The Electron's Anomalous Moment in Large-α QED: A Progress Report

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Abstract

A calculation of the anomalous moment of the electron for large coupling is underway. It is based on discretized light-cone quantization, Tamm-Dancoff truncation, and sector-dependent renormalization. Fock states with one electron and as many as two photons are included. The coupling is kept large to make the effects of two-photon Fock states discernible. Mass renormalization is carried out, including a necessarily nonperturbative renormalization for the bare electron. Results are presented for ranges of numerical parameters with α set to unity and a photon mass of one-tenth the electron mass.

1. Introduction

A nonperturbative calculation of the anomalous moment of the electron[1] is currently in progress. The intent is to demonstrate that a formalism for nonperturbative calculations can be constructed and thereby to respond to the challenge by Feynman[2] to find a better understanding of the anomalous moment. In addition, this work provides a (3+1)-dimensional gauge-theoretic setting in which to test nonperturbative renormalization of the light-cone Hamiltonian. Limitations on numerical accuracy are expected to make nonperturbative effects discernible only at large coupling.

The work is done in light-front quantization,[3] where a Fock-state expansion for the dressed electron is well defined. A Hamiltonian $H_{\rm LC}$ is constructed and the dressed state $|p,s\rangle$, with momentum p and spin s, is required to be an eigenstate:

$$H_{\rm LC}|p,s\rangle = M^2|p,s\rangle\,. \tag{1}$$

The eigenvalue is equal to the square of the physical electron mass m_e. The state $|p,s\rangle$ is expanded in a Fock basis

$$|p,s\rangle = \sum_n \int [dx]_n\,[d^2k_\perp]_n\,\psi^{(n)}_{ps}(x,\vec{k}_\perp)|n:x,\vec{k}_\perp\rangle\,, \tag{2}$$

with wave functions $\psi^{(n)}_{p,s}$. The notation for the integrations is defined by

$$[dx]_n = 4\pi\delta(1-\sum_{i=1}^{n} x_i)\prod_{i=1}^{n}\frac{dx_i}{4\pi\sqrt{x_i}} \quad \text{and} \quad [d^2k_\perp]_n = 4\pi^2\delta(\sum_{i=1}^{n}\vec{k}_{\perp i})\prod_{i=1}^{n}\frac{d^2k_{\perp i}}{4\pi^2}\,, \tag{3}$$

where x is the usual longitudinal momentum fraction and $\vec{k}_\perp$ the relative transverse momentum. The solution of (1) yields the light-front wave functions.

The anomalous moment a_e is computed from the standard form factor $F_2(q^2)$ at zero momentum transfer: $a_e = F_2(0)$. In the frame where $q = (0, q_\perp^2/p^+, \vec{q}_\perp = q_1\hat{x})$ the form factor can be computed from the spin-flip matrix element of the plus component of the current: $-\frac{q_1}{2m_e}F_2(q^2) = \frac{1}{2p^+}\langle p+q,\uparrow|J^+(0)|p,\downarrow\rangle$. Brodsky and Drell[4] have given a reduction of this matrix element to a convenient form that depends directly on the wave functions. From this we have

$$a_e = -2m_e\sum_j e_j\sum_n\int[dx]_n\,[d^2k_\perp]_n\,\psi^{(n)*}_{p\uparrow}(x,\vec{k}_\perp)\sum_{i\neq j}x_i\frac{\partial}{\partial k_{1i}}\psi^{(n)}_{p\downarrow}(x,\vec{k}_\perp)\,, \tag{4}$$

where e_j is the fractional charge of the struck constituent.

The numerical calculation is based on discretizations of (1), (2), and (4) that mostly follow the work on discretized light-cone quantization (DLCQ) by Tang, Brodsky, and Pauli.[5] The Fock space is truncated to include only one electron and at most two photons. The allowed momentum states must satisfy an invariant-mass cutoff:

$$\sum_i(m_i^2+k_{\perp i}^2)/x_i \le \Lambda^2\,. \tag{5}$$

The photon is given a small mass m_γ which reduces the errors associated with the numerical approximations to the integrals in (4). Renormalization of the electron mass is done in a sector-dependent way;[6] this is discussed in the next section. However, coupling renormalization is not yet included in the numerical calculation. The calculation of results presented here was done with additional temporary simplifications. Instantaneous fermion couplings are excluded. This eliminates self-coupling in the topmost $|e\gamma\gamma\rangle$ sector, thereby simplifying the diagonalization problem. It also reduces the complexity of the mass renormalization. Periodic boundary conditions are used for both electron and photon fields, and zero modes are ignored. The temporary selection of periodic boundary conditions for the electron was made to facilitate use of computer code used for a scalar theory.[7]

When truncated to include at most one photon in the basis, the calculation yields the Schwinger result.[1] This can be seen analytically in Ref. 4. However, to obtain this result one must take coupling renormalization into account.

2. Mass and Coupling Renormalization

The sector-dependent mass renormalization is carried out as follows. In the top $|e\gamma\gamma\rangle$ sector there can be no corrections to the bare mass; therefore, the bare mass is the physical mass. In the middle $|e\gamma\rangle$ sector, the only corrections come from single loops where one photon is emitted and absorbed. The requirement imposed is that the $|e\gamma\rangle$ scattering states have the correct threshold.[6] The contribution can be computed directly,[5] with spectator dependence taken into account both in application of the cutoff Λ^2 and in determination of the available momentum. The bottom $|e\rangle$ sector requires a nonperturbative treatment, which actually drives the method of solution for the eigenvalue problem.

To formulate the nonperturbative renormalization, we write the Fock-state expansion schematically as

$$|p,s\rangle = \psi_0|e\rangle + \vec{\psi}_1|e\gamma\rangle + \vec{\psi}_2|e\gamma\gamma\rangle\,. \tag{6}$$

The eigenvalue problem (1), on elimination of the amplitude vector $\vec{\psi}_2$, becomes a coupled set of two integral equations

$$m_0^2\psi_0 + \vec{b}^\dagger\cdot\vec{\psi}_1 = M^2\psi_0\,, \quad \vec{b}\psi_0 + A\vec{\psi}_1 = M^2\vec{\psi}_1\,, \tag{7}$$

where m_0 is the bare electron mass and $\vec{b}^\dagger$ and A are integral operators obtained from $H_{\rm LC}$. We now require that m_0 be such that $M^2 = m_e^2$ is an eigenvalue, and solve (7) for $\vec{\psi}_1/\psi_0$ and m_0^2. This approach can be generalized to cases with more Fock sectors.

Renormalization is complicated by the need to separately renormalize the couplings in different terms of the Hamiltonian.* Also, the Ward identity that usually guarantees cancellation of wave function and vertex renormalization does not hold in the truncated Fock space. For example, with truncation to include at most one photon, the $ee\gamma$ vertex experiences self-energy corrections on one leg only and no vertex correction. Thus only a $\sqrt{Z_2}$ factor appears, with no second $\sqrt{Z_2}$ and no Z_1.[9] In general, renormalization should be done from the top two sectors down to the lowest two. The Z_2 factors can be computed from the bare-electron probability amplitude ψ_0.[10] The Z_1 factors for spin-flip and no-flip $ee\gamma$ vertices should be obtainable from the no-flip transition amplitude evaluated near threshold for two photon helicities.† The Z_1 factors for instantaneous fermion and photon couplings are related to Compton scattering and fermion scattering, respectively.

3. Numerical Results

Preliminary calculations have been done to examine convergence rates. The results for truncations to one and two photons are presented in Fig. 1. The numerical parameters are

*In Yukawa theory this has been used to restore rotational invariance.[8]

†The spin-flip transition is avoided so that the anomalous moment does not become an input to the calculation.

the number of transverse points $N_\perp$ in the positive and negative x and y directions, and the resolution K of the longitudinal momentum. The transverse momentum is divided into units of $\Lambda/N_\perp$; the longitudinal momentum fraction into units of $1/K$.

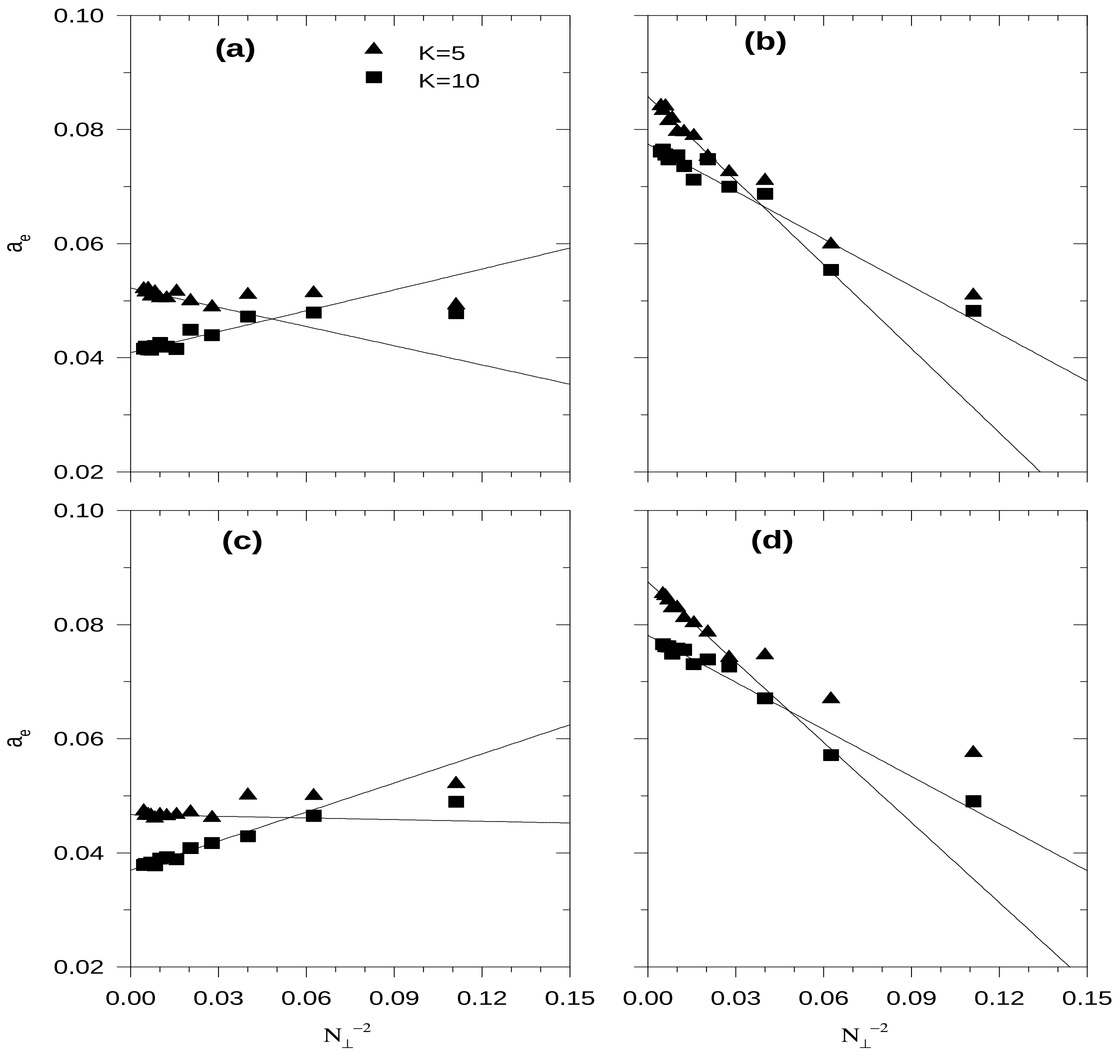

Figure 1: The anomalous moment, as a function of the number of transverse points $N_\perp$, for various values of the resolution K, and invariant-mass cutoff Λ^2. The coupling strength is $\alpha = 1.0$ and the photon mass is $m_e/10$. For (a) and (c) the Fock space is truncated to include at most one photon, and for (b) and (d), to include at most two photons. The cutoff Λ^2 is $10m_e^2$ for (a) and (b), and $20m_e^2$ for (c) and (d).

4. Future Work

There are several things left to be included as work proceeds. Coupling renormalization has already been discussed. Also needed are antiperiodic boundary conditions for the electron field and zero modes[11] for the photon field. Instantaneous fermions should be added. Cutoffs other than the invariant-mass cutoff could be considered; for example, the Hamiltonian itself can be limited by the change allowed in the invariant mass across any given matrix element.[12] Finally, the Fock basis could be expanded to include an electron-positron pair; this will require photon mass renormalization and additional coupling renormalization. Clearly, there are still important challenges to be faced, and there are interesting ways in which to extend the calculation.

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