

Condensates and Confinement in QCD Vertex Functions

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Abstract

Nonzero quark and gluon condensates are responsible for confinement of quarks and gluons. By confinement we mean: i) the absence of any asymptotic states for either quarks, or gluons, ii) the absence of any continuum spectrum for partons, iii) the absence of any colour-full parton bound systems, and iv) the presence of hadrons which are colour-less bound states of quarks (antiquarks) and gluons. Confinement does not require potentials that diverge at long distances, as opposed to recent claims by Wilson et al. in Phys.Rev. D**49**, 6720 (1994). Other “no-go theorems” for hadrons treated by LFQCD are collected.

Nonzero quark and gluon condensates¹ allow for an explicit construction of singular contributions in four, out of seven, superficially divergent QCD vertex functions. They appear in inverses of quark and gluon propagators $S^{-1}(k)$, $D_T^{-1}(p)$, in the gluon-quark vertex function $\Gamma^\mu(k, k-p, p)$, and in the triple-gluon vertex function $\Gamma^{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$, as the following physical poles, respectively,

$$\frac{\chi^3/M}{\not{k} - M + i\epsilon}; \quad \frac{\nu^4}{p^2 + i\epsilon};$$

$$\frac{\chi^3/M}{\not{k} - M + i\epsilon}, \quad \frac{\nu^4}{(k-p)^2 + i\epsilon}, \quad \frac{\chi^3/M}{\not{p} - M + i\epsilon}; \quad \frac{\nu^4}{p_i^2 + i\epsilon}, \quad \frac{\nu^4 p_i p_j}{(p_i^2 + i\epsilon)(p_j^2 + i\epsilon)}. \quad (1)$$

Quark and gluon condensates are responsible for confinement of quarks and gluons. By confinement we mean²: i) the absence of any asymptotic states for either quarks or gluons, ii) the absence of any continuum spectrum for partons, iii) the absence of any colour-full parton bound systems, and iv) the presence of hadrons which are colour-less bound states of quarks (antiquarks) and gluons.

Phenomenologically, the Local Parton-Hadron Duality of Dokshitzer and Troyan¹ supports the presence of physical singularities in three-point vertex functions (quark-gluon-quark, and triple-gluon). By Slavnov-Taylor identities the same physical singularities must appear² both in the inverse of quark propagator, and in the inverse of transverse gluon propagator. Just algebraically, these singularities in inverses of propagators lead to the absence² of quark and gluon asymptotic states. These physical singularities are also supported by the existence of

self-consistent solutions of Dyson-Schwinger equations³ which are exact solutions on the one-loop level, at these singularities.

Both quark and gluon propagators are of the Wheeler type², i.e. they are real propagators, equal to the average of retarded and advanced propagators. Nonzero values of quark and gluon condensates¹ insure that quark and gluon propagators correspond to short living quark and gluon states³. Time scales of these states are inversely proportional to inverses of mass scales of condensates, and are equal to fractions of fermi. The absence of any continuum partonic states is guaranteed² by: 1) the Wheeler character of the relative motion propagator, and 2) the vanishing of the scalar product of the Wightman-Garding relative momentum⁴ and the hadron momentum. In both of these properties a crucial role play nonzero quark and gluon condensates.

The nonzero values of quark and gluon condensates¹ are characterized by nonzero mass scales χ and ν connected with these condensates by following matrix elements of composite operators in the physical vacuum

$$-\chi^3 \equiv \langle \sqrt{\alpha} : \bar{\psi}\psi : \rangle, \quad \nu^4 \equiv \langle \frac{\alpha}{\pi} : G_{\mu\nu}^a G_a^{\mu\nu} : \rangle. \quad (2)$$

For u and d, s, c and b, and t quarks χ is equal, approximately, to the following fractions of GeV: 1/4, 1/5, 1/10, and 1/50, respectively. The gluon condensate mass scale ν is known¹ to be around 1/3 GeV.

The nonzero values of χ and ν allow for the existence of the following physical singularities² in inverses of quark and gluon propagators

$$S^{-1}(p) = \not{p} - m + i\epsilon + \frac{\chi^3/M}{\not{p} - M + i\epsilon}, \quad D_T^{-1}(p) = p^2 + i\epsilon + \frac{\nu^4}{p^2 + i\epsilon}, \quad (3)$$

where m is the current quark mass, and M is the mass of corresponding pseudoscalar mesons: π for u, d; K for s; D for c; and B for b.

It is trivial to invert algebraically inverses of propagators in Eq.(3), and the result for the gluon transverse propagator is particularly easy

$$D_T(p^2) = \frac{p^2}{p^4 + \nu^4} = \frac{1}{2} \left(\frac{1}{p^2 + i\nu^2} + \frac{1}{p^2 - i\nu^2} \right). \quad (4)$$

This equation demonstrates the absence of a real momentum pole for gluon, and therefore the absence of any asymptotic gluon state. In Eq.(4) there are two complex conjugate poles in the variable p^2 , showing the real character of the Wheeler gluon propagator. The presence of these poles is not in conflict either with causality, or with unitarity, or with analyticity².

The presence of physical poles in Eq.(3), and the numerical values of the condensate mass scales χ , and ν , can be verified² by reproducing these mass scales from vacuum-to-vacuum transitions obtained by closing up in the position space, quark, and gluon lines, respectively, represented by nonperturbative propagators. For numerical stability of such calculations anomalous

dimensions of propagators must be included. Then, the mass scales χ and ν are independent of a huge variation of an arbitrary mass scale μ , used as a normalization point in the renormalization group equation solutions. The normalization mass scale μ is varied² between 1 GeV and 100 000 GeV, and 10^{-6} stability is maintained.

Nonzero mass scales χ and ν are responsible for an exponential damping of the Fourier transform of propagators. For example, the time-dependence of the zero three-momentum gluon propagator is given² by the expression

$$\begin{aligned} \text{Fourier tr.} \left[\frac{-1}{p^2 + i\epsilon + \nu^4/(p^2 + i\epsilon)} \right]_{|\vec{p}=0} &= \frac{-1}{2\nu} e^{-\nu|\tau|/\sqrt{2}} \sin \left(\frac{\pi}{4} - \frac{\nu|\tau|}{\sqrt{2}} \right) \\ &= \frac{-1}{2\sqrt{2}\nu} e^{-\nu|\tau|/\sqrt{2}} \left[\cos \left(\frac{\nu\tau}{\sqrt{2}} \right) - \sin \left(\frac{\nu|\tau|}{\sqrt{2}} \right) \right]. \end{aligned} \quad (5)$$

This expression has the time scale of a shortly living gluon state of the value $\tau = \sqrt{2}/\nu$. This “life time” is between 0.6 and 0.8 fm, for ν between 1/3 and 1/2 GeV.

The quark propagator, calculated algebraically from Eq.(3), is²

$$S(\not{p}) \equiv \frac{Z(\not{p} + \rho)}{p^2 - \rho^2} + \frac{Z^*(\not{p} + \rho^*)}{p^2 - \rho^{*2}}, \quad (6)$$

and complex numbers Z and ρ are defined by the mass scales: m , M , and χ , as follows

$$\begin{aligned} \text{Re}Z &= \frac{1}{2}, & \text{Im}Z &= \frac{(M - m)/4}{\sqrt{\chi^3/M - \frac{1}{4}(M - m)^2}}, \\ \text{Re}\rho &= \frac{1}{2}(m + M), & \text{Im}\rho &= \sqrt{\chi^3/M - \frac{1}{4}(M - m)^2}. \end{aligned} \quad (7)$$

The Wheeler relative motion propagator in quark-antiquark system is²

$$\begin{aligned} &\frac{(\not{p}_1 + \rho)(\not{p}_2 + \rho)Z^2\sqrt{\rho^{*2} - q^2}(M^2 - \bar{P}^{*2})}{2|\rho^2 - q^2||M^2 - \bar{P}^2|^2} \delta \left(\frac{qP}{\sqrt{\bar{P}^2}} \right) \\ &+ \frac{(\not{p}_1 + \rho^*)(\not{p}_2 + \rho^*)Z^{*2}\sqrt{\rho^2 - q^2}(M^2 - \bar{P}^2)}{2|\rho^2 - q^2||M^2 - \bar{P}^2|^2} \delta \left(\frac{qP}{\sqrt{\bar{P}^{*2}}} \right). \end{aligned} \quad (8)$$

From this expression for the Wheeler relative-motion propagator we find immediately the definition of a momentum-dependent constituent quark mass $\mathcal{M}(q^2)$

$$\mathcal{M}(q^2) \equiv \sqrt{|\rho^2 - q^2| + q^2}$$

$$= \sqrt{\sqrt{(mM + \chi^3/M)^2 - q^2(m^2 + M^2 - 2\chi^3/M) + q^4} + q^2}. \quad (9)$$

For zero value of the Wightman-Garding⁴ relative momentum q in the quark-antiquark system the constituent quark mass takes the maximal value $\mathcal{M}(0)$, which is: 0.33 GeV for u and d quarks, 0.4 GeV for s quark, 1.7 GeV for c quark, and approximately 5 GeV for b quark. The explicit expression for the maximal value of the constituent quark mass is

$$\mathcal{M}(0) = (mM + \chi^3/M)^{1/2}. \quad (10)$$

For very large values of the relative momentum q the mass $\mathcal{M}(q^2)$ tends to zero, irrespective of flavour.

We note, that it is essential to demand the orthogonality of the space-like Wightman-Garding⁴ relative momentum q , to the time-like momentum $P = p_1 + p_2$ of the whole hadron. The condition $qP=0$ insures two basic properties of the relativistic, relative-motion⁴:

I. The space-like character of the relative momentum q , i.e. $q^2 < 0$, which is necessary for the proper, relativistic definition of angles⁴ between various relative momenta during the relative-motion with the cosine of these angles in the interval $[-1.0, 1.0]$, what is necessary for any sensible angular momentum and partial wave analysis⁴.

II. The cluster decomposition property⁴ in the sense of decoupling of the relative-motion dynamics described by three degrees of freedom of the constrained momentum⁴ q , $qP=0$, from the overall motion of the whole bound system with total four-momentum P being on the bound state mass-shell, $P^2 = M^2$.

It is often belived, and it is recently emphasized by Wilson et al.⁵, that confinement “requires potentials that diverge at long distances”. This is not true for non-Abelian gauge quantum field theory with mass gap. It is in conflict with cluster property, as shown by Strocchi⁶. Only for the zero mass gap an increasing potential is allowed, but in Wilson et al.⁵ there is a nonzero mass gap. Even, if the mass gap would be set equal to zero, then there would have to exist strong infrared singularity. For example, the p^{-4} singularity in the gluon propagator, but this in turn is inconsistent with Dyson-Schwinger equations, producing⁷ even more singular terms than p^{-4} in higher loops.

The constituent quark model in the Wilson et al.⁵ version, with the momentum independent constituent quark mass, is also in conflict with the decreasing d/u ratio in the deep inelastic scattering off nucleons⁸.

When dealing with hadrons in the light-front approach⁵ it is necessary to consider the relative motion of hadron constituents for the same x^+ component in the position space. This means, that solving the bound state problem in the momentum space one has to average out over the “-” component of the relative momentum. One does not take the zero value of the light-front relative energy, but instead the zero value of the relative “time”, i.e. the zero value of the relative x^- . This is one of the “no-go theorems” for doing hadrons with LFQCD⁵, since it is in conflict with the demand of obeying the Wightman-Garding orthogonality⁴ of the relative

momentum between the hadron constituents and the total hadron momentum. In the hadron rest frame, this orthogonality condition⁴ means that the time component of the relative momentum is equal to zero.

In the QED bound state computations⁹, in an equation below Eq.(2.2), in the first ref.⁹, and in Eq.'s (2.2a), (2.2d), and below, in the second ref.⁹, there is explicitly chosen the zero value of the relative energy as the appropriate way to calculate QED bound state.

There is one more “no-go theorem”, with the null-plane theory⁵. LFQCD is in conflict with Lorentz invariance¹⁰. Nakanishi and Yamawaki¹⁰ establish consistency in their “ ν -theory”, but get commutation relation in their Eq.(5.17) which does not vanish in space-like distances, i.e. it violates causality.

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