Renormalization in an Invariant Light Front Picture

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Abstract

The light front Hamiltonian approach to QCD advocated by Wilson and collaborators shows great promise, but the feasibility of the scheme still needs to be demonstrated. An invariant light front formulation of the approach, which provides an excellent framework for dealing with interactions in the light-front angular momenta, may greatly facilitate the determination of counterterms corresponding to light-front divergences and will also provide a clear signal of the restoration in observables of rotational symmetry, which is manifestly broken by cutoffs.

1. Introduction

Lest a reminder be necessary, I was part of the group that produced the lengthy but concise tome Wilson, *et al.*[1] (hereafter, WWHZPG). So while I believe that in the mountains of QCD all trails of approach toward the summits of confinement and chiral symmetry breaking (χSB) should be kept under intensive use, my bias is that the approach of WWHZPG will be the quickest and easiest for calculating low energy QCD bound state properties; and what follows — although perhaps having applications to other light front (LF) approaches — is written specifically from this point of view.

The LF Hamiltonian approach of WWHZPG (see also the contributions of Wi, H, P, and G in this volume, as well as Refs. 2 and 3) differs from other approaches to the rough QCD terrain in that it does not start from the canonical valley of first principles and carefully move up into the mountains along a restricted path that maintains all symmetries, faithful that this will lead to the desired destination. Rather, WWHZPG inverts the metaphor, speaks of the valleys of confinement and χSB and the peak of a well-defined fundamental theory, and notes that a formulation of QCD on the light front with severe momentum space cutoffs provides clear mechanisms for confinement and χSB effects. The constituent quark model (CQM) that results from this truncation of full QCD should lie near the physical valleys of interest; what remains is to connect this simple picture to the full theory. This is a job for Wilson's Renormalization Group.

To summarize WWHZPG, then, (1) breaking symmetries allows a quick and easy CQM to appear; and (2) restoring symmetries connects this picture to QCD. Under (1), LF quantization allows the identification of all vacuum degrees of freedom as zero modes, and the use of severe cutoffs and the choice of nonzero q and g masses eliminates these zero modes. Thus the vacuum is trivial, and all effects associated with a nontrivial vacuum in equal time must now arise through effective interactions. In particular, choosing a momentum-dependent gluon mass $m_g^2 = \mu^2 + k^+\Delta$, with μ small, one finds a simple confinement mechanism[3] already at order g^2. Under (2), the similarity renormalization scheme[4], which eliminates explicit interactions between states well-separated in energy, is used to bring in order by order from the full theory corrections to this simple starting point. Such corrections include finite terms necessary to counter the elimination of zero modes. At each level of approximation, the low-energy bound states can be found nonperturbatively from familiar Hamiltonian methods, and the couplings in the various counterterms may then be determined phenomenologically. Well, at least in principle one can do this: in fact, it is not yet clear how these counterterms may best be handled.

2. Renormalization Counterterms on the Light Front

We have seen time and again at this workshop that light-front physics has a very different dependence upon the transverse and longitudinal coordinates. Here one finds that the structure of counterterms needed to eliminate LF divergences includes entire functions of momenta because k^+ and $k_\perp$ scale separately. Comparing the LF and equal time (ET) free particle energies,

$$k^- = \frac{k_\perp^2 + m^2}{k^+} \quad \text{and} \quad k^0 = \sqrt{\vec{k}^2 + m^2} = \sqrt{k_\perp^2 + k_z^2 + m^2},$$

one sees that as $k_\perp \to \infty$, for example, the k_z dependence in the ET free energy is negligible, whereas the diverging LF free energy is multiplied by a function of k^+. Thus the finite parts of counterterms which eliminate transverse divergences may include functions of longitudinal momenta, and the counterterms to longitudinal divergences and the removal of zero modes may include functions of transverse momenta. These functions should be completely determined by requiring that the correct physics comes out from the diagonalization of the Hamiltonian, but *a priori* they are unknown. Fitting unknown functions in a Hamiltonian to produce physical results which obey symmetries manifestly broken by the cutoffs seems quite tedious; one would like to at least have some systematic guide for this process, a guide that is not necessarily limited to perturbation theory. The basic question at hand is: given the different behavior with respect to the transverse and longitudinal directions, how can Lorentz covariance be maintained?

As first shown by Karmanov, one can in fact develop a light front formulation that is explicitly Lorentz invariant[5,6]. The idea is to quantize on an arbitrary light front $\omega \cdot x = 0$, where $\omega^2 = 0$. The operators and wave functions of the theory will then have an explicit dependence on ω. In fact, if we write $\omega = \omega^+(1, -\hat{\mathbf{n}})$, $\omega^+ > 0$, it turns out that operators and wave functions in this formulation depend only on the two angles defining $\hat{\mathbf{n}}$. Lorentz *covariance*, however, is only recovered if no physical quantities depend on $\hat{\mathbf{n}}$. This may be formulated as constraint equations to be satisfied by the wave functions, for by allowing them to depend on ω we are making our basis overcomplete. The constraint equations merely stipulate that a rotation of the vector ω — which is kinematical in appearance — has the same effect upon the wave functions as operating with the (dynamical) angular momenta $J_\perp$. The breaking of Lorenz covariance by our light-front cutoffs manifests itself in this formulation, then, as an explicit $\hat{\mathbf{n}}$ dependence in physical quantities. Finite parts of counterterms which eliminate divergent dependence on the cutoffs must then eliminate any $\hat{\mathbf{n}}$ dependence in physical quantities, which will signal the restoration of rotational invariance.

3. Invariant Light Front Picture

Let me make all this a little more explicit. Fuda[6] has shown that the invariant LF formulation of Karmanov may be obtained from a unitary transformation of the usual LF operators and states. These may thus be thought of as expressed in a new picture, which I shall call the Invariant Light Front Picture (ILFP)*. Thus if we write the non-interacting unitary Lorentz transform operator as $U_0(a)$ and the interacting unitary Lorentz transform operator as $U(a)$, where $x'_\mu = a_{\mu\nu}x^\nu$ is a Lorentz transform of coordinate systems, and define the transform to the standard LF as $(1, 0, 0, -1) = \tilde{a}(\omega) \cdot \omega$, then the unitary operator which transforms to the ILFP is

$$C(\omega) = U_0^{-1}(\tilde{a}(\omega))U(\tilde{a}(\omega)).$$

Defining the rotation R such that $\mathbf{R}(\hat{\mathbf{n}}) \cdot \hat{\mathbf{n}} = \hat{\mathbf{e}}_3$, one sees that $\tilde{a}(\omega) = K_L(\omega^+) \cdot R(\hat{\mathbf{n}})$, with K_L a horizontal boost which leaves the light front unchanged (that is, it does not contain interactions). Thus, in fact,

$$C(\omega) = U_0^{-1}(K_L \cdot R(\hat{\mathbf{n}}))U(K_L \cdot R(\hat{\mathbf{n}})) = U_0^{-1}(R(\hat{\mathbf{n}}))U(R(\hat{\mathbf{n}})) = C(\hat{\mathbf{n}}).$$

Now one may define the ILFP states and operators:

$$|\Psi(\omega)\rangle = C(\omega)|\Psi\rangle = |\Psi(\hat{\mathbf{n}})\rangle; \qquad \mathcal{O}(\omega) = C(\omega)\mathcal{O}C^{-1}(\omega) = \mathcal{O}(\hat{\mathbf{n}}).$$

This is analogous to the interaction picture, for example, $|\Psi_I\rangle = e^{-iH_0t}e^{iHt}|\Psi\rangle$. Under an arbitrary Lorentz transform $x' = a \cdot x$, then,

$$U_0(a)|\Psi(\omega)\rangle = |\Psi'(\omega')\rangle \text{ where } |\Psi'\rangle = U(a)|\Psi\rangle,$$

$$U_0(a)\mathcal{O}(\omega)U_0^{-1}(a) = \mathcal{O}'(\omega') \text{ where } \mathcal{O}' = U(a)\mathcal{O}U^{-1}(a).$$

*Fuda writes ξ instead of ω and calls this the ξ-Picture, but I prefer to use Karmanov's ω.

So ILFP states transform like free states under rotations, and likewise for operators. It is easy, therefore, to construct states that transform correctly under rotations. These are the eigenstates of the operator

$$\mathbf{S}(\hat{\mathbf{n}}) \equiv \mathbf{J}_0 + \mathbf{L}(\hat{\mathbf{n}}) \text{ with } \mathbf{L}(\hat{\mathbf{n}}) = -i\hat{\mathbf{n}} \times \frac{\partial}{\partial \hat{\mathbf{n}}}$$

The price to be paid for giving rotations a kinematical appearance is that we now have an overcomplete set of states. There are extra states present which are degenerate in energy and classified by their eigenvalues corresponding to the operator $[\hat{\mathbf{n}} \cdot \mathbf{S}(\hat{\mathbf{n}})]^2$. These extra states may be eliminated by the constraint equations:

$$\mathbf{J}(\hat{\mathbf{n}})|\Psi(\hat{\mathbf{n}})\rangle = \mathbf{S}(\hat{\mathbf{n}})|\Psi(\hat{\mathbf{n}})\rangle \text{ with } \mathbf{J}(\hat{\mathbf{n}}) = C(\omega)\mathbf{J}C^{-1}(\omega),$$

which stipulate that rotations of the LF caused by changing ω are equivalent to dynamical rotations of the states performed by the angular momentum operators.

The states $|\Psi(\hat{\mathbf{n}})\rangle$ may be expressed in terms of the free states $|N\rangle \equiv |p_1, ..., p_N\rangle$ as

$$|\Psi(\hat{\mathbf{n}})\rangle = \sum_N \left\{ \prod_{i=1}^{N} \int [dp_i] \right\} \psi(p_1, ..., p_N; \hat{\mathbf{n}})|N\rangle \text{ with } [dp_i] = \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2).$$

Now consider the hamiltonian $H = P^-$. In the ILFP, this becomes

$$P^\mu(\hat{\mathbf{n}}) = C(\hat{\mathbf{n}})P^\mu C^{-1}(\hat{\mathbf{n}}) = P_0^\mu + \omega^\mu U_0^{-1}(\omega) H_{\text{int}} U_0(\omega).$$

The matrix elements necessary for implementing the similarity renormalization scheme are then

$$\langle M|P^\mu_{\text{int}}(\hat{\mathbf{n}})|N\rangle = \omega^\mu \langle p_{1\omega}, ..., p_{M\omega}|H_{\text{int}}|p_{1\omega}, ..., p_{N\omega}\rangle,$$

where $p_{i\omega} = \tilde{a}(\omega)\cdot p_i = K_L(\omega^+)\cdot R(\hat{\mathbf{n}})\cdot p_i \equiv K_L(\omega^+)\cdot\tilde{p}_i$. So $\tilde{p}_i^+ = p_i^0 + \mathbf{p}_i\cdot\hat{\mathbf{n}}$, $\tilde{p}_i^0 = \sqrt{\mathbf{p}_i^2 + m_i^2}$, and $\tilde{p}_{\perp i} = \mathbf{p} - \hat{\mathbf{n}}(\mathbf{p}\cdot\hat{\mathbf{n}})$. We can thus express the above matrix elements in terms of the $(\tilde{p}_i^+, \tilde{p}_i^\perp)$ or the $\mathbf{p}_i$ and $\hat{\mathbf{n}}$. The latter are more convenient for exhibiting rotational invariance, since the matrix elements will be functions of $\mathbf{p}_i \cdot \mathbf{p}_j$ and $\mathbf{p}_i \cdot \hat{\mathbf{n}}$.

Now we are ready to discuss counterterms. We need cutoffs. Let K be some reference state $K^2 = M^2$. We may then introduce momentum cutoffs in H_{int} such that $(p_i \cdot K)/M < \Lambda$. In the rest frame of K, which is denoted by a subscript (that is, $K_K^\mu = M\delta^{\mu 0}$), this becomes

$$p_{iK}^0 = \sqrt{\mathbf{p}_{iK}^2 + m_i^2} < \Lambda \quad \text{or} \quad \tilde{p}_{iK}^+ + \frac{\tilde{p}_{iK\perp}^2 + m_i^2}{\tilde{p}_{iK}^+} < \Lambda,$$

depending on which set of variables we choose. For determining the structure of counterterms, which are added to the Hamiltonian to eliminate divergent dependence on Λ when the above matrix elements are summed over, if we use the variables $\mathbf{p}_{iK}$ and $\hat{\mathbf{n}}$, then the momentum dependence of the counterterms is entirely fixed by the form of the divergence and no unknown functions of momentum arise in the finite pieces.

Conclusion

I have barely had space here to set up the formalism for this invariant LF approach, so in way of conclusion let me just hint at some of the possible benefits of this formulation of LFQCD. Clearly, the main advantage is that the structure of counterterms will be more restricted from the outset. These counterterms will not have unknown functions of momenta but will include functions of $\hat{\mathbf{n}}$. These functions must be adjusted so that no observables depend on $\hat{\mathbf{n}}$, which signals the restoration of broken symmetries. Thus one might be able to develop a nonperturbative scheme where in first approximation one averages over or minimizes with respect to the angles $\hat{\mathbf{n}}$. Finally, note that the ILFP Hamiltonian should be independent of ω^+, which may provide a means of determining the counterterms necessary to restore longitudinal boost invariance lost by the removal of states with small k^+.

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