Hadronic Matrix Elements

D. Wyler

Institute for Theoretical Physics
Universität Zürich, 8057 Zürich, Switzerland

Abstract

I discuss briefly the importance of hadronic matrix elements and the existing methods to calculate them.

Since weak interactions are essentially a short distance effect (compared to the QCD scale of about $1 GeV$), their low energy manifestations can be described by matrix elements of an effective Hamiltonian of the form

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \sum_j C_j(\mu) \, O_j(\mu) \quad , \tag{1}$$

where the $O_j(\mu)$ are a set of quark operators, like $O_1 = (\bar{s}\gamma^\mu L b) \, (\bar{c}\gamma_\mu L c)$ and μ is a renormalization scale. The short distance coefficients $C_j(\mu)$ contain the information about the fundamental parameters (like CKM matrix angles, CP violating phases and couplings and masses of new particles) and account for the perturbative QCD corrections. They can be calculated reliably, at least if the physics of the heavy quarks c, b are considered (the major problem being the choice of the scale). On the other hand, the hadronic matrix elements

$$\mathcal{M}_{fi} = \langle f | \mathcal{H} | i \rangle \tag{2}$$

must be calculated to all orders in the strong interactions and to the desired order in the electromagnetic ones. This clearly requires non-perturbative techniques[1], since the matrix elements involve physical hadrons in the confinement regime of QCD. Only when we know these latter matrix elements, we can really relate the experimentally measured matrix elements of the effective Hamiltonian to fundamental parameters.

At present, there are several phenomenological methods to estimate the matrix elements; furthermore, lattice QCD[2] provides in principle a rigorous aproach for calculating them. These calculations are almost exclusively done in the so called quenched approximation and are limited to one- or two-particle matrix elements. It is rather difficult to asses totally the corresponding systematical errors, and the recent changes in some of the quantities, most notably f_B, the B-meson decay constant, indicate that there remains room for improvement.

Many interesting matrix elements contain an operator of a heavy quark c or b, but also those with an s quark are important for studies of the weak force and interactions beyond the standard model. Among the most burning ones are

one-particle matrix elements (decay constants)
$$\langle 0|J|M\rangle$$
two-particle matrix elements (form factors or mixings)
$$\langle m|O|M\rangle$$
three or more matrix elements (decay amplitudes)
$$\langle mM'|O|M\rangle.$$

In all cases, J or O denotes a bilinear or quadrilinear operator in the light and heavy quark fields, and $M, M', m,..$ are heavy and light mesons. In principle, also baryonic matrix elements are useful, but they are usually even more difficult to calculate.

The decay constants and form factors are needed to extract the CKM matrix elements from semileptonic decays of B-mesons while the mixings are essential for determining various new physicis parameters (including the mass of the top quark) and CP violating angles from meson mixing ($B^0 - \bar{B}^0$ mixing, etc). The decay amplitudes are used to obtain prediction for certain couplings (for instance the $BB^*\pi$ coupling), the rates of non-leptonic decays, their CP violating asymmetries and for complementary determinations of the fundamental parameters.

The phenomenological techniques include:

QCD sum rules[3]

Wave functions (including light cone input)[4]

$1/N_c$ approximation[5].

The sum rule approach is based on the duality between QCD (as a basic theory of quarks and gluons) and hadrons. It allows to relate matrix elements to correlation functions calculated in terms of various QCD parameters which can be fixed from a restricted number of experiments. Mainly two versions have been used for calculating the matrix elements of heavy quark operators; in the first, the vacuum correlation functions of certain currents are considered and mesons are generally taken into account by suitable interpolating currents. In the second variant, vacuum-meson correlators are investigated, where mesons are represented by light-cone wave functions[6]. These methods apply to different regimes and have been used to calculate decay constants, certain two-particle matrix elements and most recently also decay amplitudes. The two versions give rather different results as exemplified by two recent calculations of the decay $B \to K^*\gamma$[7] ; it appears that light-cone based methods are more suitable for heavy quarks. The uncertainties of both methods are determined mainly by the accuracy of the values of the condensates or the shape of the wave functions. The QCD sum rule method may shed some light on open questions, like the validity of factorization[8] which is often used to simplify calculations of decay amplitudes. Evidently, this question is out of reach for lattice calculations.

Wave function approaches are mostly used for two particle matrix elements, but can be applied also to decays. Light-cone techniques allow to calculate form factors in certain regions of the kinematical variables (where certain diagrams vanish). It is then a question of how to

continue to the desired (physical) regions and which truncations (in the spirit of Tamm-Dancoff) are reasonable. The existing work for B-meson decays indicate that a perturbative treatment, relying on asymptotic light-cone wave functions is insufficient.

The $1/N$ approach is based on the fact that QCD becomes more simple if the number of colours is sent to infinity. Much work has been done in the Kaon sector [5], where the approximate validity of chiral perturbation theory[9] yields important additional information. The $1/N$ approximation was also applied to B-Mesons[10]. Although chiral perturbation techniques can also be used[11], the situation is less clear than in Kaon physics.

A very promising approach to matrix elements of the b quark is the observation that several simplifications occur if the quark mass becomes very large[12]. This method is particularly useful for inclusive decay processes, but gives also important conditions for all other methods discussed above. For instance, the discrepancies in ref. 7 may be resolved within the heavy quark limit. Furthermore, as mentioned above, it can be combined with chiral perturbation theory.

Although I have stressed the heavy quark operator matrix elements, there are of course various other important situations where a good matrix element is highly desirable. For instance, the (electric) form factor of the pion at high energies has received much attention[13], but also various "long distance effects" in Kaon- or D-Meson physics are of interest. For instance, the mixing between the neutral Kaons receives contributions from two local weak vertices; in D-meson decays, where purely short distance effects are often small due to unfavorable CKM matrix elements, non-local contributions are dominant. A good example is the decay $D \to \rho\gamma$.

As is obvious from this rudimentary discussion, there exist various technologies which have been applied to several problems. All of them contain certain assumptions; moreover, often simplifications (like factorization) are often made in order to arrive at a result. Since the assumptions inherent to the different pictures are likely to be different, it may be possible to combine the methods in order to understand their respective systematic uncertainties better and to establish a fruitful connection between them.

References

The following list is very incomplete. I have tried to list recent overviews and a few exemplary papers which illustrate the relevant issues.

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