

Cosmological Gravitational Particle Production: Generating Sterile Neutrinos Dark Matter

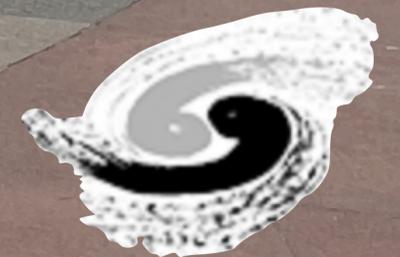
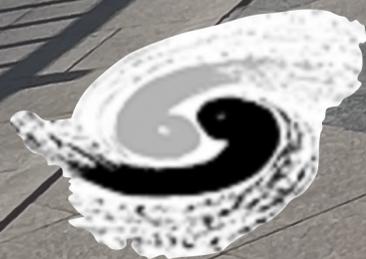
05/03/2026 @ Warsaw University

Fotis Koutroulis
Institute of High Energy Physics,
China Center of Advanced Science and Technology

*Based on **JHEP 04 (2024) 027** & **arXiv:2509.01673 (JCAP)***

*In collaboration with:
O. Lebedev, S. Pokorski*

*In collaboration with:
D. Da Silva, O. Lebedev, S. Pokorski*



CONTENTS

- **INTRO/MOTIVATION**
- **SN PRODUCTION DURING INFLATION**
- **SN PRODUCTION AFTER INFLATION**
- **CONCLUSIONS**

Intro

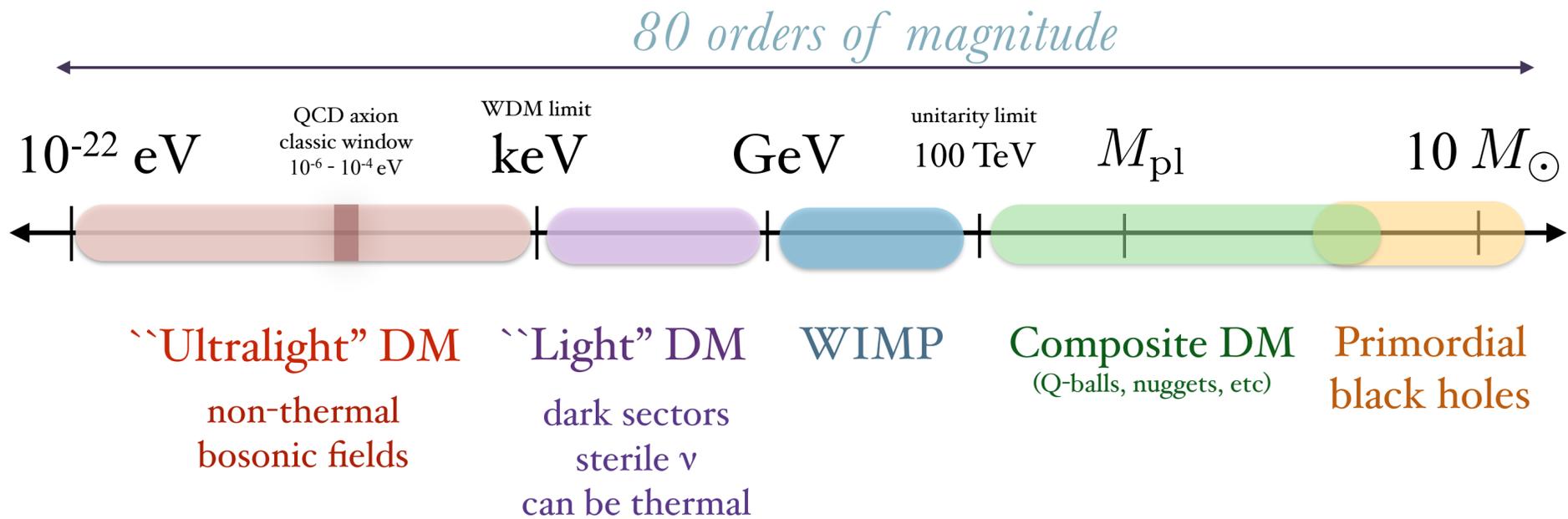
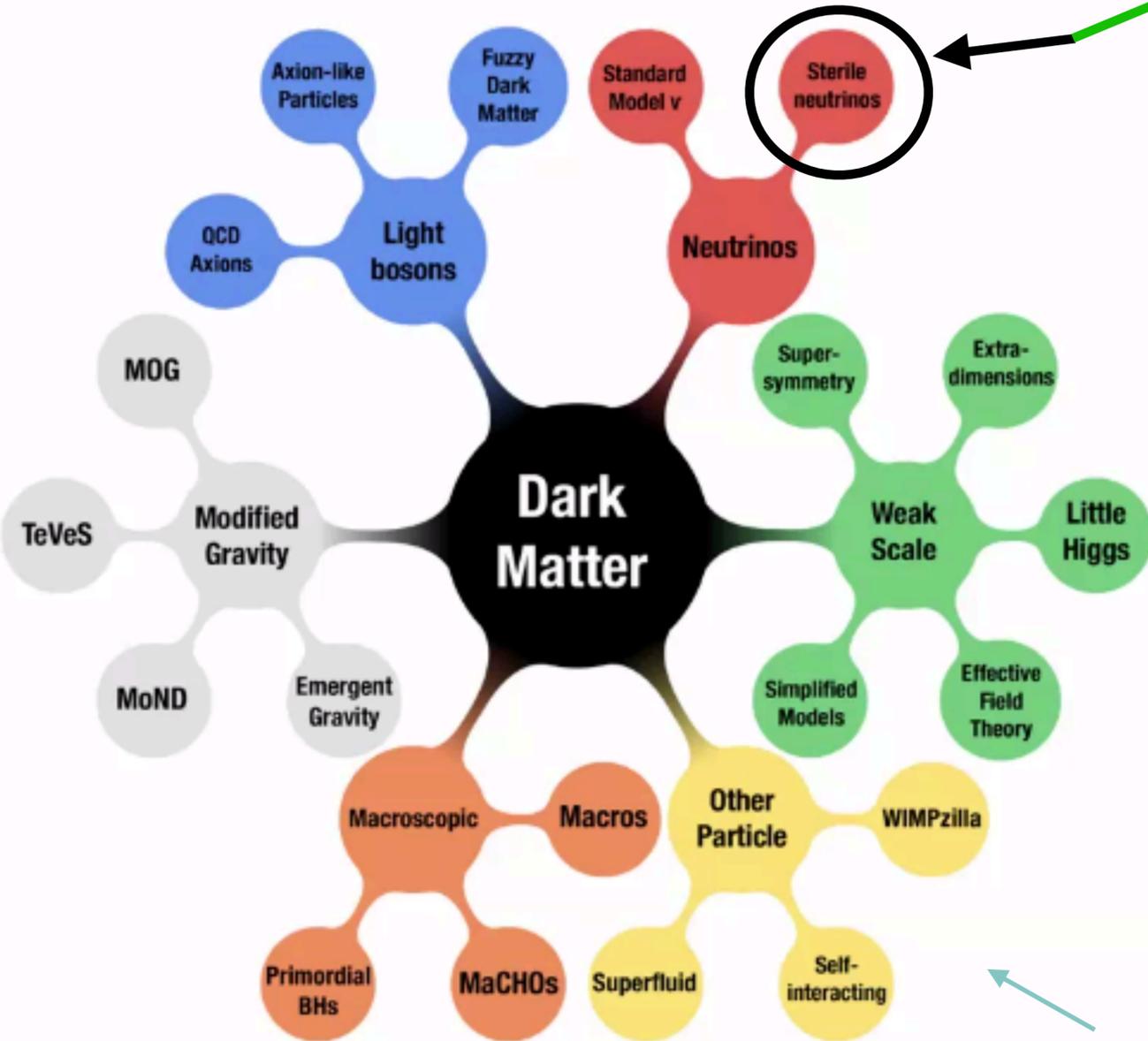
- Compelling evidence for the existence of Dark Matter (DM) on different astrophysical scales (galactic, clusters of galaxies, cosmological scale,...)
- $\sim 84\%$ of the matter in the Universe is DARK
- DM candidate: **stable** (lifetime must be long compared to cosmological timescales), (dominantly) **Non-relativistic, electrically neutral** and **colorless**. (Only?) **gravitational interactions**
- What is DM? How was produced?

Intro

Gravitationally produced Sterile Neutrinos as non-thermal Dark matter

*A. Boyarsky, O. Ruchayskiy and M. Shaposhnikov, '09
 A. Boyarsky, M. Drewes, T. Lasserre, S. Mertens and O. Ruchayskiy, '19*

DM LANDSCAPE



Bertone and Tait, Nature '18

*Tongyan Lin '19
 TASI lectures on DM*

**BASICS OF THE
MODEL
(INFLATIONARY PART)**

The basics of the model

- Starting point: A conformally invariant fermionic action of a field Ψ with mass M (breaking scale invariance) on a FRW conformally flat metric

$$ds^2 = a(x_0)^2 \eta_{\mu\nu} dx^\mu dx^\nu \quad S_\Psi = \int d^4x \sqrt{|g|} \bar{\Psi} (i\gamma^\alpha \nabla_\alpha - M) \Psi \quad \longrightarrow \quad (i\gamma^\alpha \nabla_\alpha - M) \Psi = 0$$

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}, \quad \Psi = \Omega^{-3/2} \tilde{\Psi}$$

$$e^\mu_\alpha = \Omega^{-1} \tilde{e}^\mu_\alpha \quad x_0 \equiv \eta$$

$$(i\gamma^\mu \partial_\mu - \underbrace{a(\eta)M}) \Psi = 0$$

- Flat space Dirac equation with a **time-dependent mass** causing **particle production**

V. Mukhanov, 2005

L. Parker '69, A. A. Grib and S. G. Mamaev '69

S. G. Mamaev, V. M. Mostepanenko and A. A. Starobinsky, '76

- Only thing left, however a non-trivial task, is to solve the e.o.m. revealing the particle production effect

The basics of the model

$$\Psi(x) = \sum_i \left(a_i U_i + b_i^\dagger V_i \right)$$

$$U_{\mathbf{k},s}(\eta, \mathbf{x}) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}} \begin{pmatrix} u_{A,k}(\eta) \\ s u_{B,k}(\eta) \end{pmatrix} \otimes h_s(\hat{\mathbf{k}})$$

$$k \equiv |\mathbf{k}|, \quad \hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$$

$$V_{\mathbf{k},s}(\eta, \mathbf{x}) = -\frac{e^{-i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}} \begin{pmatrix} -u_{B,k}^*(\eta) \\ s u_{A,k}^*(\eta) \end{pmatrix} \otimes h_s(-\hat{\mathbf{k}}) e^{i\phi}$$

$$(U_i, U_j) = (V_i, V_j) = \delta_{ij}, \quad (U_i, V_j) = 0$$

$$h_s^\dagger(\hat{\mathbf{k}}) h_r(\hat{\mathbf{k}}) = \delta_{rs}$$

$$i\partial_\eta \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} aM & k \\ k & -aM \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

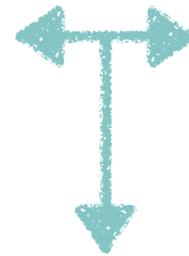
$$|u_A|^2 + |u_B|^2 = 1$$

The basics of the model

$$a_i|0\rangle = b_i|0\rangle = 0 \quad \textbf{BD-vacuum}$$

T. S. Bunch and P. C. W. Davies, '78

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\text{in}} \xrightarrow{\eta \rightarrow -\infty} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} e^{-ik\eta}$$



$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\text{out}} \xrightarrow{\eta \rightarrow \infty} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i \int \omega(\eta) d\eta}$$

$$\tilde{U}_{\mathbf{k},s} = \alpha_{\mathbf{k},s} U_{\mathbf{k},s} + \beta_{\mathbf{k},s} V_{-\mathbf{k},s}$$

**Bogolyubov
coefficient**

**Average number of
particles at the *IN-*
vacuum**

$$\beta_{\mathbf{k},s} = \text{phase} \times (u_{A,k} \tilde{u}_{B,k} - u_{B,k} \tilde{u}_{A,k})$$

$$\langle \tilde{N}_{\mathbf{k},s} \rangle \equiv \langle 0 | \tilde{a}_{\mathbf{k},s}^\dagger \tilde{a}_{\mathbf{k},s} | 0 \rangle = |\beta_{\mathbf{k},s}|^2$$

$$\beta'_{\mathbf{k},s} = 0 \quad \longrightarrow \quad \eta \sim \eta_e \equiv 0$$

$$n = \sum_s \int \frac{d^3 \mathbf{k}}{(2\pi)^3 a^3} |\beta_{\mathbf{k},s}|^2$$

Other scenarios

CGPP

N. Herring and D. Boyanovsky, *Phys. Rev. D* 101, no.12, 123522 (2020)

J. Klaric, A. Shkerin and G. Vacalis, *JCAP* 02, 034 (2023).

Edward W. Kolb et al., *Higgs Inflation: Particle Factory*, arXiv: 2510.24651 (2025)

Edward W. Kolb, Andrew J. Long et al., *Creation of spin-3/2 dark matter via cosmological gravitational particle production*, arXiv: 2512.16976 (2025)

Graviton Exchange Mechanism (GEM)

N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso and M. Pierre, *Spin-2 PortalDark Matter*, *Phys.Rev.D* 97 (2018)115020

Y. Ema, K. Nakayama and Y. Tang, *Production of Purely Gravitational Dark Matter*, *JHEP* 09 (2018)135

Y. Mambrini and K.A. Olive, *Gravitational Production of Dark Matter during Reheating*, *Phys.Rev.D* 103 (2021)115009

B. Barman, S. Clery, R. T. Co, Y. Mambrini and K. A. Olive, *JHEP* 12, 072 (2022)

INFLATIONARY PRODUCTION

The setup

- Focus on a **Radiation Dominated** universe after Inflation

$$a(\eta) = \left\{ \left(\frac{1}{a_e H_e} - \eta \right)^{-1} H_e^{-1} \text{ for } \eta \leq 0 \text{ , } a_e^2 H_e \left(\eta + \frac{1}{a_e H_e} \right) \text{ for } \eta > 0 \right\}$$

- Assume **3** right-handed **Dirac** neutrinos. **2** responsible for neutrino masses and the **lightest** is the DM. For a time-dependent fermion mass $M \equiv M(\eta)$

$$\begin{aligned} u_A'' + [i(Ma)' + a^2 M^2 + k^2] u_A &= 0, \\ u_B'' + [-i(Ma)' + a^2 M^2 + k^2] u_B &= 0 \end{aligned}$$

- The standard Higgs field h with quartic coupling λ_h **takes on a large value** in the Early Universe

$$\langle h^2 \rangle \sim 0.1 \frac{H^2}{\sqrt{\lambda_h}} \rightarrow \bar{h} = \langle h^2 \rangle \approx H_e \quad M(\eta) \sim \mathcal{Y}_R \langle h \rangle + m$$

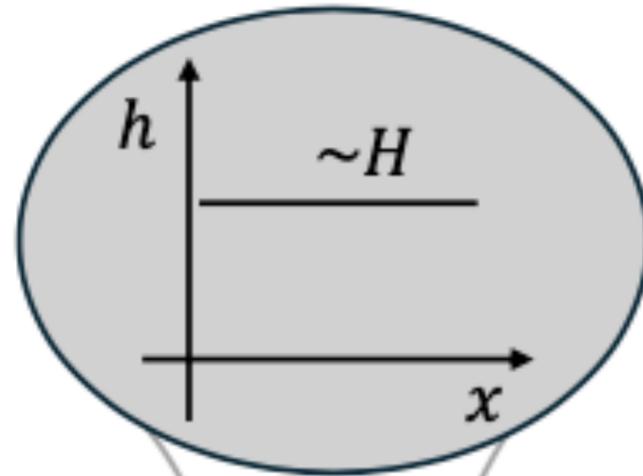
A. A. Starobinsky and J. Yokoyama,
Phys. Rev. D 50, 6357-6368 (1994)

The setup

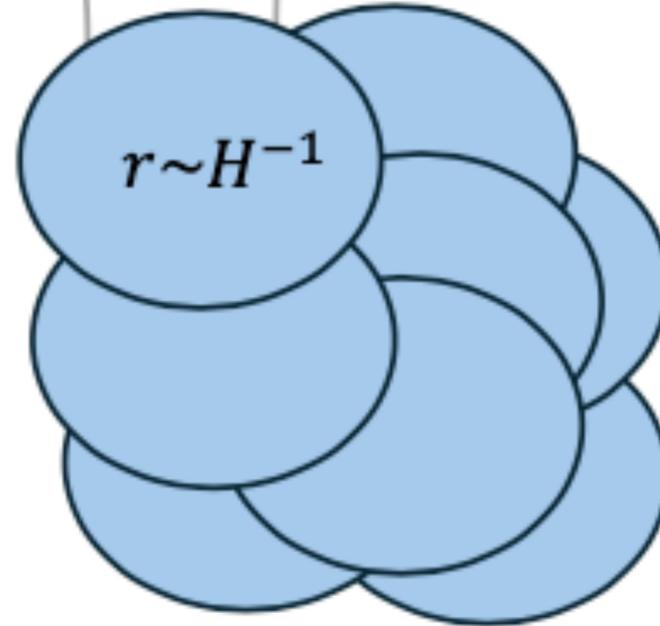
$$h(x) = \bar{h}(x) + \text{fluctuations}$$



$$\bar{h}(x) : k/a \ll H$$



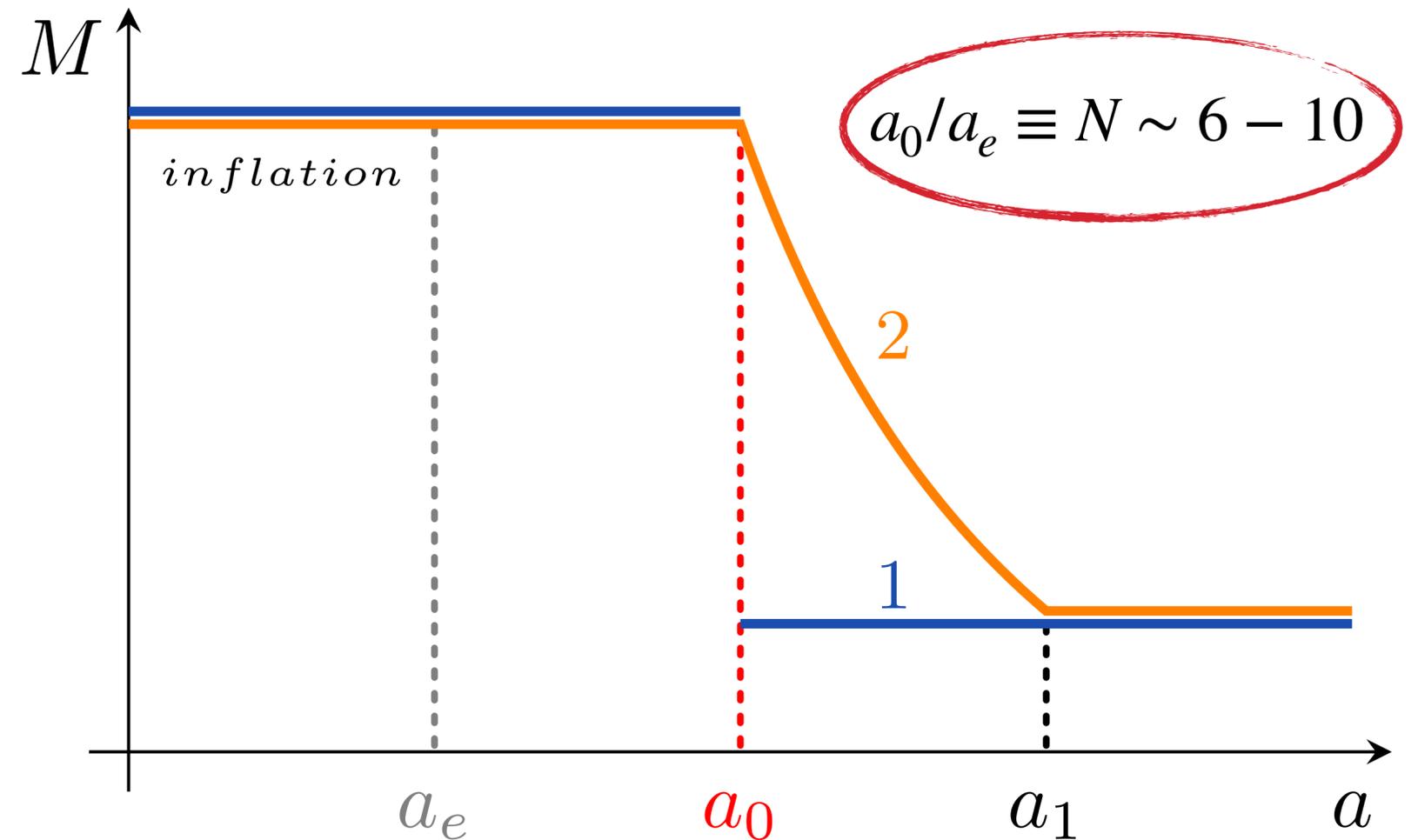
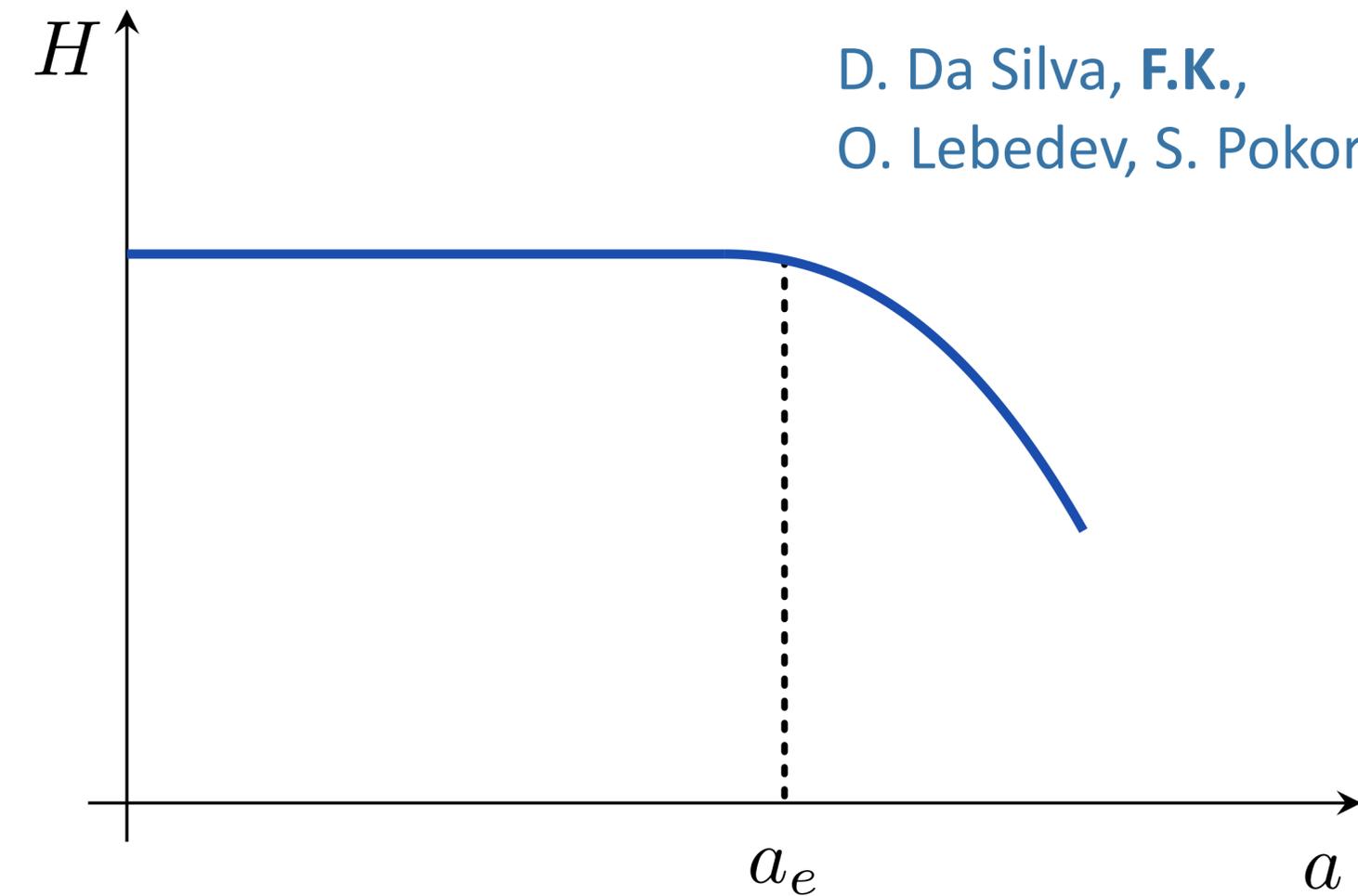
$\bar{h}(x) \simeq \text{const}$ in each Hubble patch



$$\dot{\bar{h}}(x) \simeq 0$$

The setup

D. Da Silva, F.K.,
O. Lebedev, S. Pokorski



- Two possibilities **1) Step-function mass** and **2) Smoothed from thermal effects $M \propto T \propto 1/a$**
- **3 mechanisms for SN production:** **i) Gravitational production**, ii) Higgs condensate decay, iii) Freeze-in

Step-function mass

$$M \theta(\eta_0 - \eta) + m \theta(\eta - \eta_0)$$

$$-\infty < \eta < \eta_0 \quad : \quad M(\eta) = M = \mathcal{Y}_R \langle h \rangle$$

$$\eta_0 < \eta < \infty \quad : \quad M(\eta) = m$$

In-wavefunction

$$\eta^2 u_A'' + \left(k^2 \eta^2 + \left[\frac{iM}{H_e} + \frac{M^2}{H_e^2} \right] \right) u_A = 0$$

*For u_B e.o.m.
just set
 $M \rightarrow -M$*

BC

$$\begin{pmatrix} u_A \\ u_B \end{pmatrix}^{\text{in}} \xrightarrow{\eta \rightarrow -\infty} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} e^{-ik\eta}$$

$$u_A^{\text{in}}(a) = \sqrt{\frac{\pi k}{4aH_e}} e^{i\frac{\pi}{2}(1-iM/H_e)} H_{1/2-iM/H_e}^{(1)} \left(\frac{k}{aH_e} \right)$$

$$u_B^{\text{in}}(a) = \sqrt{\frac{\pi k}{4aH_e}} e^{i\frac{\pi}{2}(1+iM/H_e)} H_{1/2+iM/H_e}^{(1)} \left(\frac{k}{aH_e} \right)$$

$$\frac{k}{a_e H_e} \ll 1$$

$$u_{A,k}^{\text{in}} \simeq \frac{1}{\sqrt{2}} \times e^{i\frac{M}{H_e} \ln \frac{k}{a_e H_e}},$$

$$u_{B,k}^{\text{in}} \simeq \frac{1}{\sqrt{2}} \times e^{-i\frac{M}{H_e} \ln \frac{k}{a_e H_e}}$$

Step-function mass

Out-wavefunction: $\eta > \eta_0$ $u_A'' + (k^2 + ima_e^2 H_e + \eta^2 m^2 a_e^4 H_e^2) u_A = 0$

BC $\begin{pmatrix} u_A \\ u_B \end{pmatrix} \xrightarrow{\eta \rightarrow \infty} \begin{pmatrix} 1 \\ \frac{k}{2am} \end{pmatrix} e^{-i \int \omega(\eta) d\eta}$

$$C = \frac{k^2}{2ma_e^2 H_e}$$

$$u_{A,k}^{out} = e^{-\frac{\pi}{4}C} D_{-iC} \left(e^{i\pi/4} \sqrt{\frac{2m}{H(\eta)}} \right) \times \text{phase}$$

$$u_{B,k}^{out} = \sqrt{C} e^{-\frac{\pi}{4}C + \frac{i\pi}{4}} D_{-1-iC} \left(e^{i\pi/4} \sqrt{\frac{2m}{H(\eta)}} \right) \times \text{phase}$$

Out-wavefunction: $\eta \leq \eta_0$ $m \rightarrow M$

$k^2 \ll Ma_e^2 H_e$

$$u_{A,k}^{out}(0) \simeq e^{-\pi C/4} 2^{-iC/2} \sqrt{\cosh \frac{\pi C}{2}} \times \left(1 + i \frac{N^2}{2} \frac{M}{H_e} + \mathcal{O} \left(\frac{k}{a_e H_e} \right) \right)$$

$$u_{B,k}^{out}(0) \simeq e^{-\pi C/4} 2^{-iC/2} \sqrt{\sinh \frac{\pi C}{2}} \times \left(1 - i \frac{N^2}{2} \frac{M}{H_e} + \mathcal{O} \left(\frac{k}{a_e H_e} \right) \right)$$

Step-function mass

$$|\beta_{\mathbf{k}}| = |u_{A,k}^{in} u_{B,k}^{out} - u_{B,k}^{in} u_{A,k}^{out}|$$



$$|\beta_k| \simeq \frac{1}{2} \frac{M}{H_e} \left| N^2 - 2 \ln \frac{k}{a_e H_e} \right|$$

Cut-offs

$$k_* = \sqrt{2ma_e^2 H_e}$$

$$\tilde{k}_* = \sqrt{2Ma_e^2 H_e}$$



At $H(a_M) = M \rightarrow \tilde{k}_ = a_M M$*

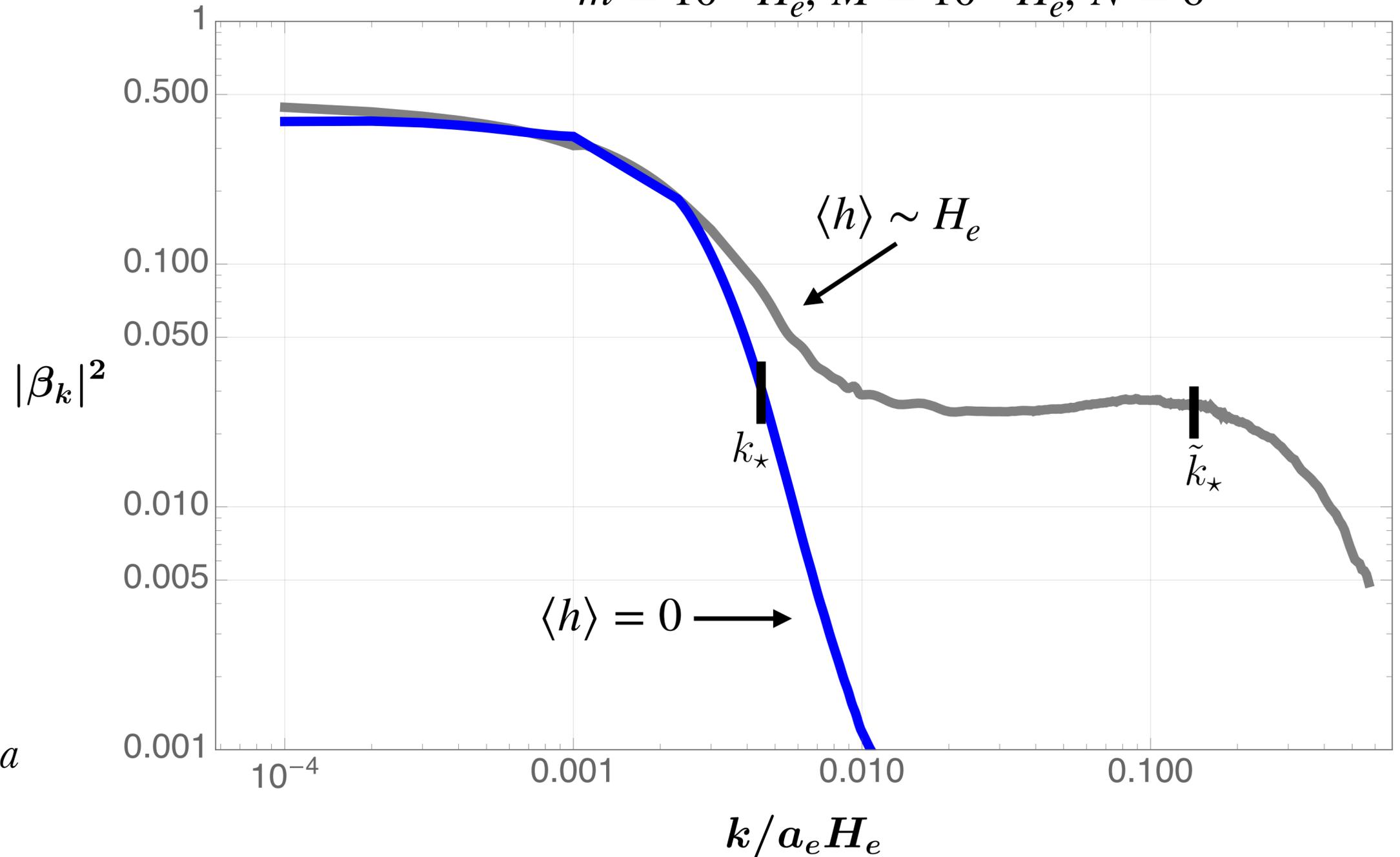
Only particles with physical 3-momenta

$\tilde{k}/a < M$ are created

Non-relativistic

Particle number

$$m = 10^{-5} H_e, M = 10^{-2} H_e, N = 6$$



Step-function mass

Particle number

$$n \sim 4 \times \int \frac{d^3 \mathbf{k}}{(2\pi)^3 a^3} \theta(\tilde{k}_* - k) |\beta_k|^2 \longrightarrow n \sim \frac{2}{3\pi^2} \tilde{k}_*^3 \frac{1}{a^3} |\beta_k|^2 = \frac{\sqrt{2}}{3\pi^2} N^4 \frac{M^{7/2}}{H_e^{1/2}} \frac{a_e^3}{a^3}$$

Particle abundance

$$Y = \frac{n}{s_{\text{SM}}}, \quad s_{\text{SM}} = \frac{2\pi^2 g_*}{45} T^3$$

$$Y_{\nu_R}^{\text{step}} = 10^{-3} \times N^4 \frac{M^{7/2}}{M_{\text{Pl}}^{3/2} H_e^2}$$

*DM observational
constraint*

$$Y \leq 4.4 \times 10^{-10} \frac{\text{GeV}}{M}$$

$$M \lesssim 6 \times 10^9 \text{ GeV} \rightarrow Y_R \lesssim 6 \times 10^{-4}$$

*Sterile Neutrino production during inflation is **insignificant** unless it is very heavy. Also $Y_{\nu_R}^{\text{step}} \ll Y_{\nu_R}^{\text{Higgs}} \ll Y_{\nu_R}^{\text{FI}}$*

Slow effective mass decrease

Particle number

$$n \sim \frac{1}{3\pi^2} N^3 M^3 \frac{a_e^3}{a^3}$$

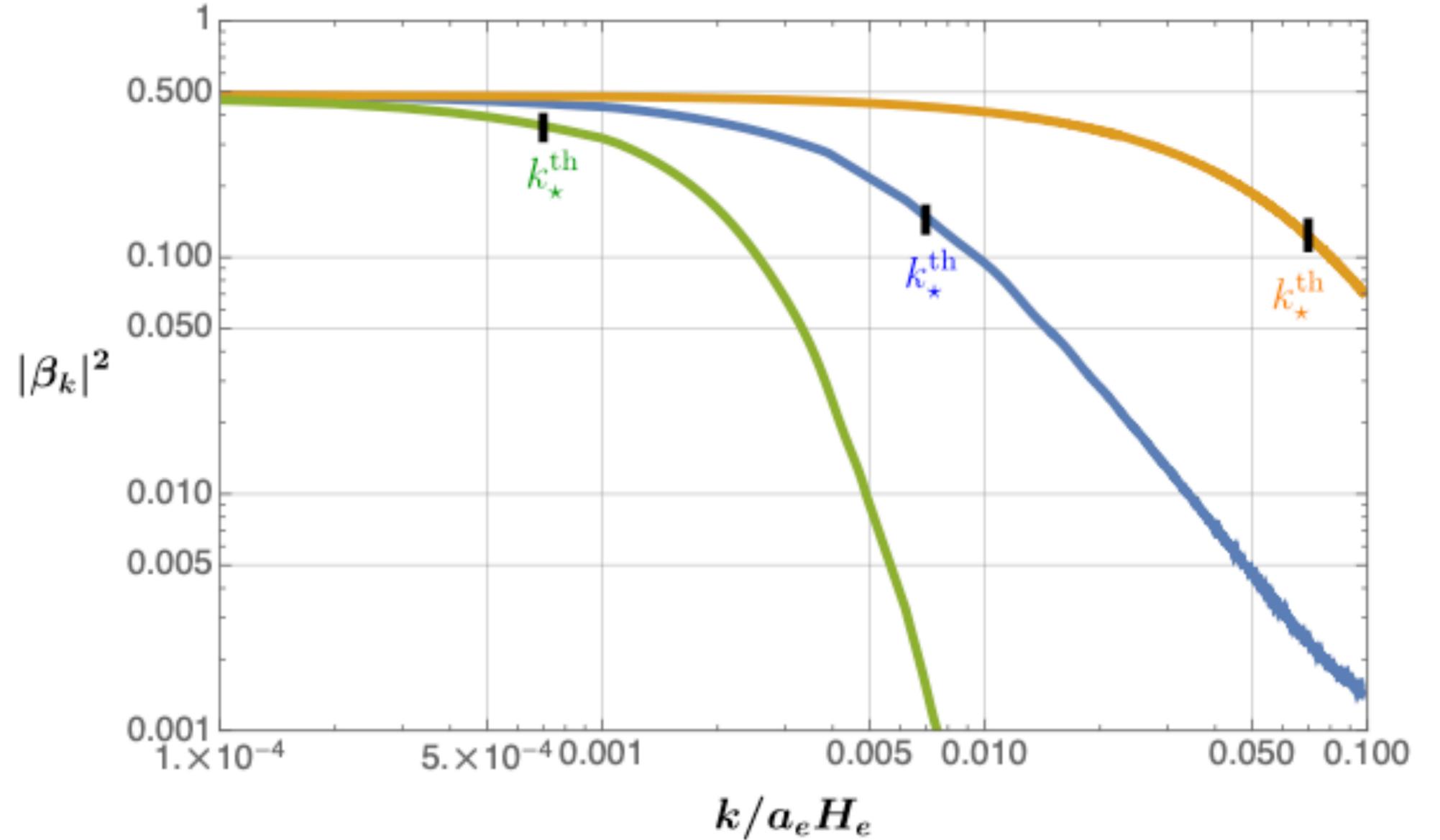
Thermal cut-off

$$k_*^{\text{th}} = a_0 M$$

Particle abundance

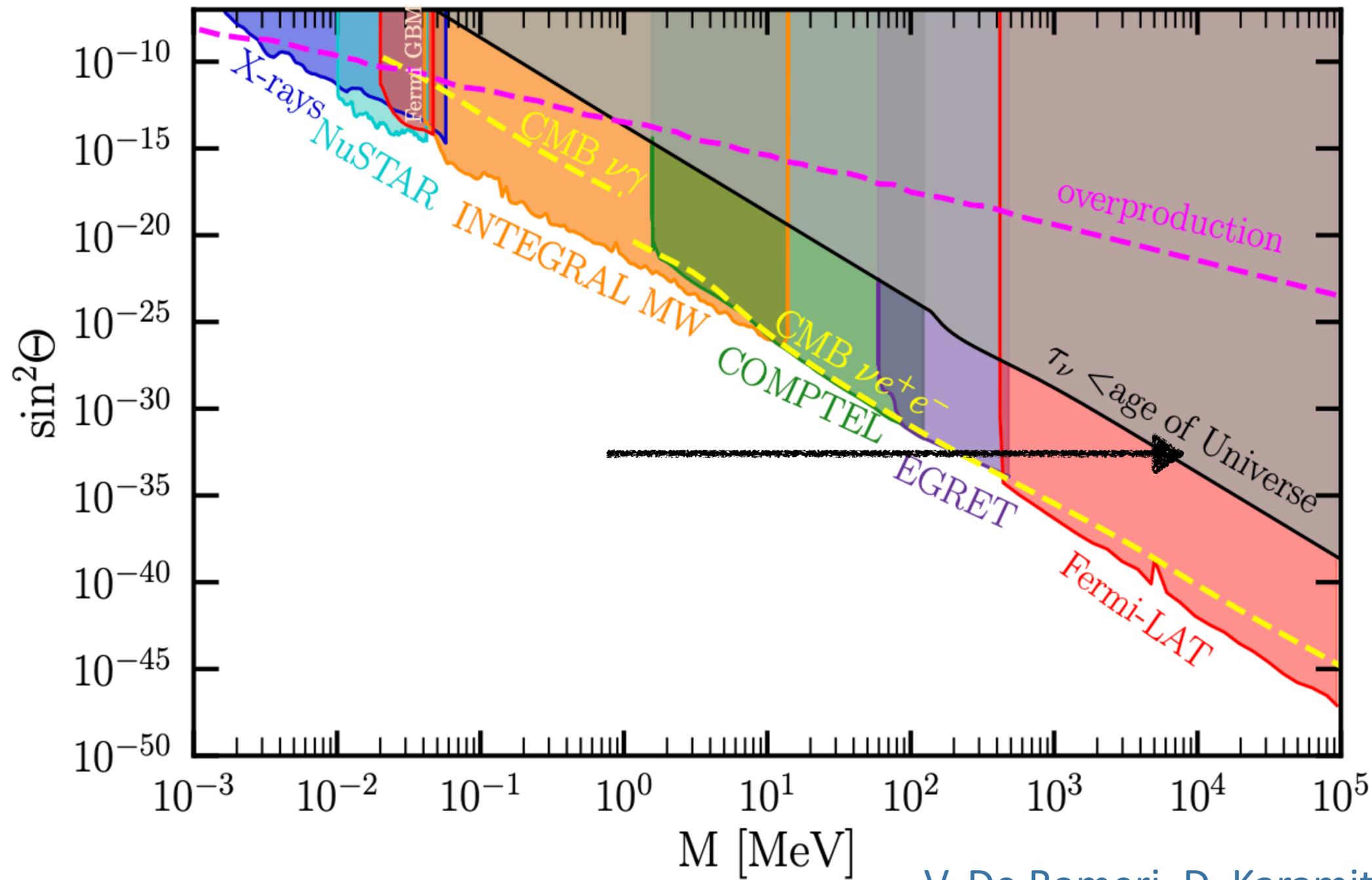
$$Y_{\nu_R}^{\text{slow}} \sim 10^{-3} \times N^3 \frac{M^3}{(M_{\text{Pl}} H_e)^{3/2}}$$

$$M/H_e = 10^{-2}, 10^{-3}, 10^{-4}, m/H_e = 10^{-5}$$



$$Y_{\text{fast}} \sim Y_{\text{slow}} \times N \sqrt{\frac{M}{H_e}} \ll Y_{\text{slow}}$$

The constrains



V. De Romeri, D. Karamitros, O. Lebedev and T. Toma, JHEP 10 (2020) 137

POST-INFLATIONARY PRODUCTION

Oscillating period

- Inflation completes and the Inflaton field ϕ oscillates around its minimum

Classical time-dependent background which naturally leads to particle production

- Quantum (and classical) gravity effects induces gauge invariant **Planck-suppressed operators** among various fields including the Inflaton

Particle production takes place even in the absence of direct renormalizable couplings between the inflaton and other fields

- For $\phi_0 < M_{\text{Pl}}$ after Inflation an EFT description is on giving in leading order the **dim-5 operator**

$$\frac{\mathcal{C}}{M_{\text{Pl}}} \phi^2 \bar{\Psi} \Psi$$

under $\phi \rightarrow -\phi$ and P conservation

~~$$\phi \bar{\Psi} \Psi$$~~

~~$$\phi^2 \bar{\Psi} \gamma_5 \Psi$$~~

Oscillating period

- The Wilson coefficient C

1) *Demands a complete quantum gravity theory, e.g. via an n -point function in string theory*

W. Buchmuller, K. Hamaguchi,
O. Lebedev and M. Ratz, Phys.
Rev. Lett. 96 (2006)

2) *Different structure compared to those generated by graviton exchange, no relation to $T_{\mu\nu}$*

3) *Dim-5 operator breaks CFT as classical gravity does but C **is not** proportional to M*

Y. Ema, R. Jinno, K. Mukaida and
K. Nakayama, Phys. Rev. D 94
(2016) 063517

- There are no general arguments to naturally suppress C and is treated as free parameter under $C \leq 1$

Oscillating period

Fermion production rate

$$\phi^2(t) = \sum_{n=-\infty}^{\infty} \zeta_n e^{-in\omega t}$$

ω is the oscillation frequency and ζ_n are slow functions of time

$$\underbrace{-i \int_{-\infty}^{\infty} dt \langle f | V(t) | i \rangle}_{\mathcal{M}} = -i \frac{\mathcal{C}}{M_{\text{Pl}}} (2\pi)^4 \delta(\mathbf{p} + \mathbf{q}) \sum_{n=1}^{\infty} \zeta_n \delta(E_p + E_q - n\omega) \bar{u}v$$

\mathcal{M}

$$\sum_{\text{spin}} |\mathcal{M}_n|^2 = \frac{\mathcal{C}^2}{M_{\text{Pl}}^2} 2(n\omega)^2 |\zeta_n|^2 \quad \longrightarrow$$

$$\Gamma = \sum_{n=1}^{\infty} \Gamma_n = \sum_{n=1}^{\infty} \int \left(\sum_{\text{spin}} |\mathcal{M}_n|^2 \right) d\Pi$$

$$= \frac{\mathcal{C}^2}{4\pi M_{\text{Pl}}^2} \omega^2 \sum_{n=1}^{\infty} n^2 |\zeta_n|^2$$

Oscillating period

Relic abundance

The particle density for Dirac fermions is found via the Boltzmann equation

$$\dot{n} + 3Hn = 2\Gamma \quad \longrightarrow \quad n a^3 = \int da a^2 \frac{2\Gamma}{H}$$

Quadratic Inflaton potential

$$V = \frac{1}{2} m_\phi^2 \phi^2, \quad \phi(t) = \frac{\phi_0}{a^{3/2}} \cos m_\phi t$$

$$\Gamma = \frac{c^2 m_\phi^2}{16\pi M_{\text{Pl}}^2} \frac{\phi_0^4}{a^6}$$

$$H = \frac{m_\phi \phi_0}{\sqrt{6} M_{\text{Pl}} a^{3/2}}$$

$$n(t) = \frac{c^2 m_\phi}{2\sqrt{6}\pi M_{\text{Pl}}} \frac{\phi_0^3}{a^3}$$

Defining the dilution factor $\Delta_{\text{NR}} = \left(\frac{H_e}{H_R} \right)^{1/2} = a_R^{3/4} \simeq T_R^{\text{inst}} / T_R$

$$Y_{\nu_R}^{\text{QG}} = 10^{-1} c^2 \frac{H_e^{3/2} M_{\text{Pl}}^{1/2}}{\Delta_{\text{NR}} m_\phi^2}$$

Oscillating period

Constraints and implications

$$\begin{array}{ccc}
 \text{For } Y_{\nu_R}^{\text{QG}} \lesssim Y & & \\
 \mathcal{C} \lesssim 10^{-4} \Delta_{\text{NR}}^{1/2} \frac{M_{\text{Pl}}^{3/4} H_e^{1/4}}{\phi_0} \sqrt{\frac{\text{GeV}}{M}} & \xrightarrow[\phi_0 \sim 0.1 M_{\text{Pl}}]{H_e \sim 10^{-5} M_{\text{Pl}}} & \mathcal{C} \leq 10^{-5} \Delta_{\text{NR}}^{1/2} \sqrt{\frac{\text{GeV}}{M}}
 \end{array}$$

Unless $\Delta_{\text{NR}} \gg 1$, $\mathcal{C} \ll 1$ for $M \geq \text{GeV}$ otherwise the Universe would be too dark

Quartic Inflaton potential

$$V = \frac{1}{4} \lambda_\phi \phi^4 \quad \omega \simeq 0.85 \sqrt{\lambda_\phi} \phi_0 / a \quad \zeta_1 \simeq 0.25 \phi_0^2 / a^2$$

$$Y_{\nu_R}^{\text{QG}} \simeq 1.2 \times 10^{-2} \mathcal{C}^2 \frac{\phi_0^2}{M_{\text{Pl}}^{3/2} H_e^{1/2}}$$

No dependence on reheating temperature!!! (or $\equiv \Delta_{\text{NR}} = 1$)

Results

- For both cases $\mathcal{C}(M \sim \text{keV}) \simeq 10^{-2} - 10^{-1}$

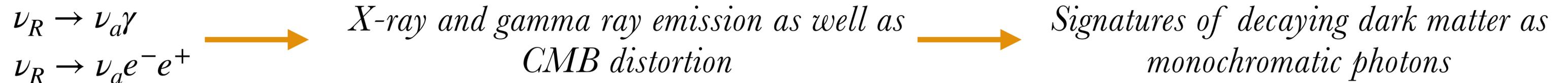
*The QG dim-5 operator with a **small** Wilson coefficient can generate **all of the dark matter** even in the keV regime*

- SN are **cold DM** in contrast to the Dodelson-Widrow mechanism (E_{ν_R} related to T_{SM})

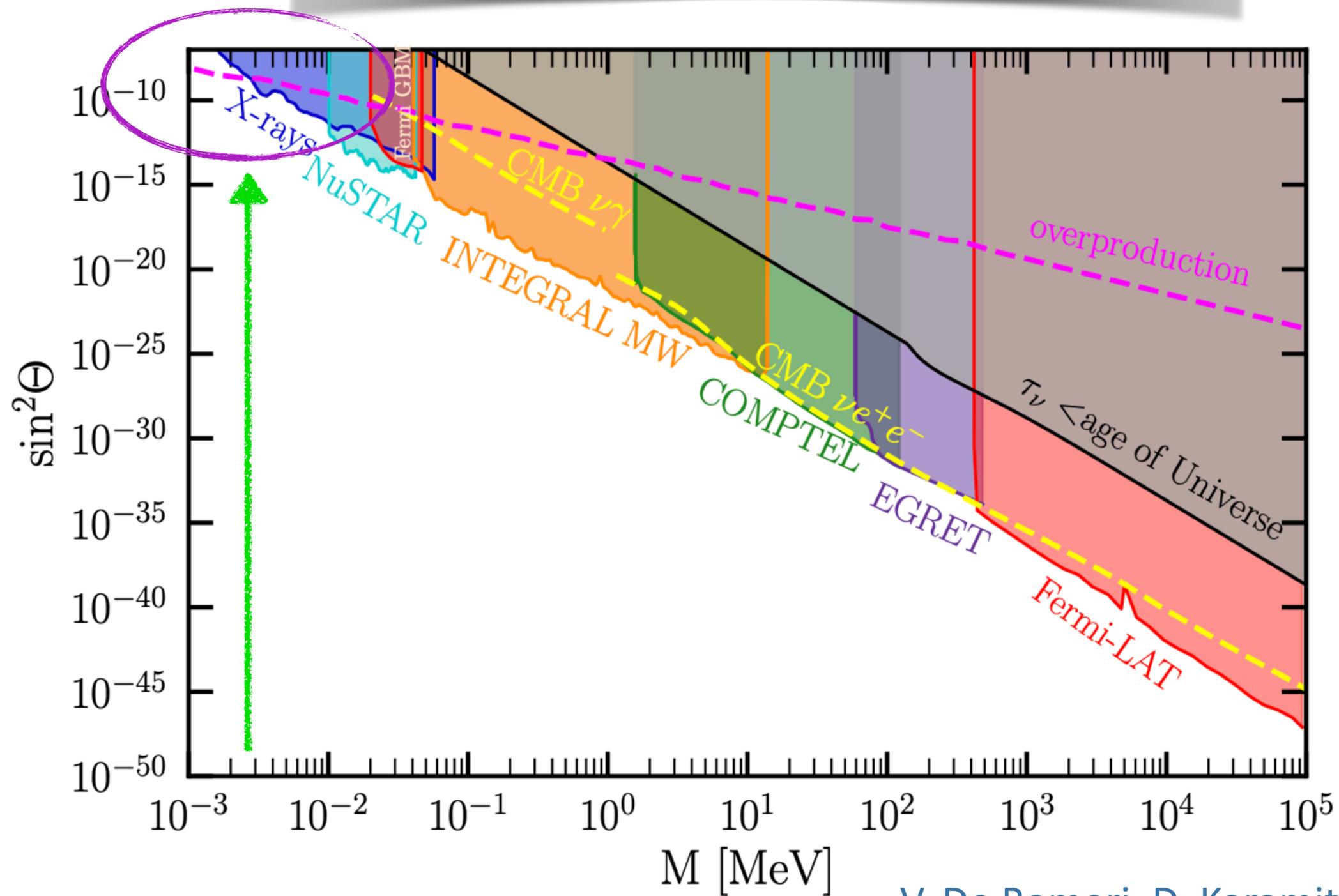
$$E_{\nu_R} \sim m_\phi \ll V^{1/4} \sim T_{\text{SM}}$$

*SN become non-relativistic at $T_{\text{SM}} \gg M$ and are **cold** at the stage of structure formation*

- The gravitational production mechanism is operative **irrespective** of the **active-sterile** mixing angle Θ



The message



V. De Romeri, D. Karamitros, O. Lebedev and T. Toma, JHEP 10 (2020) 137

Conclusions

- DM in the form of SN is produced **only** gravitationally via Inflaton **during** and **after** Inflation
- **During** Inflation the relic density of the produced (cold) Dirac SN, even though enhanced due to the Higgs condensate, is extremely suppressed for $M \ll 6 \times 10^9$ GeV
- At the **post**-inflationary regime **quantum gravity** operators break conformal invariance and couple the SN with the Inflaton. At leading order a dim-5 operator with a relatively small Wilson coefficient can generate **all of the dark matter** of the Universe even in the keV regime with observable signatures
- Future plans: Stay tuned for the upcoming work including SM particle production and reheating only via the Higgs condensate (decoupling T_R from Inflaton dynamics) and Majorana SN production among others

THANK YOU!!!