

# Topology and Confinement In Light-Front QCD

Stephen Pinsky  
Department of Physics, The Ohio State University  
Columbus OH 43210

## Abstract

In 1+1 dimensional compact QCD the zero modes of  $A^+$  give the theory a non-trivial topological structure. We examine the effects of these topological structures on the confining infrared structure of the theory. We show that the ground state wavefunction of the topological excitation smears the infrared behavior sufficiently to eliminate confinement for some matter currents. We review the work of Franke et. al. [1] which shows that the zero modes of  $A^+$  in  $QCD_{3+1}$  give rise to instantons. The relation of zero modes of  $A^+$ , instantons and confinement in  $QCD_{3+1}$  is discussed.

## 1. Introduction

It is almost universally accepted that Quantum Chromodynamics (QCD) is the correct theory of the strong interactions. There are numerous reasons for this wide-spread acceptance of QCD. In particular the comparison between perturbative QCD and experiments is generally very convincing. There are few if any tests however that probe the strong coupling regime of the theory. It is crucial that a successful non-perturbative solution expose the three important long range properties of QCD: confinement, spontaneous chiral symmetry breaking, and the topological structure. In the limit of a vanishing mass matrix, the chiral  $U(N_f) \otimes U(N_f)$  symmetry is explicit in the QCD Lagrangian. To avoid parity doublets in the spectrum, this chiral symmetry must be spontaneously broken to  $SU(N_f)$  leaving  $N_f^2 - 1$  Goldstone particles to be identified with the pions. Any successful calculation must produce this symmetry breaking. QCD appears to have an extra  $U_A(1)$  symmetry and if QCD actually had this additional symmetry it would lead to one more Goldstone particle the  $\eta$  and force  $m_\eta \leq \sqrt{3}m_\pi$  [2]. To avoid this  $U(1)_A$  problem the solution of QCD must have the third long range property, topological structure, that breaks the  $U_A(1)$  symmetry [3].

The calculation of the pseudo-scalar meson spectrum from first principles is a true test of QCD because it requires all three of these long range properties. The light-meson calculation should be considered the “hydrogen-atom calculation” for QCD. It is not sufficient to simply put the QCD action into a large computer and then look at the answer. We must have a calculation that provides some overall physical insight into the

way nonperturbative, nonlinear processes work in this theory [5]. It is the objective of the light-front (LF) QCD program to provide such a formalism. We believe that in LF formalism it is possible to isolate the degrees of freedom that are responsible for the long-range properties of QCD, and obtain a deeper understanding of QCD. All of the long range physics required by QCD have been discussed in great detail in this workshop. There is every reason to expect a very strong interplay between these long range properties. If topology strongly affects the mass of the  $\eta$  then it is logical that it plays a role in confinement as well. We will illustrate here how topology can play a role in confinement in LF QCD.

Perry and Wilson (to appear in these proceedings) argue that the confinement mechanism of  $QCD_{1+1}$  can be promoted to a three dimensional confinement mechanism through non-perturbative renormalization in a LF quantized formulation of QCD. Their point is that  $QCD_{3+1}$  already has a confining interaction term in the LF Hamiltonian, the instantaneous four Fermion interaction, which is the confining interaction in  $QCD_{1+1}$ . The issue, as they see it, is to show that the second order quark glue interaction does not cancel the instantaneous interaction as it does in perturbation theory. I will point out that the instantaneous interaction of the off diagonal currents is modified by the topological properties of the theory, and confinement is destroyed for these currents in  $QCD_{1+1}$  on a cylinder. In  $QCD_{1+1}$  on a cylinder the topological structure is carried by the zero mode (ZM) of  $A^+$  [4]. I argue that a similar effect is expected in  $QCD_{3+1}$  where the topology is just that associated with the instanton and is not an artifact of the space in which the theory is formulated.

## 2. Topology and Confinement in Two Dimensions

In a two dimensional  $SU(N)$  gauge field theory in flat space the gauge field provides a confining interaction for the matter degrees of freedom. This is evident from the form of the light-front Hamiltonian

$$P^- = -g^2 \int dx Tr(J^+ \frac{1}{(\partial_-)^2} J^+) \quad (1)$$

The operator  $1/(\partial_-)^2$  becomes  $1/(k^+)^2$  in momentum space and provides the linearly confining potential.

The theory develops a topological gluon degree of freedom when the system is put in a light-front spatial box with period boundary conditions. This degree of freedom is the ZM of  $A^+$  [4]. The existence of the topological degree of freedom hinges on the fact that the space is in compact  $x^-$ . When one attempts to completely fix the gauge one finds that the allowed gauge transformation must be periodic, up to an element of  $Z_N$ , to maintain the period boundary conditions on the fields. Thus the field can not be brought to  $A^+ = 0$ . The best that one can do is  $\partial_- A^+ = 0$ . An additional global gauge rotation

can be made on  $A^+$  to bring it to diagonal form. Thus for  $SU(2)$ , we have in the end a single color comonet as a degree of freedom

$$\frac{q(x^+)}{2L} \delta_{a,3} = \frac{1}{2L} \int_0^{2L} dx^- A_a^+(x^+, x^-). \quad (2)$$

This seemingly harmless rotation to a diagonal basis has significant consequences. With this gauge fixing only the off diagonal part of Gauss's Law can be solve strongly. The diagonal part must be imposed as a condition on the states. In  $QCD_{3+1}$  this is not always desirable. In  $QCD_{1+1}$ ,  $q(x^+)$  is the only gluon degree of freedom and the exponential of  $q$  is just the Wilson loop around the cylindrical space that the theory lives in. This degree of freedom enters the Hamiltonian in several places. There is a kinematic term  $\pi_q^2/2$  and the potential  $1/(k^+)^2$  between the off-diagonal matter currents becomes

$$\frac{1}{(k^+ \pm g(q/2L))^2}. \quad (3)$$

The interaction potential between the diagonal currents is unaffected. The states of the theory are the normal Fock states of the matter fields tensored with the states associated with the  $\pi$  and  $q$  operators. The states associated with  $\pi_q$  and  $q$  are described in the Schordinger picture by wavefunctions and energy levels (see Kalloniatis in these proceedings)

$$\phi_n = C_n \sin((n+1)gq) \quad E_n = ((n-1)^2 - 1)g^2\pi^2 2L/8. \quad (4)$$

Thus the complete states can be written

$$|n; N_i \rangle = C_n \sin((n+1)gq) |N_i \rangle \quad (5)$$

where  $N_i$  denotes the Fock space states of the matter field which are operated on by the matter currents. Very similar results are found inn equal time quantization[7]. Matrix elements are now of the form

$$\int_0^{2\pi/g} dq \langle n; N_i | \text{"operator"} | m; N_j \rangle \quad (6)$$

where the  $q$  integral runs over one fundamental modular domain [6].

The energy level splitting of the  $q$  states are of order  $2L$ , and in the large  $L$  limit we expect only the ground state to contribute. The ground state wavefunction is not trivial and the matter fields will be affected by it. Consider the matrix element of the off diagonal currents in the Hamiltonian between states with  $N_i$  and  $N_j$  matter excitations. In momentum space the matrix element takes the form;

$$\sum_n \langle N_i | J(k_n) J(-k_n) | N_j \rangle \left\{ \int_0^{2\pi} \frac{\sin(x)^2 dx}{(2\pi n + x)^2} + \int_0^{2\pi} \frac{\sin(x)^2 dx}{(2\pi n - x)^2} \right\} \quad (7)$$

where  $x = gq$  and  $k_n = 2\pi n/2L$ . Doing the integral we find for small momentum

$$\sum_n \frac{\langle N_i | J(k_n) J(-k_n) | N_j \rangle}{.67 + O(n^2)} \quad (8)$$

The potential now has a form similar to that of the exchange of a massive particle. The topological ground state has smeared out the infrared behavior and we do not have a confining interaction for the off diagonal currents. The diagonal currents are unaffected by the topological degree of freedom.

### The 3. $A^+$ ZM in 3+1 Dimensions

Many years ago by Franke, Novozhilov and Prokhvatilov [1] showed that there must be a ZM of  $A^+$  in LC  $QCD_{3+1}$  for the same reason that there must be a ZM of  $A^+$  in LC  $QCD_{1+1}$ . Again the ZM of  $A^+$  is a dynamical variable and again it carries the topological properties of the theory. They only imposed periodic boundary conditions in the  $x^-$  direction and their gauge condition was  $\partial_- A^+ = 0$ . Now the topological properties are those of the well known instanton.

The instanton winding number can be written in terms of the topological current  $K^\mu$  (which is a function of  $A^\mu$ ) as;

$$\nu = \frac{g^2}{8\pi^2} \int (K^+(T) - K^+(0)) dx^- d^2 x^\perp \quad (9)$$

where  $T$  is a large LC time. The winding number shows the relation between two pure gauge configurations separated by a large LC time  $T$ . These two pure gauge configuration must be related by a gauge transformation. Thus we are looking for transformations that take one from one pure ZM configuration to another pure ZM configuration. All such gauge transforms give an integer result for the winding number. Those transformations that give non-zero results are essentially the instantons of the theory. An example of such a gauge transformation is

$$G(x^+, x^-, x^\perp) = \text{Exp}\left(\frac{iN\pi}{L} \vec{\sigma} \cdot \vec{n} x^-\right) \quad (10)$$

where  $\vec{n}$  is a space dependent unit vector and

$$\vec{\sigma} \cdot \vec{n} = \frac{-2ax_2\sigma_1 + 2ax_1\sigma_2 + (a^2 - x_1^2 - x_2^2)}{(a^2 + x_1^2 + x_2^2)} \quad (11)$$

We see the usual instanton structure that mix color and space indices and "a" can be interpreted as the instanton size.

The ZM of  $A^+$  enters the Hamiltonian in  $QCD_{3+1}$  through structures similar to those we saw in  $QCD_{1+1}$ . Again the the ZM of  $A^+$  is related to the topology. In 3+1 the

topology is not related to the compact spatial manifold but is related to the fact that there are non-trivial gauge transforms that leave us in the gauge  $\partial_- A_a^+ = 0$ . These transformations do not preserve the diagonal gauge for  $A^+$ . Within a given winding number sector one can use the diagonal gauge but if one wants to consider processes that change winding number one must relax this condition.

We have seen that the two dimensional confinement mechanism seen in  $QCD_{1+1}$  can be profoundly changed by the topological structure of the theory. We saw that the topological structure of  $QCD_{1+1}$  on a cylinder smears the infrared confinement mechanism for some of the matter currents. We argued that if one attempts to promote this one dimensional confinement mechanism to a three dimensional confinement mechanism via the LF four Fermion instantaneous interaction, as Wilson and Perry have suggested, one must consider the topological effects. The result in two dimension was particular to the cylinder topology of the space, tempting one to argue that this effect is irrelevant, but in  $QCD_{3+1}$  the topology is unavoidable. Instantons exist and are known to be essential in the solution of the  $U(1)_A$  problem. Furthermore the instantons that appear in LF  $QCD_{3+1}$  produce structure similar to those that smeared out the confining instantaneous interaction in two dimensions.

## References

- [1] V.A. Franke, Yu.A. Novozhilov, and E.V. Prokhvatilov, *Lett.Math.Phys.* **5** (1981) 239; V.A. Franke, Y.V. Novozhilov, E.V. Prokhvatilov, *Lett. Math. Phys.* **5**, 437 (1981)
- [2] S. Weinberg *Phys. Rev. D* **11**, 3583 (1975).
- [3] G. 't Hooft *Phys. Rev. Lett.* **37**, 8 (1976); *Phys. Rev. D* **14**, 3432 (1976).
- [4] A. C. Kalloniatis, H. C. Pauli, and S. S. Pinsky, OHSTPY-HEP-TH-94-001 to appear in *Phys. Rev. D*; A. C. Kalloniatis, H. C. Pauli, *Z. Phys. C* **60**, 255 (1993); Heidelberg preprint MPIH-V2-94.
- [5] C. M. Bender, S. S. Pinsky, and B. van de Sande, *Phys. Rev. D* **48**, 816 (1993). S. S. Pinsky, and B. van de Sande, *Phys. Rev. D* **49**, 2001 (1994). J. Hiller, S. S. Pinsky, and B. van de Sande, OHSTPY-HEP-TH-94-014
- [6] P. van Baal, *Nucl. Phys. B* **369**, 259 (1992).
- [7] See, for example: N. Manton, *Ann. Phys. (NY)* **159**, 220 (1985); J.E. Hetrick, Y. Hosotani, *Phys. Lett.* **B30** (1989) 88; S.G. Rajeev, *Phys. Lett.* **B212** (1988) 203 and UR-1283 (1992); E. Langmann, G.W. Semenoff, *Phys. Lett.* **B303**, 303 (1993); J.E. Hetrick, *Nucl. Phys. B (Proc. Suppl.)* **30** 228 (1993).