Zadanie 1. 15|15 Anna Socha

zadanie 1.

(A) $|l,m\rangle$ $\hat{L}^2$ oraz $\hat{L}_z$ $l=1$ $|1,1\rangle$ $|1,0\rangle$ $|1,-1\rangle$ 1

4|4

$l=1$ $\underline{m=1 \vee m=0 \vee m=-1}$

$L^2|l,m\rangle = \hbar^2 \cdot l(l+1)|l,m\rangle$ $L_z|l,m\rangle = m\hbar|l,m\rangle$

$L^2|1,1\rangle = \hbar^2 \cdot 1(2)|1,1\rangle$ $L_z|1,1\rangle = \hbar|1,1\rangle$ 1

$L^2|1,1\rangle = 2\hbar^2|1,1\rangle$ $L_z|1,0\rangle = 0 \cdot \hbar |1,0\rangle$

$L_z|1,-1\rangle = -\hbar|1,-1\rangle$

$L^2|1,0\rangle = 2\hbar^2|1,0\rangle$

$L^2|1,-1\rangle = 2\hbar^2|1,-1\rangle$

$\langle l',m'|\hat{L}_z|l,m\rangle$ 1

$\langle 1,1|\hat{L}_z|1,1\rangle = \hbar$ $\langle 1,0|\hat{L}_z|1,1\rangle = 0$ $\langle 1,-1|\hat{L}_z|1,1\rangle = 0$

$\langle 1,1|\hat{L}_z|1,0\rangle = 0$ $\langle 1,0|\hat{L}_z|1,0\rangle = 0$ $\langle 1,-1|\hat{L}_z|1,0\rangle = 0$

$\langle 1,1|\hat{L}_z|1,-1\rangle = 0$ $\langle 1,0|\hat{L}_z|1,-1\rangle = 0$ $\langle 1,-1|\hat{L}_z|1,-1\rangle = -\hbar$

$$\begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix} = \underline{\hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$$ 1

(B) B=8|8

$\hat{L}_+|l,m\rangle = \hbar \cdot \sqrt{l(l+1)-m(m+1)}\,|l,m+1\rangle$

$\hat{L}_-|l,m\rangle = \hbar \cdot \sqrt{l(l+1)-m(m-1)}\,|l,m-1\rangle$

$L_+|1,1\rangle = \hbar \cdot \sqrt{1\cdot 2 - 1\cdot 2}\,|1,2\rangle = 0$

$$\boxed{\begin{aligned} L_+|1,0\rangle &= \hbar \cdot \sqrt{1\cdot 2 - 0}\,|1,1\rangle = \sqrt{2}\hbar|1,1\rangle \\ L_+|1,-1\rangle &= \hbar \cdot \sqrt{1\cdot 2 + 1(-0)}\,|1,0\rangle = \sqrt{2}\hbar|1,0\rangle \\ L_-|1,1\rangle &= \hbar \cdot \sqrt{1\cdot 2 - 1(\cdot 0)}\,|1,0\rangle = \sqrt{2}\hbar|1,0\rangle \\ L_-|1,0\rangle &= \hbar \cdot \sqrt{1\cdot 2 - 0}\,|1,-1\rangle = \sqrt{2}\hbar|1,-1\rangle \end{aligned}}$$

$L_-|1,-1\rangle = \hbar \cdot \sqrt{1\cdot 2 + 1(-2)}\,|1,-2\rangle = \underline{\underline{0}}$

$\hat{L}_+ = \hat{L}_x + i\hat{L}_y$ $\hat{L}_+ - \hat{L}_- = 2i\hat{L}_y$

$L_- = L_x - iL_y$ $\hat{L}_y = \frac{1}{2i}(\hat{L}_+ - \hat{L}_-)$

$2\hat{L}_x = \hat{L}_+ + \hat{L}_-$

$\hat{L}_x = \frac{1}{2}(\hat{L}_+ + \hat{L}_-)$

$\hat{L}_x|l,m\rangle = \left(\frac{1}{2}(\hat{L}_+ + L_-)\right)|l,m\rangle = \frac{1}{2}L_+|l,m\rangle + \frac{1}{2}L_-|l,m\rangle$

$\hat{L}_y|l,m\rangle = \frac{1}{2i}\left(\hat{L}_+ - \hat{L}_-\right)|l,m\rangle = \frac{1}{2i}\hat{L}_+|l,m\rangle - \frac{1}{2i}\hat{L}_-|l,m\rangle$

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zadanie 1

$\hat{L}_x |1,1\rangle = \frac{1}{2} L_+ |1,1\rangle + \frac{1}{2} L_- |1,1\rangle = 0 + \frac{\sqrt{2}}{2}\hbar |1,0\rangle$

$\hat{L}_x |1,0\rangle = \frac{1}{2} L_+ |1,0\rangle + \frac{1}{2} L_- |1,0\rangle = \frac{\sqrt{2}}{2}\hbar |1,1\rangle + \frac{\sqrt{2}}{2}\hbar |1,-1\rangle$

$\hat{L}_x |1,-1\rangle = \frac{1}{2} L_+ |1,-1\rangle + \frac{1}{2} L_- |1,-1\rangle = \frac{\sqrt{2}}{2}\hbar |1,0\rangle + 0$

$\hat{L}_y |1,1\rangle = \frac{1}{2i} L_+ |1,1\rangle + \frac{1}{2i} L_- |1,1\rangle = 0 - \frac{1}{2i}\sqrt{2}\hbar |1,0\rangle$

$\hat{L}_y |1,0\rangle = \frac{1}{2i} L_+ |1,0\rangle - \frac{1}{2i} L_- |1,0\rangle = \frac{\sqrt{2}}{2i}\hbar |1,1\rangle - \frac{\sqrt{2}}{2i}\hbar |1,-1\rangle$

$\hat{L}_y |1-1\rangle = \frac{1}{2i} L_+ |1,-1\rangle - \frac{1}{2i} L_- |1,-1\rangle = \frac{\sqrt{2}}{2i}\hbar |1,0\rangle + 0$

$\langle 1,1|\hat{L}_x|1,1\rangle = \frac{\sqrt{2}}{2}\hbar \langle 1,1|1,0\rangle = 0$ $\qquad \langle 1,1|1,1\rangle = 1$

$\langle 1,1|1,-1\rangle = 0$

$\langle 1,1|1,0\rangle = 0$

$\langle 1,1|\hat{L}_x|1,0\rangle = \frac{\sqrt{2}}{2}\hbar$

$\langle 1,1|\hat{L}_x|1-1\rangle = 0$

$\langle 1,0|\hat{L}_x|1,1\rangle = \frac{\sqrt{2}}{2}\hbar$

$\langle 1,0|\hat{L}_x|1,0\rangle = 0$

$\langle 1,0|\hat{L}_x|1-1\rangle = \frac{\sqrt{2}}{2}\hbar$

$\langle 1,-1|\hat{L}_x|1,1\rangle = 0$

$\langle 1,-1|\hat{L}_x|1,0\rangle = \frac{\sqrt{2}}{2}\hbar$

$\langle 1,-1|\hat{L}_x|1,-1\rangle = 0$

$$\hat{L}_x = \begin{pmatrix} 0 & \frac{\sqrt{2}}{2}\hbar & 0 \\ \frac{\sqrt{2}}{2}\hbar & 0 & \frac{\sqrt{2}}{2}\hbar \\ 0 & \frac{\sqrt{2}}{2}\hbar & 0 \end{pmatrix} = \frac{\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$-\frac{1}{i}^2$

$\langle 1,1|\hat{L}_y|1,1\rangle = 0$

$\langle 1,1|\hat{L}_y|1,0\rangle = \frac{\sqrt{2}}{2i}\hbar$

$\langle 1,1|\hat{L}_y|1-1\rangle = 0$

$\langle 1,0|\hat{L}_y|1,1\rangle = -\frac{\sqrt{2}}{2i}\hbar$

$\langle 1,0|\hat{L}_y|1,0\rangle = 0$

$\langle 1,0|\hat{L}_y|1,-1\rangle = \frac{\sqrt{2}}{2i}\hbar$

$\langle 1,-1|\hat{L}_y|1,1\rangle = 0$

$\langle 1,-1|\hat{L}_y|1,0\rangle = -\frac{\sqrt{2}}{2i}\hbar$

$\langle 1,-1|\hat{L}_y|1-1\rangle = 0$

$$\hat{L}_y = \begin{pmatrix} 0 & -\frac{\sqrt{2}}{2i}\hbar & 0 \\ \frac{\sqrt{2}}{2i}\hbar & 0 & -\frac{\sqrt{2}}{2i}\hbar \\ 0 & \frac{\sqrt{2}}{2i}\hbar & 0 \end{pmatrix} = \frac{\sqrt{2}}{2i}\hbar \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$[\hat{L}_x, L_y] = \hat{L}_x\hat{L}_y - \hat{L}_y\hat{L}_x =$

$\frac{\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \frac{\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} -$

$\frac{\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \frac{\sqrt{2}}{2i}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} =$

$\frac{1}{2}\hbar^2 \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix} - \frac{1}{2}\hbar^2 \begin{pmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} =$

$\frac{1}{2}\hbar^2 \begin{pmatrix} 2 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \hbar^2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

			0	-1	0
			1	0	-1
			0	1	0
0	1	0	1	0	-1
1	0	1	0	0	0
0	1	0	1	0	-1

			0	1	0
			1	0	1
			0	1	0
0	-1	0	-1	0	-1
1	0	-1	0	0	0
0	1	0	1	0	1

3.

$$\hat{L}_x = \frac{\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{L}_y = \frac{-i^2\sqrt{2}}{2i}\hbar \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{-i\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$[\hat{L}_y, \hat{L}_x] = \hat{L}_x\hat{L}_y - \hat{L}_y L_x = \frac{\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \left(-\frac{i\sqrt{2}}{2}\hbar\right) \cdot \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$+ \frac{i\sqrt{2}}{2}\hbar \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \frac{\sqrt{2}}{2}\hbar = -\frac{i\hbar^2}{2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

$$+ \frac{i\hbar^2}{2} \begin{pmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \frac{i\hbar^2}{2} \begin{pmatrix} -2 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$$

$$-i\hbar^2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -i\hbar \hat{L}_z$$

$$\begin{array}{c|c} & \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{matrix} \\ \hline \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & \begin{matrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{matrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \\ \hline \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{matrix} & \begin{matrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{matrix} \end{array}$$

3/3 = C

Komutator jest proporcjonalny do macierzy reprezentującej operator $\hat{L}_z$, stałą proporcjonalności jest $\underline{-i\hbar}$

Jeśli byłby spełniony ten związek to:

$$[\hat{L}_i, \hat{L}_j] = i\hbar \varepsilon_{ijk} \hat{L}_k =$$

$$[\hat{L}_y, \hat{L}_x] = i\hbar \varepsilon_{213} \hat{L}_3 = -i\hbar \hat{L}_z$$

jest to zgodne z tym, co otrzymałam powyżej

~~$[\hat{L}_x, \hat{L}_y] = i\hbar \varepsilon_{123} \hat{L}_3 = i\hbar \hat{L}_z \neq -i\hbar \hat{L}_z$~~

~~więc nie jest spełniony ten związek~~

~~(coś mi nie pasuje, ale~~

zadanie 2

Anna Socha

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$$|\psi\rangle = \frac{1}{\sqrt{3}}|\phi_0\rangle + \sqrt{\frac{2}{3}}|\phi_1\rangle$$

$$\mathcal{E}_0 = \frac{\hbar\omega}{2} \qquad \mathcal{E}_1 = \frac{3\hbar\omega}{2}$$

Prawdopodobieństwo:

A) $$|\langle\phi_0|\psi\rangle|^2 = \left|\frac{1}{\sqrt{3}}\langle\phi_0|\phi_0\rangle + \sqrt{\frac{2}{3}}\langle\phi_0|\phi_1\rangle\right|^2 = \frac{1}{3}$$

2/2

Prawdopodobieństwo, że cząstka kd w stanie $|\psi\rangle$ jest w stanie podst $= \frac{1}{3}$

b) $\langle\psi|\hat{H}|\psi\rangle$

Hamiltonian oscylatora harmonicznego dany jest wzorem

$$\hat{H} = \frac{\hat{p}^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}kx^2 =$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2$$

$$\langle\psi|\hat{H}|\psi\rangle = \langle\psi| -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2|\psi\rangle$$

Równanie Schrödingera

$$H|\psi\rangle = \mathcal{E}|\psi\rangle$$

$\mathcal{E}$ - wartość własna tj. energia w określonym stanie

$$H|\phi_0\rangle = \mathcal{E}_0 \cdot |\phi_0\rangle \qquad H|\phi_0\rangle = \frac{\hbar\omega}{2}|\phi_0\rangle$$

$$H|\phi_1\rangle = \mathcal{E}_1 \cdot |\phi_1\rangle \qquad H|\phi_1\rangle = \frac{3\hbar\omega}{2}|\phi_1\rangle$$

$$\langle\psi|\hat{H}|\psi\rangle = \langle\psi|\left(\frac{1}{\sqrt{3}}H|\phi_0\rangle + \sqrt{\frac{2}{3}}H|\phi_1\rangle\right) =$$

$$\langle\psi|\frac{1}{\sqrt{3}}H|\phi_0\rangle + \sqrt{\frac{2}{3}}\langle\psi|H|\phi_1\rangle =$$

2/2

$$\frac{1}{\sqrt{3}}\langle\psi|\frac{\hbar\omega}{2}|\phi_0\rangle + \sqrt{\frac{2}{3}}\langle\psi|\frac{3\hbar\omega}{2}|\phi_1\rangle = \frac{\hbar\omega}{2\sqrt{3}}\left(\frac{1}{\sqrt{3}}\langle\phi_0|\phi_0\rangle + \right.$$

$$\left.\sqrt{\frac{2}{3}}\langle\phi_1|\phi_0\rangle\right) + \sqrt{\frac{2}{3}}\frac{3\hbar\omega}{2}\left(\frac{1}{\sqrt{3}}\langle\phi_0|\phi_1\rangle + \sqrt{\frac{2}{3}}\langle\phi_1|\phi_1\rangle\right) = \frac{\hbar\omega}{6} + \hbar\omega = \boxed{\frac{7}{6}\hbar\omega}$$

1

c) $|\psi\rangle$ — średnie położenie

$$\langle x'|\hat{x}|x\rangle = x\,\delta(x-x')$$

$$|\phi_0\rangle = \sqrt{\frac{1}{2^0\cdot 0!}}\left(\frac{m\omega}{\hbar\pi}\right)^{\frac14} e^{-\frac{m\omega x^2}{2\hbar}} H_0\left(\sqrt{\frac{m\omega}{\hbar}}x\right) = \underline{\left(\frac{m\omega}{\hbar\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\cdot 1}$$

$$|\phi_1\rangle = \sqrt{\frac{1}{2\cdot 1!}}\left(\frac{m\omega}{\hbar\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}} H_1\left(\sqrt{\frac{m\omega}{\hbar}}\cdot x\right) = \frac{1}{\sqrt2}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}$$

$$2\cdot\sqrt{\frac{m\omega}{\hbar}}\cdot x = \frac{2}{\sqrt2}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\cdot\sqrt{\frac{m\omega}{\hbar}}\cdot x$$

$$\langle\psi|\hat{x}|\psi\rangle = \int_{-\infty}^{+\infty}\psi^*\cdot x\cdot\psi\,dx = \int_{-\infty}^{+\infty}\Bigg(\frac{1}{\sqrt3}\cdot\left(\frac{m\omega}{\hbar\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}$$

$$+\sqrt{\frac23}\cdot\frac{2}{\sqrt2}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\cdot\sqrt{\frac{m\omega}{\hbar}}\cdot x\Bigg)\cdot x\cdot\Bigg(\frac{1}{\sqrt3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}$$

$$+x\cdot\sqrt{\frac{m\omega}{\hbar}}\sqrt{\frac23}\cdot\frac{2}{\sqrt2}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\Bigg)dx =$$

$$\int_{-\infty}^{+\infty}\left(\frac{1}{\sqrt3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\left(e^{-\frac{m\omega x^2}{2\hbar}} + 2\sqrt{\frac{m\omega}{\hbar}}\cdot x\cdot e^{-\frac{m\omega x^2}{2\hbar}}\right)\right)\cdot x\left(\frac{1}{\sqrt3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac14}\left(e^{-\frac{m\omega x^2}{2\hbar}}\right.\right. +$$

$$\left.\left. e^{-\frac{m\omega x^2}{2\hbar}}\cdot 2x\cdot\sqrt{\frac{m\omega}{\hbar}}\right)\right) = \frac13\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac12}\int_{-\infty}^{+\infty}\left(e^{-\frac{m\omega x^2}{2\hbar}} + 2\sqrt{\frac{m\omega}{\hbar}}\,x\cdot e^{-\frac{m\omega x^2}{2\hbar}}\right)\cdot$$

$$\left(x\cdot e^{-\frac{m\omega x^2}{2\hbar}} + 2x^2\sqrt{\frac{m\omega}{\hbar}}e^{-\frac{m\omega x^2}{2\hbar}}\right) = \frac13\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac12}\left(\int_{-\infty}^{+\infty} e^{-\frac{m\omega x^2}{\hbar}}\cdot x\right. +$$

$$\left.\int_{-\infty}^{+\infty}\sqrt{\frac{m\omega}{\hbar}}\,2x^2\cdot e^{-\frac{m\omega x^2}{\hbar}}dx + \int_{-\infty}^{+\infty}\sqrt{\frac{m\omega}{\hbar}}\,2x^2 e^{-\frac{m\omega x^2}{\hbar}}dx + 4\int_{-\infty}^{+\infty}\left(\frac{m\omega}{\hbar}\right)\cdot x^3\cdot e^{-\frac{m\omega x^2}{\hbar}}dx\right) =$$

$$\left\{\begin{array}{l}\frac{m\omega}{\hbar}x = u\\ dx = \frac{\hbar}{m\omega}du\end{array}\right\}$$

dla wielomianu Hermite'a $H_1(x) = 2x$

Poprzeglem [illegible]

Ełżbieta Łoche
zadanie 2

$$\frac{1}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\cdot\Big[\int_{-\infty}^{+\infty}\frac{\hbar^2}{m^2\omega^2}\cdot u\cdot e^{-u^2}du + 4\int_{-\infty}^{+\infty}\sqrt{\frac{m\omega}{\hbar}}\cdot\frac{\hbar^3}{m^3\omega^3}u^2\cdot e^{-u^2}du$$

$$+4\int_{-\infty}^{+\infty}\frac{m\omega}{\hbar}\cdot\frac{\hbar^4}{m^4\omega^4}u^3\cdot e^{-u^2}du\Big] = \frac{1}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\cdot\Big(\frac{\hbar^2}{m^2\omega^2}\underbrace{\int_{-\infty}^{+\infty}u\cdot e^{-u^2}du}_{=0} + \left(\frac{\hbar}{m\omega}\right)^{\frac{5}{2}}\underbrace{\int_{-\infty}^{+\infty}u^2\cdot e^{-u^2}du}_{=\frac{\sqrt{\pi}}{2}}$$

$$+\frac{4\hbar^3}{m^3\omega^3}\underbrace{\int_{-\infty}^{+\infty}u^3\cdot e^{-u^2}du}_{=0}\Big) = \frac{1}{3}\cdot\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\cdot\left(\frac{\hbar}{m\omega}\right)^{\frac{5}{2}}\cdot 2\frac{\sqrt{\pi}}{2}\cdot 2$$

$$= \frac{1}{3}\cdot\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}\cdot\left(\frac{\hbar}{m\omega}\right)^{\frac{5}{2}} = \frac{2\hbar^2}{3m^2\omega^2}$$

1/2

ten wzór może nie być dobry, ale na pewno wystąpuje w nim zależność od t i E w danej chwili

$$|\psi(t)\rangle = |\psi\rangle\cdot e^{-\frac{iEt}{\hbar\omega}}$$

~~(ten wzór nie j~~

(D) $t=0 \to |\psi\rangle$ ewolucja czasowa $|\psi(t)\rangle$

$$|\psi(t)\rangle = \frac{1}{\sqrt{3}}|\phi_0(t)\rangle + \sqrt{\frac{2}{3}}|\phi_1(t)\rangle$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{3}}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{4}}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\cdot e^{-\frac{i\hbar\omega t}{2\hbar\omega}} + \frac{2}{\sqrt{2}}\sqrt{\frac{2}{3}}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{4}}e^{-\frac{m\omega x^2}{2\hbar}}e^{-\frac{3}{2}\frac{i\hbar\omega t}{\hbar\omega}}$$

$$= \frac{1}{\sqrt{3}}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{4}}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\left(e^{-\frac{it}{2}} + 2e^{-\frac{3it}{2}}\right)$$

$$\cancel{= \frac{1}{\sqrt{3}}\left(\frac{m\omega}{\hbar\cdot\pi}\right) e^{\frac{m\omega x^2}{2\hbar}}}$$

$$\langle\psi(t)|x|\psi(t)\rangle = \int_{-\infty}^{+\infty}\frac{1}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}e^{-\frac{m\omega x^2}{\hbar}}\left(e^{\frac{it}{2}} + 2e^{\frac{3it}{2}}\right)\left(xe^{-\frac{it}{2}} + 2xe^{-\frac{3it}{2}}\right)dx$$

$$\cancel{= \frac{1}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\int_{-\infty}^{+\infty}e^{-\frac{m\omega x^2}{\hbar}}}$$

$$= \frac{1}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\int_{-\infty}^{+\infty}e^{-\frac{m\omega x^2}{\hbar}}\left(x + 2x\cdot e^{-\frac{it}{2}} + 2xe^{\frac{it}{2}} + 2x\right)dx =$$

$$\cancel{\frac{1}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\int_{-\infty}^{+\infty}e^{-\frac{m\omega x^2}{\hbar}}\cdot 5x\,dx = \frac{5}{3}\left(\frac{m\omega}{\hbar\cdot\pi}\right)^{\frac{1}{2}}\int_{-\infty}^{+\infty}x\cdot e}$$

Anna Socha zadanie 3

6/6 +E

(3.) $\psi(r,\theta,\phi) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \cdot e^{-r/a_0}$ a_0 - promień Bohra

(A) $r = a_0$

Dany jest stan ~~$\theta(a_0,\theta,\phi)$ tak~~ $\theta(r,\theta,\phi)$ taki, że $r = a_0$

$$\theta(a_0,\theta,\phi) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \cdot e^{-1} = \frac{1}{e\sqrt{\pi}\, a_0^{3/2}}$$

Prawdopodobieństwo, że elektron znajdzie się w stanie θ wynosi

$$|\langle\theta|\psi\rangle|^2 = \frac{16}{a_0^2 \cdot e^4}$$

1/1

$$\langle\theta|\psi\rangle = \int d^3r\ \theta^*(r,\theta,\varphi)\cdot\psi(r,\theta,\varphi) =$$ $\boxed{R^2 = a_0^2}$

$$\int_0^{2\pi} d\varphi \cdot \int_0^{\pi} d\theta \cdot \sin\theta \cdot \frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_0}\right)^{3/2}\cdot e^{-1} \cdot \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \cdot e^{-r/a_0} \cdot a_0^2 =$$

$$2\pi\cdot\frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 \cdot a_0^2\, \frac{1}{e}\int_0^{\pi} e^{-r/a_0} \cdot \sin\theta\, d\theta = \frac{2}{e\, a_0}\cdot e^{-r/a_0}\cdot(-\cos\theta)\Big|_0^{\pi} =$$

$$\frac{2}{e\cdot a_0\, e^{r/a_0}}\cdot 2 = \frac{4}{a_0\, e^{\frac{a_0+r}{a_0}}} = \frac{4}{a_0 e^2}$$

1/1

(B) Prawdopodobieństwo tego, że elektron znajduje się w odl. nie większej niż jeden promień Bohra

Zakładam, że chodzi o to, że elektron znajduje się w sferze $(0, a_0)$

$$\theta(r_0,\theta,\phi) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0} =$$ $0 < r \leqslant a_0$

1/1

$$|\langle\theta|\psi\rangle|^2 = \int d^3r\ \theta^*\ \psi = \int_0^{2\pi} d\varphi \cdot \int_0^{\pi} d\theta \cdot \sin\theta \cdot \int_0^{a_0} \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 r^2 \cdot e^{-r/a_0} \cdot e^{-r/a_0}\, dr$$

$$= \frac{2\pi}{\pi}\cdot\left(\frac{1}{a_0}\right)^3 \cdot (-\cos\theta)\Big|_0^{\pi} \int_0^{a_0} e^{-\frac{2r}{a_0}}\, r^2 \cdot dr = 4\left(\frac{1}{a_0}\right)^3 \cdot \int_0^{a_0} e^{-\frac{2r}{a_0}} \cdot r^2 dr$$

$$\left\{\begin{array}{l} \frac{2r}{a_0} = u \\ dr = \frac{a_0}{2} du \end{array}\right\} = 4\cdot\left(\frac{1}{a_0}\right)^3 \cdot \int_0^2 e^{-u}\, u^2\left(\frac{a_0}{2}\right)^2 \cdot \frac{a_0}{2}\, du = 4\cdot\frac{1}{a_0^3}\,\frac{a_0^3}{8}\int_0^2 e^{-u}\cdot u^2\, du$$

$$\frac{1}{2}\int_0^2 e^{-u} u^2\,du = \frac{1}{2}\cdot e^{-u}\left(\frac{x^2}{(-1)} - \frac{2x}{(-1)^2} + \frac{2}{(-1)^3}\right)\Big|_0^2 =$$

$$\frac{1}{2}\left[e^{-2}\left(-4-4-2\right) - \left(0-0-2\right)\right] =$$

1/1

$$\frac{1}{2}\left(e^{-2}\cdot(-10)+2\right) = \underline{\underline{1-5\cdot e^{-2}}}$$ | Prawdopodobieństwo $|\langle\theta|\psi\rangle|^2 = \underline{\underline{(1-5\cdot e^{-2})^2}}$

(c) $$\langle\psi|\hat{r}|\psi\rangle = \int d^3r\,\psi^*\,r\,\psi = \int_0^{2\pi} d\varphi \cdot \int_0^{\pi} d\theta \sin\theta \cdot \int_0^{+\infty} r^2 \cdot \left(\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_0}\right)^{3/2} \cdot e^{-r/a_0}\right.$$

$$\left.\cdot\, r \cdot \frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}\right) dr = 2\pi\cdot(-\cos\theta)\Big|_0^{\pi} \cdot \int_0^{+\infty} \frac{1}{\pi}\cdot\left(\frac{1}{a_0}\right)^3 \cdot r^3 \cdot e^{-\frac{2r}{a_0}}\,dr$$

$$= 4\left(\frac{1}{a_0}\right)^3 \cdot \int_0^{+\infty} r^3 \cdot e^{-\frac{2r}{a_0}}\,dr \quad \left\{\begin{array}{l}\frac{2}{a_0}r = u \\ dr = \frac{a_0}{2}du\end{array}\right\} =$$

2/2

$$4\left(\frac{1}{a_0}\right)^3 \cdot \int_0^{+\infty} \frac{a_0^4}{16}\cdot u^3 \cdot e^{-u}\,du = \frac{a_0}{4}\cdot\int_0^{+\infty} u^3\cdot e^{-u}\,du = \frac{6a_0}{4} = \frac{3}{2}a_0$$

(d) dla największej prawdopodobnej odległości max. wartość osiąga $|\langle\theta|\psi\rangle|^2$

$$\langle\theta|\psi\rangle = \int d^3r\,\theta^*\,\psi = \int_0^{2\pi} d\varphi \cdot \int_0^{\pi} \sin\theta\,d\theta \cdot r^2 \cdot \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 \cdot e^{-\frac{2r}{a_0}} =$$

$$4\cdot\left(\frac{1}{a_0}\right)^3 \cdot e^{-\frac{2r}{a_0}} \cdot r^2\,dr = \quad \left\{\begin{array}{l}\frac{2r}{a_0} = u \\ dr = \frac{a_0}{2}du\end{array}\right\} =$$

$$4\left(\frac{1}{a_0}\right)^3 \cdot \int_0^{+\infty} e^{-u}\,u^2\,\frac{a_0^3}{8}\,du = \frac{1}{2}\cdot\int_0^{+\infty} u^2\cdot e^{-u}\,du = 1$$

to jest prawdopodobieństwo, że ~~atom~~ elektron znajduje się gdzieś

zgodnie ze wzorem

$$\int x^2 e^{cx} = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right) = \frac{1}{e^x}\left(-x^2-2x+2\right)\Big|_0^{+\infty}$$

dominującym wkładem bd. $\frac{1}{e^x} \rightarrow$ ~~największe prawdopodobieństwo~~