mu(y|x)),$$

$$< P|\sqsubseteq^\mu(y|0)|P >= P^\mu V_1(y^2, P.y) + y^\mu V_2(y^2, P.y),$$
$$< P|\bar{\sqsubseteq}^\mu(y|0)|P >= P^\mu \bar{V}_1(y^2, P.y) + y^\mu \bar{V}_2(y^2, P.y).$$

Note that $V(y^2, P.y) \rightarrow V(\frac{1}{2}P^+y^-) = V(\eta)$ since $y^2 = 0$, and $P.y = \frac{1}{2}P^+y^- = \eta$ at $y^+ = 0, y^\perp = 0$.

3. Scaling Function as Fourier Transform of Bilocal Matrix Element

The hadron tensor relevant for spin-averaged electron-nucleon scattering is given by

$$W^{\mu\rho} = \frac{1}{4\pi}\int d^4y\, e^{iq.y} \; < P|[J^\mu(y), J^\rho(0)]|P >$$

$$= [-g^{\mu\rho} + \frac{q^\mu q^\rho}{q^2}]W_1 + (P^\mu - \frac{P.qq^\mu}{q^2})(P^\rho - \frac{P.qq^\rho}{q^2})W_2.$$

Consider forward virtual Compton scattering amplitude

$$T^{\mu\rho}(P,q) = i\int d^4y\, \theta(y^+)\, e^{iq.y} < P|[J^\mu(y), J^\rho(0)]|P > .$$

We also have

$$W^{\mu\rho}(P,q) = \frac{1}{2\pi} Im\, T^{\mu\rho}(P,q).$$

Write a fixed q^2 dispersion relation

$$T^{\mu\rho} = \int \frac{d\nu'}{\nu' - \nu} W^{\mu\rho} \;\text{ where } \nu = P.q \; . \qquad (1)$$

Consider Bjorken-Johnson-Low limit of Compton amplitude

$$Limit_{q^-\to\infty}\, T^{\mu\rho} = -\frac{1}{q^-}\int dy^- d^2y^\perp e^{i(\frac{q^+y^-}{2} - q^\perp.y^\perp)} < P|[J^\mu(y), J^\rho(0)]_{y^+=0}|P > .$$

Using the integral representation for the antisymmetric step function

$$\epsilon(x^-) = -\frac{i}{\pi}\int \frac{dq'}{q'} e^{\frac{i}{2}q'x^-}$$

and taking the $q^- \to \infty$ of eq. (1), take absorptive part on both sides and compare coefficients. From "$+-$" component,

$$Limit_{q^-\to\infty}\, \nu W_2(q^2, \nu) = F_2(x)$$

with $x = \frac{-q^2}{2\nu}$ and $\frac{F_2(x)}{x} = \frac{i}{2\pi}\int d\eta e^{-i\eta x}\bar{V}_1(\eta)$. Thus scaling function is the Fourier transform of the bilocal matrix element. Just as matrix elements of local currents are measured in elastic scattering deep inelastiic scattering measures matrix elements of bilocal currents. From "++" component we get $Limit_{q^-\to\infty} W_2, W_L = 0$, where $W_L = W_1 + \frac{(P.q)^2}{q^2}W_2$.

Making a Fock space expansion for $|P >$, i.e.,

$$|P >= \int \; \phi(k_1)b^\dagger(k_1)d^\dagger(P - k_1)|0 > \; +....$$

the parton picture with probabilistic interpretation emerges:

$$\frac{F_2(x)}{x} = \int d^2k^\perp |\phi(x, k^\perp)|^2 + ...$$

4. Trouble with Canonical Picture: Renormalization Aspects

In the naive canonical manipulations (even though they have lead to an intuitive physical picture of scaling) renormalization effects are completely ignored. The structure function we obtained has no dependence on a mass scale whereas in the real world we do need a scale dependence. Once renormalization effects are taken in to acccount, we encounter divergent loop integrals when loop momenta also tend to infinity. Thus $q^- \to \infty$ limit is valid a priori only for a cutoff theory. So the question remains whether the intuitive parton based picture still survives after renormalization effects are taken into account.

5. Dual Scaling Analysis, Light-front Power Counting, Consequences

To start tackling the renormalization problem which is forced upon us from physical considerations, let us begin with light-front canonical reasonings. The starting point of renormalization analysis is the study of behaviour of operators under scale transformations. In light-front dynamics we consider separate scaling analysis[3] in the longitudinal (x^-) coordinate and transverse ($x^\perp$) coordinate. For the fermion field operators we have $\psi^+ \sim \frac{1}{\sqrt{x^-}}\frac{1}{x^\perp}$ and $\psi^- \sim \frac{\sqrt{x^-}}{(x^\perp)^2}$. Thus $J^+ \sim \frac{1}{x^-}\frac{1}{(x^\perp)^2}$ and canonically J^+ has a unique scaling behaviour whether or not masses are present. On the other hand $J^\perp \sim \frac{1}{(x^\perp)^3}$ and $J^- \sim \frac{x^-}{(x^\perp)^4}$. When masses are present, $J^\perp$ and J^- have no unique transverse scaling behaviour and only dimensional analysis applies in the transverse coordinate.

First let us recall the consequences for scale breaking which follow from canonical reasonings. In light-front dynamics longitudinal scale transformation corresponds to longitudinal boost transformation and hence longitudinal scale invariance is a Lorentz symmetry of the theory. Hence canonical reasonings indicate that the longitudinal scale invariance cannot be broken by masses or renormalization process. This implies that a mass scale can get generated only through transverse divergences. This inference is corroborated in perturbation theory; for example, the standard asymptotic freedom result in light-front QCD arises through transverse momentum divergences.

Consider deep inelastic process where relevant distance scales are short transverse separations and medium to large longitudinal separations. The dual power counting is ideally suited to study this phenomena. The behaviour of bilocal matrix elements for large y^- determines the small x behaviour of structure functions. On the other hand, power correction to scaling is determined by the scaling behaviour of operator product of currents under transverse scale transformations.

To study power corrections to scaling one can classify operators on the basis of their transverse mass dimension. For example, J^+ has transverse mass dimension 2. According to the terminology of Gell-Mann and Fritzsch $+, \perp,$ and $-$ components correspond to good, bad and terrible operators respectively. For good operators we notice that twist and transverse mass dimension coincide. However, by the same token, bad and terrible operators corresponds to higher twist! From different considerations similar conclusions have been arrived at before by parton theorists[4].

Acknowledgement

The author acknowledges helpful conversations with the members of the Theory Group, SINP. He is also grateful to the participants of the Polana Zgorzelisko Workshop, especially Prof. Leonard Susskind for many illuminating discussions. The author thanks Prof. Jozef Namysłowski for his kind hospitality in Warsaw, Tatra mountains and places in between.

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