# Exact Renormalization Group methods in Particle Physics and Quantum Gravity

KBFI
NICPB

**Daniele Rizzo**

Based on

PRD 045020 (2026) & *ArXiv:2603.xxxxx*

in collaboration with

**Gabriele Giacometti, Kamila Kowalska, Enrico Sessolo & Dario Zappalà**

March 12th 2026

Seminar "Theory of Particle Physics and Cosmology"

UNIVERSITY OF WARSAW

# Plan for the talk

Perturbation theory and when we need non-perturbative physics

Exact Renormalization Group formulation(s): Wetterich FRG & Wilsonian Propertime FRG

The importance of reproducing 2-loop β-functions from non-perturbative FRG *(PRD 045020)*

Fixed point of Quantum Gravity as a cure for the Landau Pole of the Standard Model

Heuristic approach to Asymptotically Safe Quantum Gravity

Asymptotically Safe Quantum Gravity in Wilsonian Propertime FRG *(ArXiv:2603.xxxxx)*

# Perturbative and non-perturbative physics

Perturbation theory is ~ Taylor expansion for small coupling

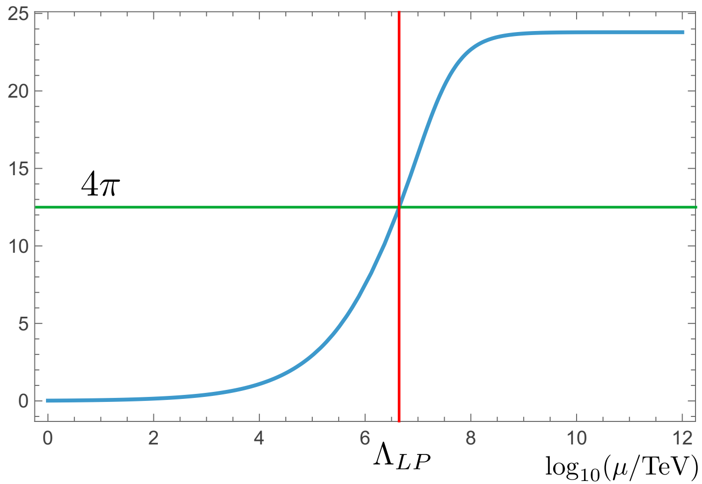
$$\mathcal{L} = \sum \lambda^n \mathcal{L}^{(n)} \longrightarrow \lambda << 4\pi$$

Perturbation theory works greatly at describing the Standard Model of particle physics

Why and when do we need non-perturbative physics?

Strongly coupled system → λ is not small!

Non-polynomial physics → $1/\lambda$ or $\ln \lambda$




Examples: QCD at low energy QED at high energy Quantum Gravity

# Exact Renormalization Group formulation(s): Wetterich FRG & Wilsonian Propertime FRG

# Wilson vs Wetterich renormalization group

Compute the path integral gives the "trace-log" result. Then use Wilson's idea of integrating only the modes above the energy scale you are interested in (for example integrate only for *p>k*):

$$S_k[\phi] = S_{\text{tree}}[\phi] + \frac{1}{2}\,\text{Tr}_{|p|>k}\ln\left(S^{(2)}_{\text{tree}}[\phi]\right)$$

This defines a theory at the energy scale k where only mode above k are integrated out

Divergencies in the trace-log are later regulated using **Schwinger proper-time representation**

$$\text{Tr}(\log A) = -\int_0^{\infty}\frac{ds}{s}\,f_k(s,\Lambda)\,\text{Tr}\left(e^{-sA}\right)$$

Choose a truncation for your initial action

$$\boxed{k\partial_k\,S_k = -\frac{1}{2}\,k\partial_k\int_{1/\Lambda^2}^{1/k^2}\frac{ds}{s}\,\text{TR}\left[e^{-s\,S_k^{(2)}}\right]}$$

Compute 1PI generator in path integral

$$\Gamma^{(1)}_k[\phi] = S[\phi] + \frac{1}{2}\,\text{Tr}\,\ln\left(S^{(2)}_{\text{tree}}[\phi] + R_k\right)$$

This defines a theory at the energy scale k, where all momenta are integrated and IR fluctuations are simply suppressed

Regularize the IR divergences

$$\Delta S_k[\varphi] = \frac{1}{2}\int\varphi\,R_k\,\varphi$$

$$R_k(p \ll k) \sim k^2$$
$$R_k(p \gg k) \to 0$$

Choose a truncation for the effective average action

$$\boxed{k\partial_k\Gamma_k = \frac{1}{2}\,\text{STr}\left[k\partial_k R_k\cdot\left(\Gamma^{(2)}_k + R_k\right)^{-1}\right]}$$

# On the universal content of the proper time flow in scalar and Yang-Mills theories

Phys. Rev. D: 113, 045020 (2026)

in collaboration with

**Gabriele Giacometti & Dario Zappalà**

# What do I mean by "universal content"?

The 2-loop β-function of massless theories is regulator independent $\longrightarrow$ **UNIVERSAL**

The goal is studying proper time flow and compute the universal coefficients

RG improvement

The proper time flow equation

$$\boxed{k\partial_k\, S_k = -\frac{1}{2}\, k\partial_k \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\, \mathrm{TR}\, \left[e^{-s\, S_k^{(2)}}\right]}$$

($n$+1)-loop flow **Nesting** $n$-loop action

Integrate some dof from Λ until an intermediate scale k' and use the obtained action to integrate until 0. $\longrightarrow$ **This reproduces correct 2-loop β-functions**

$$\int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s} \int_{1/\Lambda^2}^{s} \frac{dt}{t}$$

# Wetterich equation and 2-loop β-functions

Papenbrock, Wetterich (1995) reproduce the correct 2-loop β-functions of the scalar theory using **nesting** of FRG flow equation.

Next station: Gauge theories, e.g. Yang-Mills theory

In pure FRG with Wetterich equation, the regulator breaks gauge invariance

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[ \partial_t R_k \cdot \left( \Gamma_k^{(2)} + R_k \right)^{-1} \right]$$

**No 2-loop Yang-Mills**

| | | |
|---|---|---|
| Reuter, Wetterich (1994) | $\beta_2 = \frac{110}{9}$ | 93% correct |
| Gies (2002) | $\beta_2 = \frac{77}{6} - \text{cut-off dependent part}$ | 99% correct |
| Wetterich (2018) | $\beta_2 = \frac{77}{6}$ | 88% correct |

*And many more attempts…*

Correct Result $\beta_2 = \frac{34}{3} = \frac{102}{9}$

Can the proper time flow succeed at reproducing 2-loop β-functions?

# Perturbative expansion and the proper time flow

The flow equation for the Wilsonian action in (sharp) proper time regularization

$$\boxed{k\partial_k\, S_k = -\frac{k\partial_k}{2}\int_{1/\Lambda^2}^{1/k^2}\frac{ds}{s}\,\mathrm{TR}\,\left[e^{-s\,S_k''}\right]}$$

(*n*+1)-loop flow

*n*-loop action

Concretely, the action is expanded in powers of ħ and same powers are collected

$$S_k = S_0 + \hbar S_1 + \hbar^2 S_2 + \mathcal{O}(\hbar^3)$$

The 1-loop correction to the Wilsonian action is easily recovered

$$S_1 = -\frac{1}{2}\int_0^{\infty}\frac{ds}{s}\,\mathrm{TR}\,\left[e^{-s\,S_0''}\right] = \frac{1}{2}\,\mathrm{TR}\,\log\left[S_0''\right]$$

# Perturbative expansion and the proper time flow

Iterate the process → Insert 1-loop Wilsonian action and obtain 2-loop action

$$S_1 + S_2 = -\frac{1}{2}\int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\,\mathrm{TR}\,\left[e^{-s\,(S_0''+S_1'')}\right] \approx -\frac{1}{2}\int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\,\mathrm{TR}\,\left[e^{-s\,S_0''}\,(1-s\,S_1'')\right]$$

$$= S_1 \underbrace{-\frac{1}{2}\int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\,\mathrm{TR}\,\left[e^{-s\,S_0''}\,(-s\,S_1'')\right]}_{S_2}$$

The running action entering the PT flow equation is a local action

de Alwis (2018) but also Bonanno, Lippoldt, Percacci, and Vacca (2020)

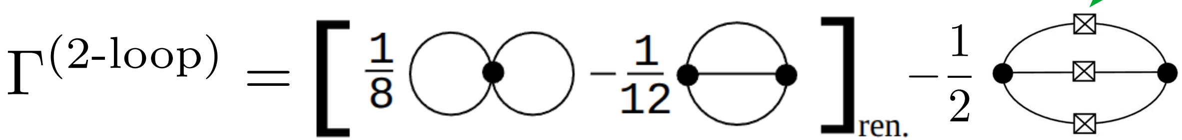
↓

Diagrammatically **wrong!**

Litim and Pawlowski (2002)

Non-local structure

Regulator insertion




The β-function do not see the spurious diagram

↓

β-functions should be **correct!**

# O(N) Scalar Theory – 1-loop β-function

$$S_0 = \int d^4x \left[\frac{1}{2} Z_k \, (\partial\phi)^2 + \frac{\lambda}{4!} \, (\phi^2)^2\right]$$

$$\beta_\lambda = \cancel{2\eta\lambda} + \partial_t (P_{\phi^4} \, S_k)$$

$$\partial_t (P_{\phi^4} \, S_1) = -\frac{1}{2} \partial_t P_{\phi^4} \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s} \, \mathrm{TR} \left[ e^{-s \, S_0''} \right] =$$

$$\boxed{\partial_t (P_{\phi^4} \, S_1) = \frac{\lambda^2}{(16\pi^2)} \, \frac{(N+8)}{3}}$$

$$= -\frac{1}{2} \partial_t \, \mathrm{P}_{\phi^4} \left\{ \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s} \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} \left[ e^{-s\left[\left(p^2 + \frac{\lambda}{6}\phi_j\phi_j\right)\delta_{ab} + \frac{\lambda}{3}\phi_a\phi_b\right]} \right] \right\} \Bigg|_{\phi=0}$$

$$= -\frac{\lambda^2}{(16\pi^2)} \, \frac{(N+8)}{6} \, \partial_t \log \frac{\Lambda^2}{k^2}$$

# O(N) Scalar Theory – 2-loop β-function

$$S_0 = \int d^4x \left[\frac{1}{2} Z_k \, (\partial\phi)^2 + \frac{\lambda}{4!} \, (\phi^2)^2\right] \qquad\qquad \beta_\lambda = 2\,\eta\,\lambda + \partial_t (P_{\phi^4}\, S_k)$$

$$\partial_t (P_{\phi^4}\, S_2) = -\frac{1}{2} \partial_t P_{\phi^4} \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s} \, \mathrm{TR}\left[ e^{-s\, S_0''} \, (-s\, S_1'') \right] \qquad \boxed{\partial_t (P_{\phi^4}\, S_1) = \frac{\lambda^2}{(16\pi^2)} \, \frac{(N+8)}{3}}$$

$$= \frac{\lambda^3}{(16\pi^2)^2} \, \partial_t \left\{ 1 + \frac{1}{4} \left[ \frac{(N+8)}{3} \log\left(\frac{\Lambda^2}{k^2}\right)\right]^2 + \frac{(10N+44)}{18} \log\left(\frac{\Lambda^2}{k^2}\right) \right\}$$

# O(N) Scalar Theory – Anomalous dimension

$$S_0 = \int d^4x \left[\frac{1}{2} Z_k \,(\partial\phi)^2 + \frac{\lambda}{4!}\,(\phi^2)^2\right]$$

$$\beta_\lambda = 2\,\eta\,\lambda + \partial_t(P_{\phi^4}\, S_k)$$

$$S_2 = -\frac{1}{2}\int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\ \mathrm{TR}\ \left[e^{-s\,S_0''}\,(-s\,S_1'')\right]$$

$$\boxed{\partial_t(P_{\phi^4}\, S_1) = \frac{\lambda^2}{(16\pi^2)}\,\frac{(N+8)}{3}}$$

$$\boxed{\partial_t(P_{\phi^4}\, S_2) = -\frac{\lambda^3}{(16\pi^2)^2}\,\frac{(10N+44)}{9}}$$

$$Z_2 = \left(\frac{\partial}{\partial\, q^2}\left[\frac{\delta^2 S_2}{\delta\widetilde{\phi}_i(q)\delta\widetilde{\phi}_i(-q)}\right]_{\phi=0}\right)_{q^2=0} =$$

$$= \left[\frac{\partial}{\partial\, q^2}\left(-\int_{1/\Lambda^2}^{1/k^2}\frac{ds}{4}\int\frac{d^4p_e}{(2\pi)^4}\int_{1/\Lambda^2}^{s} dt\, t\int\frac{d^4p_i}{(2\pi)^4}\int_0^1 du\ \frac{\lambda^2\,(2N+4)}{3}\right.\right.$$

$$\left.\left.\times e^{-sp_e^2}\ e^{-tp_i^2(1-u)}\ e^{-su(p_i+q)^2}\right)\right]_{q^2=0} = \frac{\lambda^2}{(16\pi^2)^2}\,\frac{(N+2)}{36}\ \log\frac{\Lambda^2}{k^2}$$

# O(N) Scalar Theory – 2-loop β-function

$$S_0 = \int d^4x \left[\frac{1}{2} Z_k \, (\partial\phi)^2 + \frac{\lambda}{4!} \, (\phi^2)^2\right]$$

$$\beta_\lambda = 2\,\eta\,\lambda + \partial_t(P_{\phi^4}\, S_k)$$

$$S_2 = -\frac{1}{2} \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s} \, \mathrm{TR} \left[e^{-s\, S_0''} \, (-s\, S_1'')\right]$$

$$\partial_t(P_{\phi^4}\, S_1) = \frac{\lambda^2}{(16\pi^2)} \, \frac{(N+8)}{3}$$

$$\partial_t(P_{\phi^4}\, S_2) = -\frac{\lambda^3}{(16\pi^2)^2} \, \frac{(10N+44)}{9}$$

$$Z_2 = \left( \frac{\partial}{\partial\, q^2} \left[ \frac{\delta^2 S_2}{\delta\widetilde{\phi}_i(q)\delta\widetilde{\phi}_i(-q)} \right]_{\phi=0} \right)_{q^2=0} =$$

$$= \left[ \frac{\partial}{\partial\, q^2} \left( - \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{4} \int \frac{d^4 p_e}{(2\pi)^4} \int_{1/\Lambda^2}^{s} dt\, t \int \frac{d^4 p_i}{(2\pi)^4} \int_0 du \; \cdots \; \frac{\cdots}{3} \right. \right.$$

$$\eta^{2L} = \frac{\lambda^2}{(16\pi^2)^2} \, \frac{N+2}{18}$$

$$\beta^{2L}_{\mathrm{Total}} = \frac{\lambda^2}{(16\pi^2)} \, \frac{(N+8)}{3} - \frac{\lambda^3}{(16\pi^2)^2} \, \frac{3N+14}{3}$$

**Correct!**

# Background field method in gauge theories

Abbott (1981) computes the 2-loop β-function of YM, **perturbatively**, in background field method

Ghost $\theta_a = \chi_a + \cancel{c_a}$

Gauge $\widehat{A}_\mu = A_\mu + \cancel{Q_\mu}$

Enters $F_{\mu\nu}$ Background Fluctuation

Without background field, the 1-loop β-function is **wrong**

Theory with field $\widehat{A}_\mu$

↓

Split Background + Fluctuations

↓

Integrate the Fluctuations in path integral

↓

You are left with a theory with only the Background

↓

Call it $\widehat{A}_\mu$

After integrating the fluctuations Q and c, and setting **χ=0**, the effective action is **gauge invariant!**

$$A_R = \sqrt{Z_A} A_0 \qquad g_R = Z_g \, g_0$$

Ward Identity in background field

$$Z_g = Z_A^{-1/2} \xrightarrow{\partial_t} \beta(g) = g \, \eta_A / 2$$

# SU(N) Yang-Mills Theory

$$S_A = S_{\mathrm{YM}} + S_{gf} = \int d^4x' \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \left( D^{ab}_\mu(A)\, Q^b_\mu \right) \left( D^{ac}_\nu(A)\, Q^c_\nu \right) \right\}$$

$$S_G = \int d^4x' \left\{ -\,\overline{\theta}_a \left[ D^{ab}_\mu(A)\, D^{bc}_\mu(A+Q) \right] \theta_c \right\}$$

$$Z_A = 1 + \left[ \frac{g^2 N}{16\pi^2}\, \beta_1 + \left( \frac{g^2 N}{16\pi^2} \right)^2 \beta_2 + O(g^6) \right] \log \frac{k^2}{\Lambda^2} + O\left( \log^2 \frac{k^2}{\Lambda^2} \right) + O\left( \frac{k^2}{\Lambda^2} \right)$$

1-loop β-function

2-loop β-function

From Perturbation theory: $\beta_1 = +\dfrac{11}{3}$ $\beta_2 = +\dfrac{34}{3}$

$$\beta(g) = -\frac{11}{3} \frac{N}{16\pi^2} g^3 - \frac{34}{3} \frac{N^2}{(16\pi^2)^2} g^5$$

2-loop Yang-Mills β-function

# SU(N) Yang-Mills Theory

$$S_A = S_{\rm YM} + S_{gf} = \int d^4x' \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \left(D^{ab}_\mu(A)\, Q^b_\mu\right) \left(D^{ac}_\nu(A)\, Q^c_\nu\right) \right\}$$

$$S_G = \int d^4x' \; \left\{ -\bar\theta_a \left[D^{ab}_\mu(A)\; D^{bc}_\mu(A+Q)\right] \theta_c \right\}$$

There are both bosons (gluons) and fermions (ghosts)

$$S_1 = -\frac{1}{2} \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\; \mathrm{TR} \left[ e^{-s\, S_0''} \right] \longrightarrow S_{\rm YM1} = -\frac{1}{2} \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s}\; \mathrm{TR} \left[ e^{-s\, K^{ab}_{\mu\nu}} - 2\, e^{-s\, H^{ab}} \right]$$

Liao (1995)

**Correct!**

$$\boxed{\beta_1 = +\frac{11}{3}}$$

Compute either 2-points or 4-points function

Two derivatives wrt gauge field

Two derivatives wrt ghost field

The 2-loop action can be obtained similarly

$$S_{\rm YM2} = \int_{1/\Lambda^2}^{1/k^2} \frac{ds}{s} \left\{ -\frac{1}{2} \mathrm{TR} \left[ -s\; S''_{\rm YM1}\; e^{-s\, K} \right] + \mathrm{TR} \left[ -s\; \ddot S_{\rm YM1}\; e^{-s\, H} \right] \right\}$$

$$\boxed{\beta_2 = \frac{34}{3}}$$

*Giacometti, **DR**, Zappala (2025)*

This action is **gauge invariant**: the nesting operation generates **no mass terms!**

# Summary of this work

- Massless theories, in mass-independent regularization schemes, have **universal** one- and two-loop coefficients of the β-function.

- *de Alwis (2018)* claimed that the Wilsonian RG flow with proper time sharp cutoff is **exact** and **gauge invariant**. If he is right, this flow equation **must** reproduce the universal content.

- For the scalar massless O(N) theory, we (re-)computed one- and two-loop coefficients of the β-function for quartic coupling and anomalous dimension and reproduced the well-known result.

- For the Yang-Mills theory, we employed the **background field** formalism, which has proven (see *Abbott (1981)*) to be able to reproduce one- and two-loop β-function coefficients of the gauge coupling, within perturbation theory, in a path integral formalism.

- We (re-)computed (part of) the one-loop action, obtaining the correct one-loop β-function coefficient, and importantly **no gauge breaking operators** at dimension two (mass) and six.

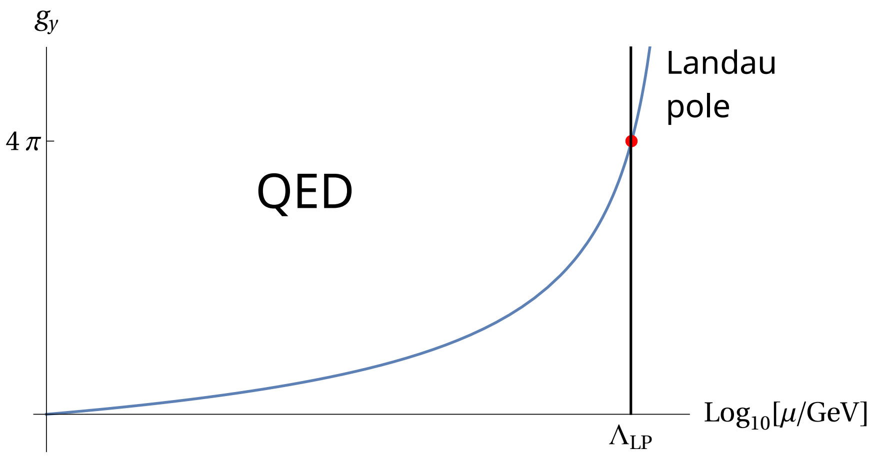
- Finally, by **reinserting** the one loop action into the RG flow in the background gauge, we recovered the **universal two loop coefficient of the beta function**, which shows the level of consistency of the PT Flow equation.

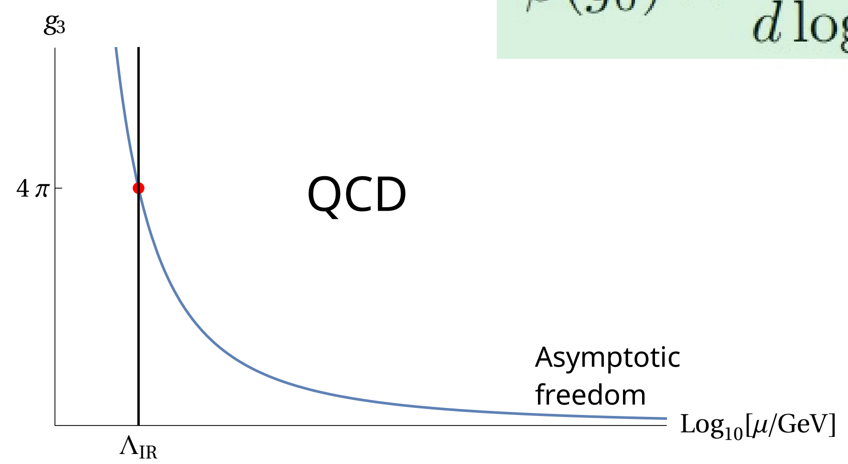
## Fixed point of Quantum Gravity as a cure for the Landau Pole of the Standard Model

# Asymptotic Behaviors

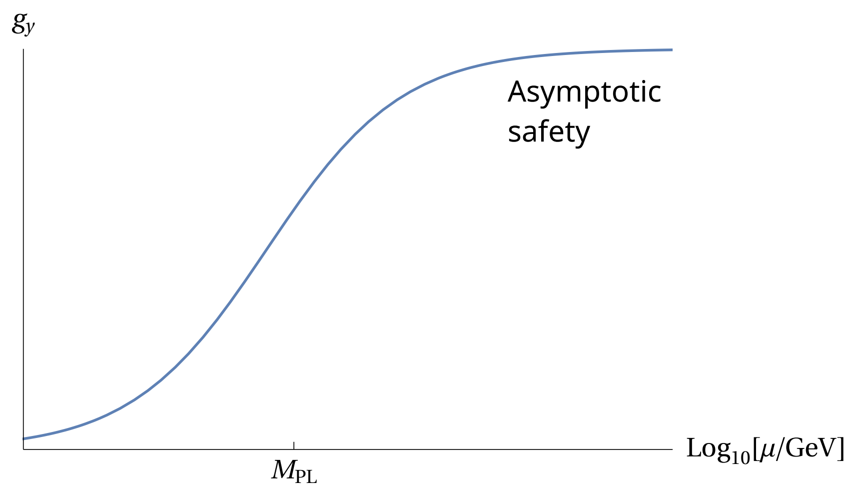
In the Standard Model, two possible asymptotic behavior:

$$\beta(g_0) \equiv \frac{dg_0}{d\log\mu}$$



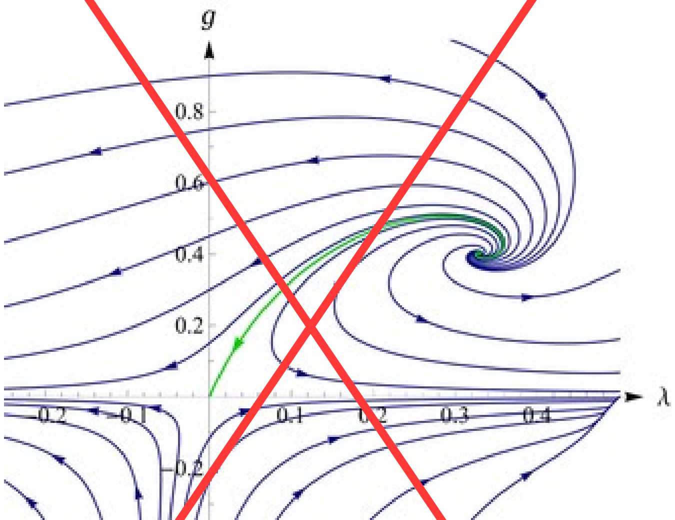

In a generic QFT a third asymptotic behaviour can be found:



# Asymptotically Safe QG: the heuristic approach

## Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(-R(g) + 2\Lambda)$$




Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

Large uncertainties when computed analytically.

[ Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, … ]

## Renormalization Group Equations in the Sub-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}}$$

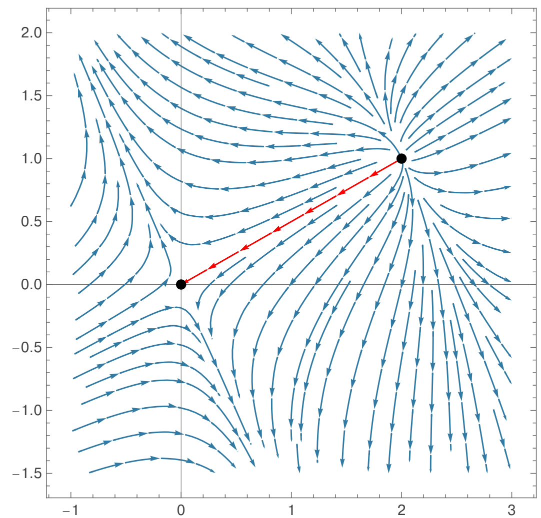
$$\beta_y = \beta_y^{\mathrm{SM+NP}}$$

## Renormalization Group Equations in the Trans-Planckian regime

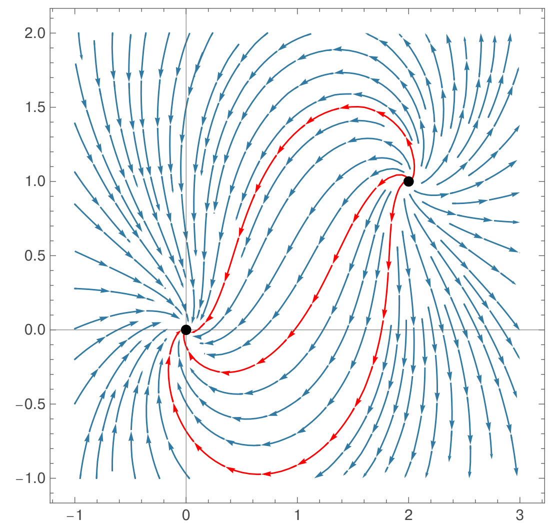
$$\beta_g = \beta_g^{\mathrm{SM+NP}} - g\, f_g$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}} - y\, f_y$$

fg and fy are determined by matching the low-energy data.




Relevant → Free parameter



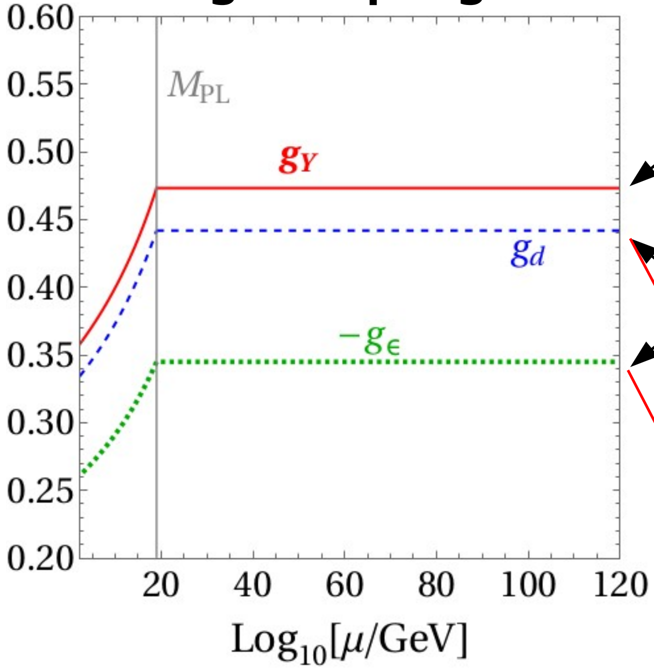

Irrelevant → Prediction

# Asymptotically Safe QG: the heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\,(\tilde{\epsilon}H^*)^\dagger L - \frac{1}{2}Y_N SNN + \text{H.c.}$$

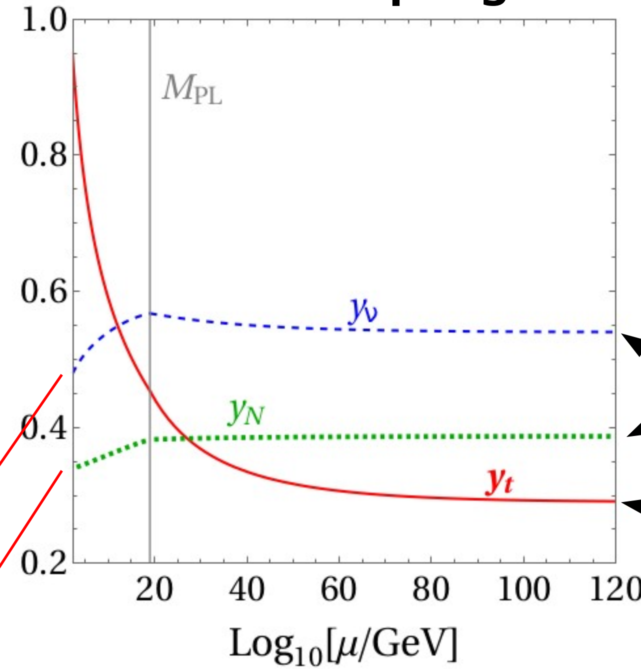
**Gauge couplings**




Get $f_g$

Predict

**Yukawa couplings**




Predict

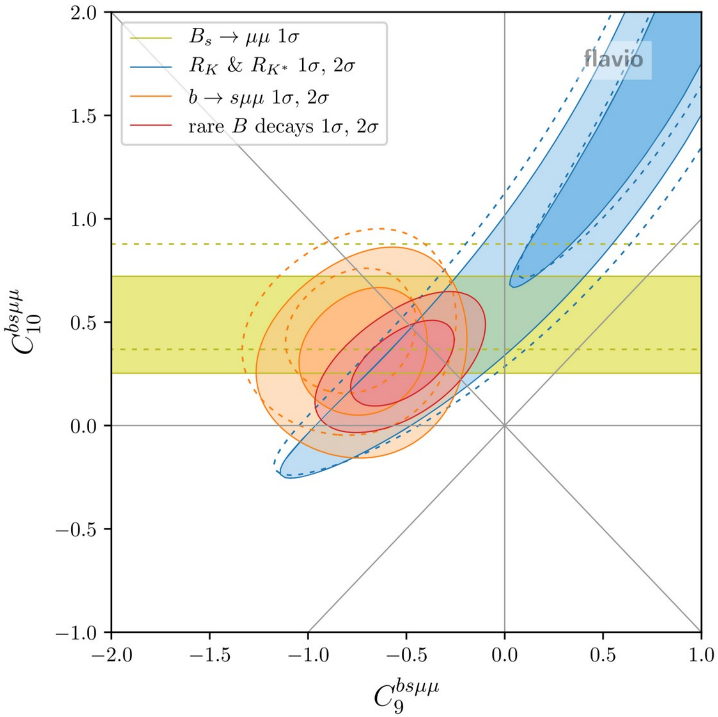
Get $f_y$

**Phenomenology!**

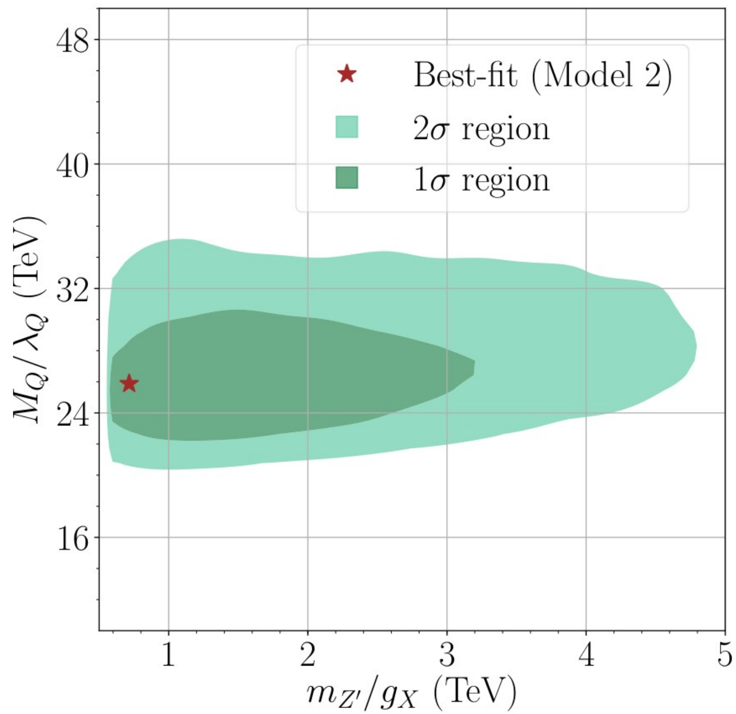
Source: Kamila Kowalska & Enrico Sessolo

# Heuristic ASQG on Z' solutions to the flavor anomalies

Explain flavor anomalies → Z' vector-like quark vector-like lepton → Wilson coefficients constrain ratio coupling/mass → **ASQG determines uniquely the couplings**




*Altmannshofer, Stangl, Eur. Phys. J. C 81 (2021)*




*Kowalska, Kumar, Sessolo, Eur. Phys. J. C (2019)*

At 2 TeV:

| | $\mathrm{FP}_{1\mathrm{A},a}$ | $\mathrm{FP}_{1\mathrm{A},b}$ |
|---|---|---|
| $g_D$ | 0.305 | 0.305 |
| $g_\epsilon$ | 0 | 0 |
| $\lambda_{Q,3}$ | -0.381 | 0.034 |
| $\lambda_{Q,2}$ | 0.016 | 0.803 |
| $\lambda_{L,2}$ | 0.823 | 0.606 |

| | $\mathrm{FP}_{1\mathrm{B},a}$ | $\mathrm{FP}_{1\mathrm{B},b}$ |
|---|---|---|
| $g_D$ | 0.318 | 0.318 |
| $g_\epsilon$ | 0.110 | 0.110 |
| $\lambda_{Q,3}$ | -0.612 | 0.004 |
| $\lambda_{Q,2}$ | 0.296 | 0.874 |
| $\lambda_{L,2}$ | 0.652 | 0.499 |

*Chikkaballi, Kotlarski, Kowalska, **DR**, Sessolo JHEP 01 (2023) 164*

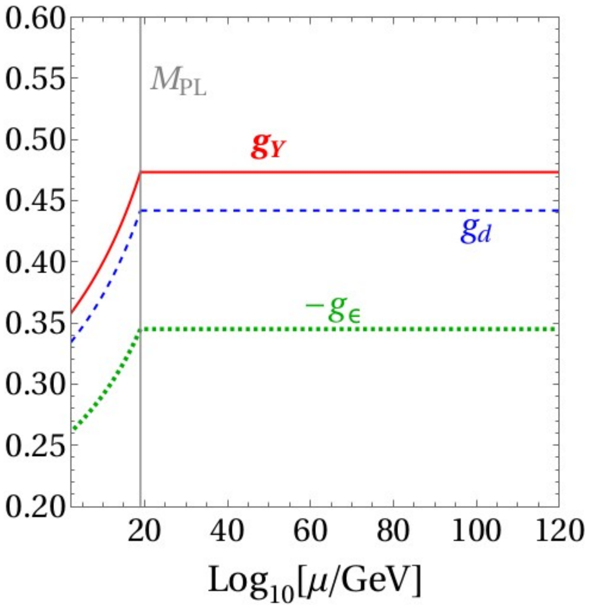
The heuristic approach to ASQG has assumptions. What happens when we relax them?

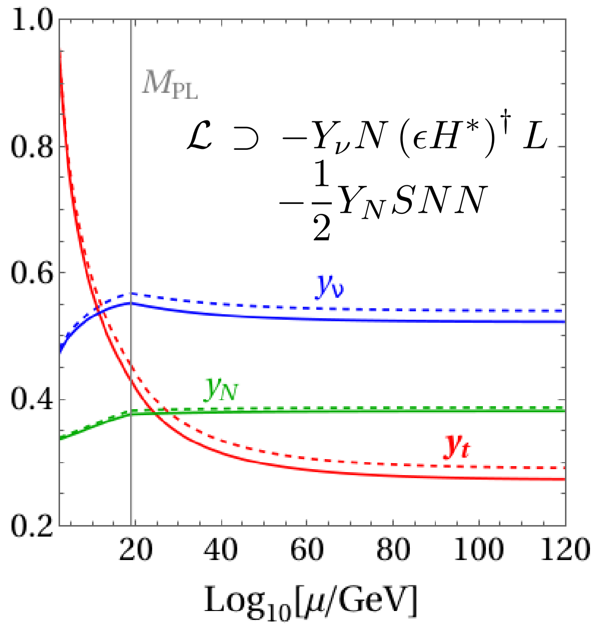
Sources of uncertainties →

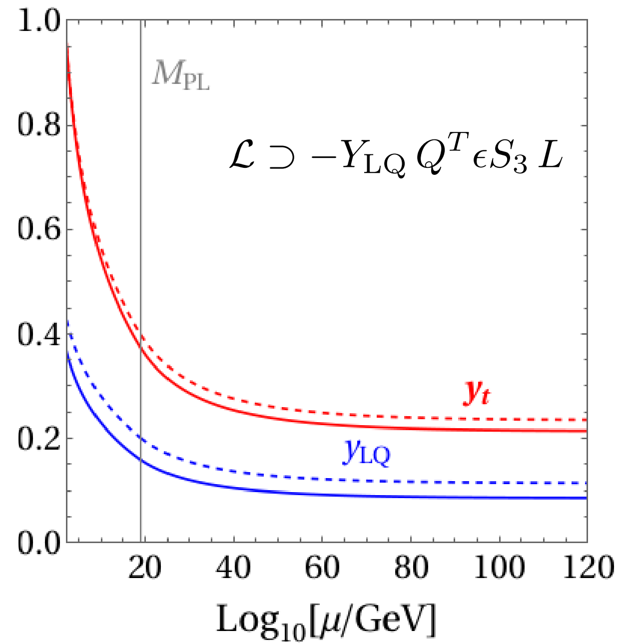
**1 - Computations of the beta functions are performed at 1-loop level.**

2 - Planck scale is set arbitrarily at $10^{19}$ GeV.

3 - Gravity decouples instantaneously at the Planck scale.




Shifting gy shifts the new-physics → $\sim\ 0.1\%$

New-physics Yukawa big → $\sim\ 1\%$

New-physics Yukawa small → $\sim\ 10\%$

*Kotlarski, Kowalska, **DR**, Sessolo* *Eur.Phys.J.C 83 (2023)*

# Asymptotically Safe Einstein-Hilbert Quantum Gravity in Wilsonian Propertime FRG

*ArXiv:2603.xxxxx*

in collaboration with

**Gabriele Giacometti, Kamila Kowalska, Enrico Sessolo & Dario Zappalà**

# Asymptotically Safe Einstein-Hilbert QG in Propertime

$$\beta_g = \beta_g^{\mathrm{SM+\cancel{N}P}} - g\, f_g$$

$$\beta_y = \beta_y^{\mathrm{SM+\cancel{N}P}} - y\, f_y$$

Minimal coupling between gravity and matter
Gravitational <u>gauge fixing</u> dependency
Propertime calculation & <u>regulator dependency</u>

The full gravity + matter action

$$S = -\frac{1}{16\pi G}\int d^4x\,\sqrt{g}\,(R-2\Lambda) + \frac{1}{32\pi G\,\alpha}\int d^4x\sqrt{\bar{g}}\,\bar{g}^{\mu\nu}F_\mu F_\nu$$
$$+\frac{1}{4}\int d^4x\,\sqrt{g}g^{\mu\nu}g^{\kappa\lambda}F_{\mu\kappa}F_{\nu\lambda} + \frac{1}{2\xi}\int d^4x\,\sqrt{\bar{g}}\,\left(\bar{g}^{\mu\nu}\bar{D}_\mu a_\nu\right)^2$$

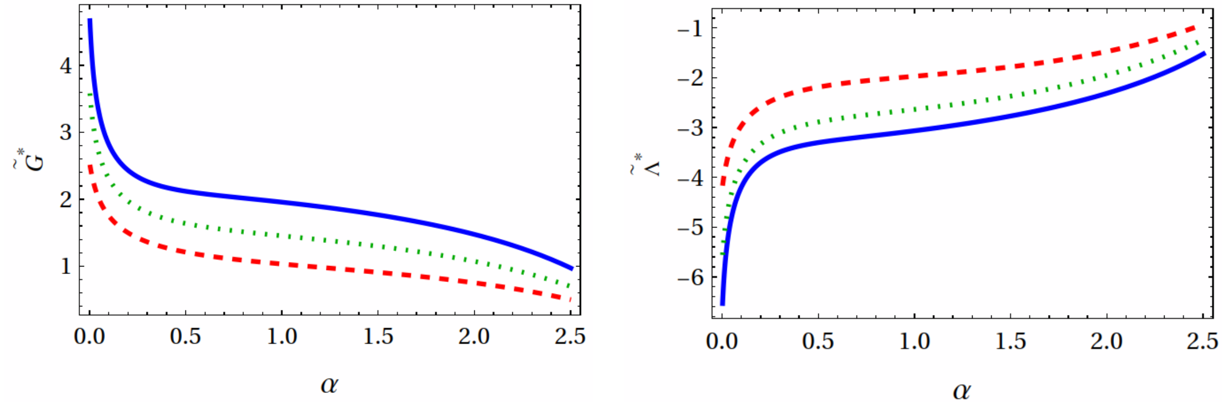
The propertime equation with generic regulator

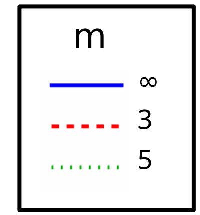
$$k\partial_k S_k = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\mathrm{Tr}\left[\left(k\,\partial_k\,\rho_{k,k_{\mathrm{UV}}}\right)\ e^{-s\,S_k''}\right] \qquad k\,\partial_k\rho_{k,k_{\mathrm{UV}}} = -\frac{2\,(m\,Z_k\,s\,k^2)^m}{\Gamma(m)}e^{-m\,Z_k\,s\,k^2}$$

# Asymptotically Safe Einstein-Hilbert QG in Propertime

$$\beta_g = \beta_g^{\mathrm{SM}+\cancel{\mathrm{NP}}} - g\, f_g$$

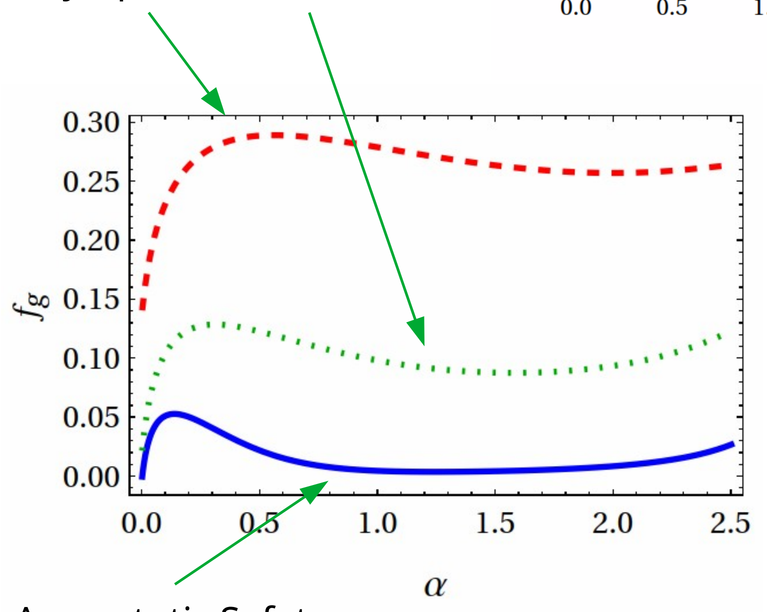
$$\beta_y = \beta_y^{\mathrm{SM}+\cancel{\mathrm{NP}}} - y\, f_y$$

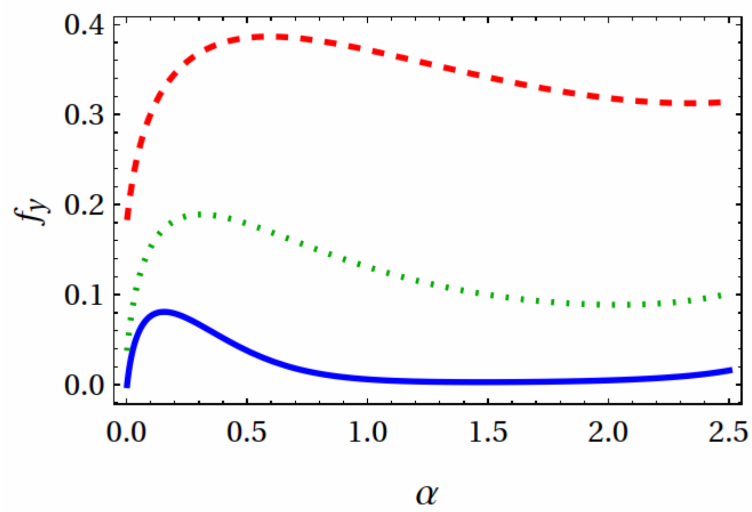



2504.07877
Bonanno *et al.*

Asymptotic Freedom




Asymptotic Safety

$f_g(\mathrm{SM}) \approx 0.0095$

Different values of fy can probe different fixed points and therefore different predictions!

PRELIMINARY RESULTS

Strong regulator dependency
Small gravitational gauge fixing dependency

# Conclusions

- While the Standard Model and other theories are perfectly described by perturbation theory, strong coupled regime and non-renormalizable theories call for **non-perturbative** calculations.

- I have given an introduction on exact renormalization group techniques, focusing on **Wetterich's flow** equation and on the **Wilsonian propertime** flow equation. The first is the most commonly used as it is in the same universality class as the Callan-Symanzik equation, while the second is more convenient in calculations where gauge invariance must be preserved.

- I have discussed how, despite not reproducing the full 2-loop effective action, the propertime flow equation is able to reproduce 2-loop β-functions for both the scalar O(N) theory and the gauge Yang-Mills theory. Most importantly, it keeps the flow gauge invariant.

- As a physically relevant application, I have discussed the calculation of the contribution from quantum gravity to the running of gauge (fg) and Yukawa (fy) couplings. The **heuristic approach** is able to make unique predictions for the IR value of BSM couplings.

- Finally, the **propertime** flow equation was used to compute the Einstein-Hilbert gravity contributions to fg and fy, showing a small dependency on the gravitational gauge fixing and a strong dependency on the regulator. For small values of the regulator (smooth cutoff), the gauge sector is asymptotically free, while for higher values (and importantly in the sharp cutoff case) we obtain Asymptotic safety or a Landau Pole, depending on the gauge fixing.

**Thank you for the attention!**