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1 (A)

$|e_1\rangle, |e_2\rangle$ - baza ortonormalna dwuwymiarowej przes. wek.

$|e_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |e_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\hat{\sigma}_y$ - operator wzdłuż osi y

$\hat{\sigma}_y |e_1\rangle = -i|e_2\rangle = \begin{bmatrix} 0 \\ -i \end{bmatrix}$ $\qquad \hat{\sigma}_y$

$\hat{\sigma}_y |e_2\rangle = i|e_1\rangle = \begin{bmatrix} i \\ 0 \end{bmatrix}$

- macierz operatora

$$\begin{bmatrix} {}_y\sigma_{11} & {}_y\sigma_{12} \\ {}_y\sigma_{21} & {}_y\sigma_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -i \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix} \Rightarrow \begin{matrix} {}_y\sigma_{11} = 0 \\ {}_y\sigma_{21} = -i \end{matrix}$$

$$\begin{bmatrix} {}_y\sigma_{11} & {}_y\sigma_{12} \\ {}_y\sigma_{21} & {}_y\sigma_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} i \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} {}_y\sigma_{12} = i \\ {}_y\sigma_{22} = 0 \end{matrix}$$

zatem

$$\hat{\sigma}_y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$ 1/1

- czy macierz jest hermitowska? 1/1

$$\Big(\underbrace{\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}}_{\hat{\sigma}_y}{}^T\Big)^* = \left(\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\right)^* = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = \hat{\sigma}_y$$

więc macierz jest hermitowska

- wartości i wektory własne

$$\det(\hat{\sigma}_y - \lambda \mathbb{1}) = \det\left(\begin{bmatrix} -\lambda & -i \\ i & -\lambda \end{bmatrix}\right) = \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$$

$$(\lambda + 1)(\lambda + 1) = 0$$

1/1

$$\text{zatem } \underbrace{\lambda_1 = 1 \qquad \lambda_2 = -1}_{\text{wartości własne}}$$

Dla $\lambda_1 = 1$ $|v_1\rangle = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

$$\left(\hat{\sigma}_y - \lambda_1 \mathbb{1}\right)\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & i \\ -i & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x_1 + iy_1 = 0 \\ -ix_1 - y_1 = 0 \end{cases} \Rightarrow x_1 = iy_1$$

$$|v_1\rangle = \begin{bmatrix} iy_1 \\ y_1 \end{bmatrix}$$

Dla $\lambda_2 = -1$ $|v_2\rangle = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$$\left(\hat{\sigma}_y - \lambda_2 \mathbb{1}\right)\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x_2 + iy_2 = 0 \\ -ix_2 + y_2 = 0 \end{cases} \Rightarrow y_2 = ix_2$$

$$|v_2\rangle = \begin{bmatrix} x_2 \\ ix_2 \end{bmatrix}$$ 1/1

- unormowane wektory własne

$$\sqrt{\langle v_1 | v_1\rangle} = 1$$

$$\langle v_1 | v_1\rangle = \begin{bmatrix} -iy_1 & y_1 \end{bmatrix}\begin{bmatrix} iy_1 \\ y_1 \end{bmatrix} = y_1^2 + y_1^2 = 2y_1^2$$

$$\sqrt{2y_1^2} = 1 \Rightarrow y_1 = \frac{1}{\sqrt{2}} \Rightarrow |v_1\rangle = \begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

1/1

$$\sqrt{\langle v_2 | v_2\rangle} = 1$$

$$\langle v_2 | v_2\rangle = \begin{bmatrix} x_2 & -ix_2 \end{bmatrix}\begin{bmatrix} x_2 \\ ix_2 \end{bmatrix} = x_2^2 + x_2^2 = 2x_2^2$$

$$\sqrt{2x_2^2} = 1 \Rightarrow x_2 = \frac{1}{\sqrt{2}} \Rightarrow |v_2\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

- czy wektory są ortogonalne?

$$\langle v_1 | v_2\rangle = 0$$

$$\langle v_1 | v_2\rangle = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} = \frac{-i}{2} + \frac{i}{2} = 0$$ 1/1

$$\langle v_2 | v_1\rangle = 0$$

$$\langle v_2 | v_1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{i}{2} - \frac{i}{2} = 0$$

Zatem wektory są ortogonalne

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Zad 1 (B)

$\hat{\sigma}_z|e_1\rangle = |e_1\rangle$ $\quad |e_1\rangle |e_2\rangle$ - baza

$\hat{\sigma}_z|e_2\rangle = -|e_2\rangle$ $\quad |e_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |e_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

• macierz operatora

$$\begin{bmatrix} {}_z\sigma_{11} & {}_z\sigma_{12} \\ {}_z\sigma_{21} & {}_z\sigma_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow {}_{(z)}\sigma_{11} = 1 \quad {}_{(z)}\sigma_{21} = 0$$

$$\begin{bmatrix} {}_{(z)}\sigma_{11} & {}_{(z)}\sigma_{12} \\ {}_{(z)}\sigma_{21} & {}_{(z)}\sigma_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \Rightarrow {}_{(z)}\sigma_{12} = 0 \quad {}_{(z)}\sigma_{22} = -1$$

$$\hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$ 1/1

• czy macierz jest hermitowska

$$\Bigg(\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\hat{\sigma}_z}^{T}\Bigg)^{*} = \left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right)^{*} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$ 1/1

Tak, macierz jest hermitowska.

• wartości i wektory własne 1/1

$$\det\begin{pmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{pmatrix} = -(1-\lambda)(1+\lambda) = 0 \Rightarrow \lambda_1 = 1 \quad \lambda_2 = -1$$

Dla $\lambda_1 = 1$ $\quad |v_1\rangle = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ 1/1

$$\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow y_1 = 0 \quad x_1 \text{ - dowolne} \quad |v_1\rangle = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$$

Dla $\lambda_2 = -1$ $\quad |v_2\rangle = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_2 = 0 \quad y_2 \text{ - dowolne} \quad |v_2\rangle = \begin{bmatrix} 0 \\ y_2 \end{bmatrix}$$

· unormowane wektory własne

$\sqrt{\langle v_1|v_1\rangle} = 1$

$\langle v_1|v_1\rangle = x_1^2 \Rightarrow x_1 = 1 \qquad |v_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\sqrt{\langle v_2|v_2\rangle} = 1$

$\langle v_2|v_2\rangle = y_2^2 \Rightarrow y_2 = 1 \qquad |v_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

wektory bazy!

1/1

· czy wektory są ortogonalne?

1/1

$\langle v_1|v_2\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \langle v_2|v_1\rangle = 0$ wek. są ortogonalne

(c) komutator

$[\hat{\sigma}_y, \hat{\sigma}_z] = \hat{\sigma}_y \hat{\sigma}_z - \hat{\sigma}_z \hat{\sigma}_y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} =$

$= \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} - \begin{bmatrix} 0 & i \\ +i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2i \\ -2i & 0 \end{bmatrix}$ 1/1

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2 z 3

(4/4)

$$V_\infty(x) = \begin{cases} \infty & x < -a \\ 0 & -a \le x \le a \\ \infty & a < x \end{cases}$$

$$\psi(x, t=0) \equiv \psi(x) = \frac{1}{\sqrt{3a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{3a/2}} \cos\left(\frac{\pi x}{2a}\right)$$

$$\psi(x,t) = \hat{U}(t, t=0) \cdot \psi(x, t=0)$$ – dla st. własnych

$$\hat{U}(t, t=0) = e^{-iEt/\hbar}$$

$\hat{H}$ hamiltonian niezależny od czasu

równanie stacjonarne:

$$\hat{H}_n |\varphi_n\rangle = E_n |\varphi_n\rangle$$

$$|\psi(\varphi)\rangle = \sum c_n |\varphi_n\rangle$$

Szukam rozkładu $|\psi\rangle$ w $|\varphi_n\rangle$ (1/1)

ogólnie

$$|\psi(x,0)\rangle = c_1 |\varphi_1(x,0)\rangle + c_2 |\varphi_2(x,0)\rangle$$

$$\varphi_{2k}(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{2k\pi x}{2a}\right) = \left\{ \begin{matrix} 2k = n \\ \text{parzysta} \\ n = 2 \end{matrix} \right\} = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right)$$

$$\varphi_{2k+1}(x) = \frac{1}{\sqrt{a}} \cos\left(\frac{(2k+1)\pi x}{2a}\right) = \left\{ \begin{matrix} 2k+1 = n \\ \text{nieparzysta} \\ n = 1 \end{matrix} \right\} = \frac{1}{\sqrt{a}} \cos\left(\frac{\pi x}{2a}\right)$$

$$\cancel{|\psi(x,0)\rangle = \frac{1}{\sqrt{3}} |\varphi_1(x)\rangle + \frac{1}{\sqrt{\frac{3}{2}}} |\varphi_2(x)\rangle}$$

$$|\psi(x,0)\rangle = \frac{1}{\sqrt{3}} |\varphi_2(x)\rangle + \frac{1}{\sqrt{\frac{3}{2}}} |\varphi_1(x)\rangle$$ (1/1) !

EXTRA +0.5 !

ROZKŁAD ZNALEZIONY

wiem, że dla stanów własnych

$$|\varphi_1(x,t)\rangle = e^{-iE_1 t/\hbar} \cdot |\varphi_1(x)\rangle$$ (1/1)

$$|\varphi_2(x,t)\rangle = e^{-iE_2 t/\hbar} \cdot |\varphi_2(x)\rangle$$

(0.5/1)

WŁAŚCIWY WZÓR NIE PRZEDSTAWIONY DO KOŃCA