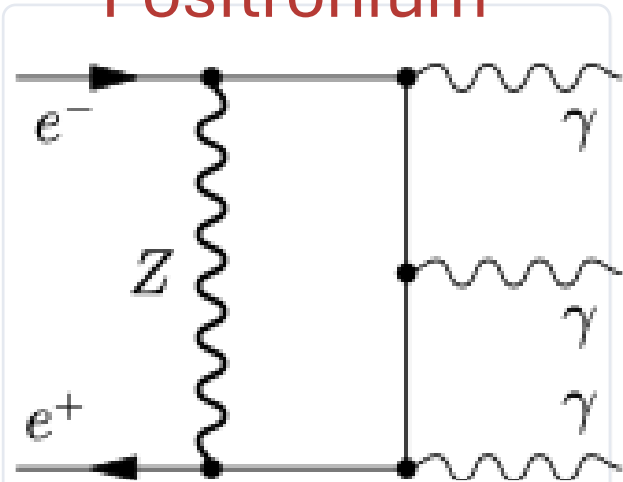


Radiative Effects in Bound States

Positronium



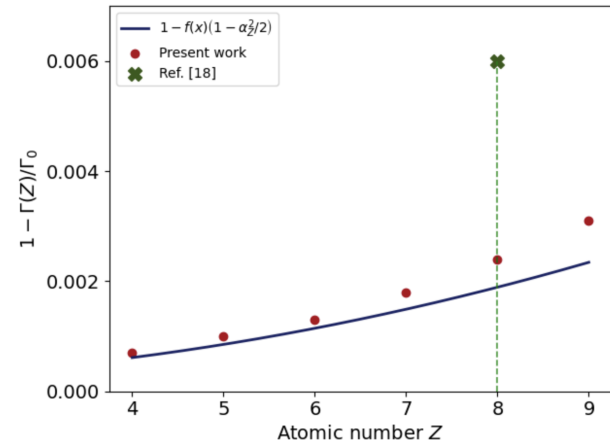
QED-forbidden channel;
full SM rate now complete

PHYSICAL REVIEW D **113**, 073002 (2026)

Parapositronium decay into three photons and implications
for the neutral pion

Andrzej Czarnecki^{1,*}, Divyesh Dagia[†], Ting Gao[‡], and Ripanjeet Toor[§]

Muonic atoms



Bound muon: lifetime;
resolved old discrepancy

PHYSICAL REVIEW D **113**, 036028 (2026)

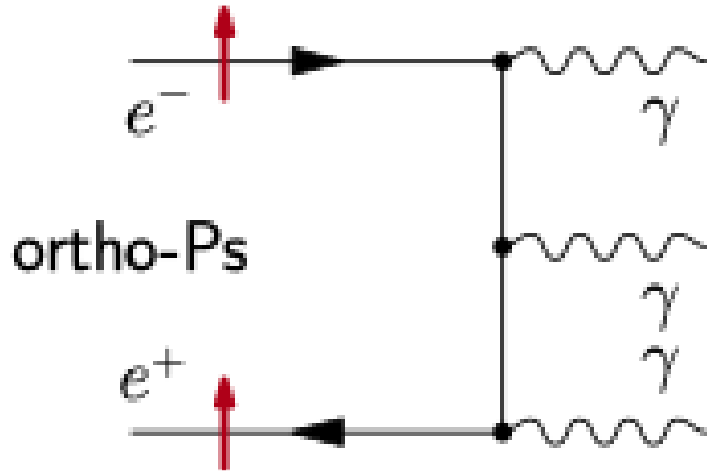
Total decay rate of a muon bound to a light nucleus

A. Czarnecki^{1,*}, A. O. Davydov¹, and M. Y. Kaygorodov

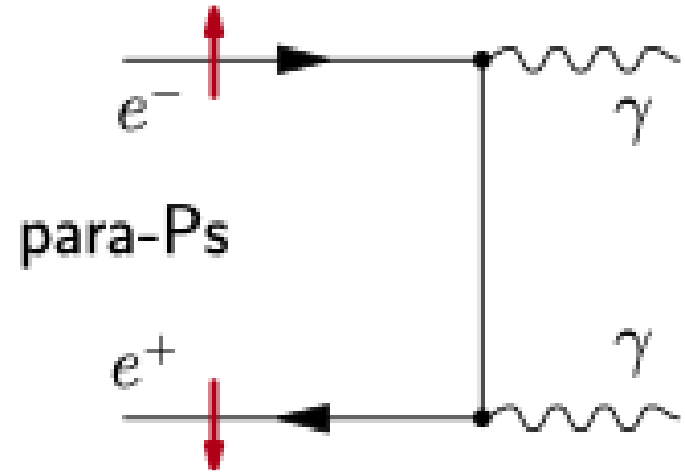
Seminar, Warsaw University

Andrzej Czarnecki, University of Alberta

April 30, 2026



o-Ps decay into 2 photons
is forbidden: Landau-Yang
theorem: Bose statistics



Can p-Ps decay
into 3 photons?

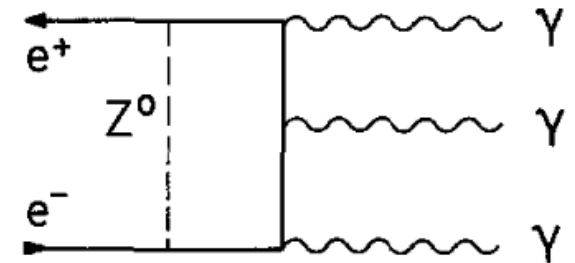
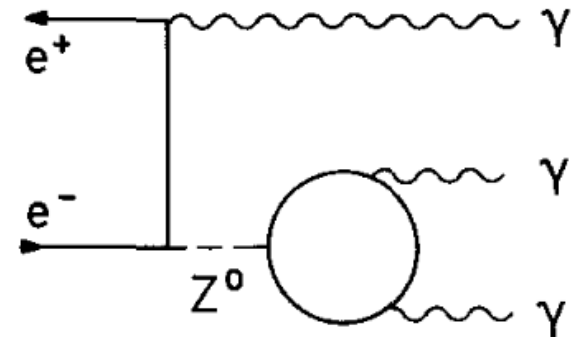
Para-positronium to 3γ : C-violating channel

1981 Estimate

Weak Interaction Effects in Positronium

W. Bernreuther and O. Nachtmann

Z. Phys. C – Particles and Fields 11, 235–245 (1981)



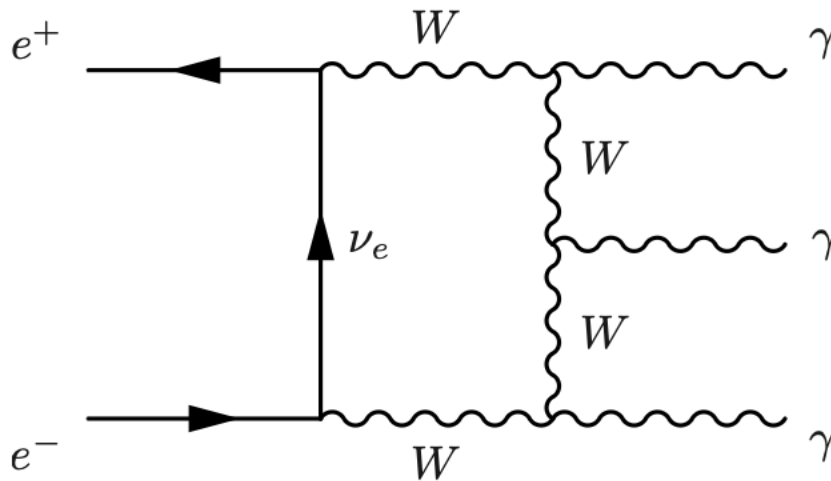
$$\frac{\Gamma(\text{pPs} \rightarrow 3\gamma)}{\Gamma(\text{pPs} \rightarrow 2\gamma)} \simeq \frac{\Gamma(\text{oPs} \rightarrow 4\gamma)}{\Gamma(\text{oPs} \rightarrow 3\gamma)} \simeq \alpha (G_F m_e^2 g_V)^2 \simeq 10^{-27}$$

$$g_V \sim 1 - 4 \sin^2 \theta_W \ll 1$$

PHYSICAL REVIEW D **96**, 093002 (2017)

Parapositronium can decay into three photons

Andrzej Pokraka^{*} and Andrzej Czarnecki[†]

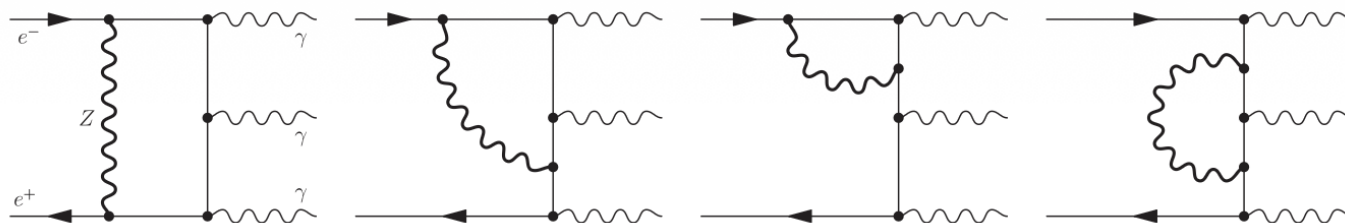


Extra $1/m_W$ factors
needed to build
a gauge-invariant
momentum structure

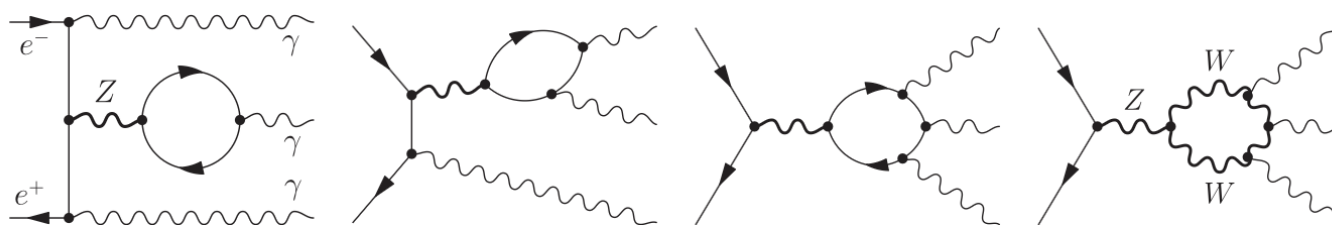
$$\frac{\Gamma(p\text{-Ps} \rightarrow 3\gamma; W\text{-loops only})}{\Gamma(p\text{-Ps} \rightarrow 2\gamma)} \approx 4.4 \times 10^{-77}$$

10^{50} times smaller than previously estimated

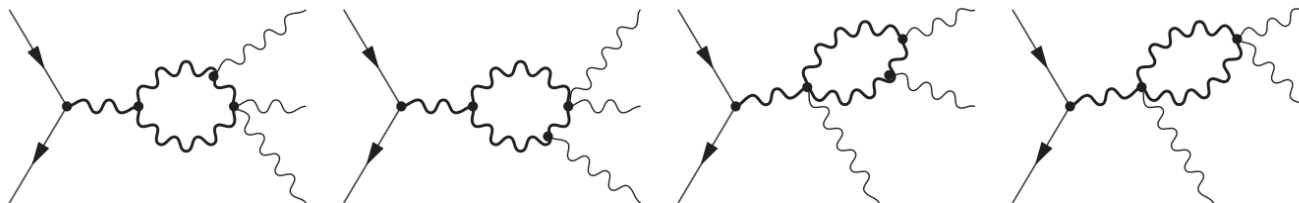
Z-boson contribution to pPs $\rightarrow 3\gamma$



contribute



vanish



PHYSICAL REVIEW D **113**, 073002 (2026)

Parapositronium decay into three photons and implications for the neutral pion

Andrzej Czarnecki^{Ⓜ*}, Divyesh Dagia,[†] Ting Gao,[‡] and Ripanjeet Toor[§]

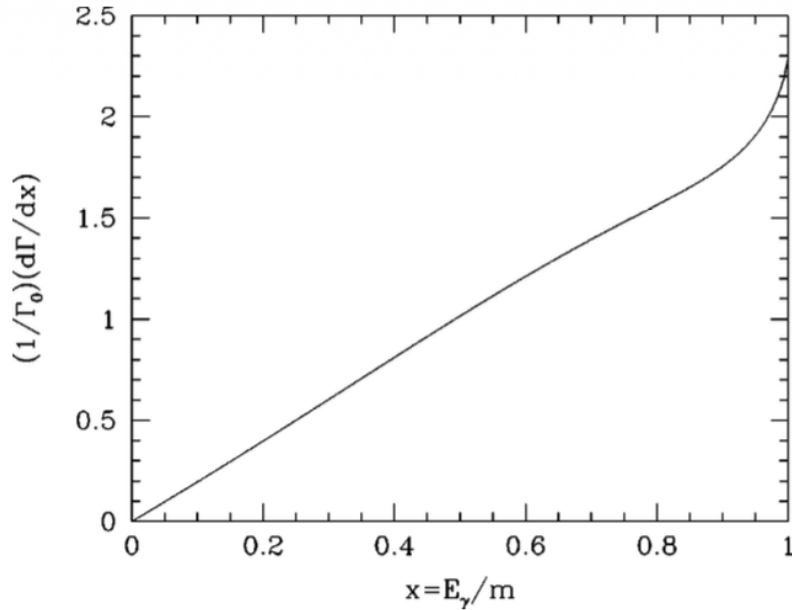
Two mass scales in Z-diagrams: m_Z (hard) and m_e (soft)

- To emit three photons requires four powers of photon momenta (for gauge invariance)
- The **hard** contribution like in W-diagrams, suppressed by $1/m_Z^6$
- The **soft** contribution, for less clear reasons, has no $1/m_Z^2$, $1/m_Z^4$; also only $1/m_Z^6$
- Amplitude enhanced only by $\ln(m_Z/m_e)$ relative to W

$$\begin{aligned} \text{BR}(p\text{-Ps} \rightarrow 3\gamma) &\simeq \frac{\alpha G_F^2 m_e^{12}}{m_W^8} \frac{14700\pi^2 - 145081}{1428840000\pi^5} \\ &\quad \times \left[7 + 30 \cos^4\theta_W (1 - 4 \sin^2\theta_W) \ln \frac{m_Z^2}{m_e^2} \right]^2 \\ &= 2.6 \times 10^{-75} \quad \text{Smaller by } 10^{47} \text{ than estimated} \\ &\quad \text{by Nachtmann+Bernreuther} \end{aligned}$$

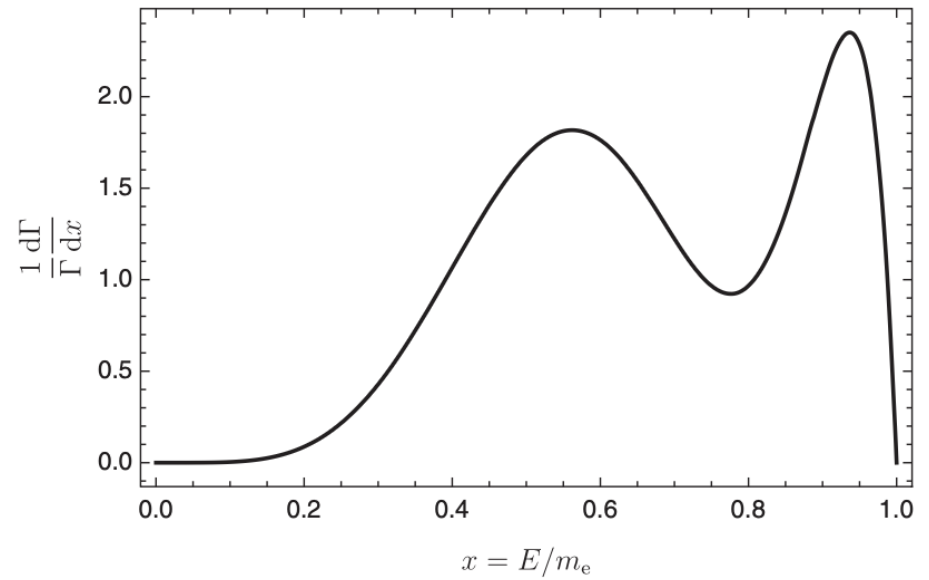
Spectra of 3-photon positronium decays

ortho-Ps



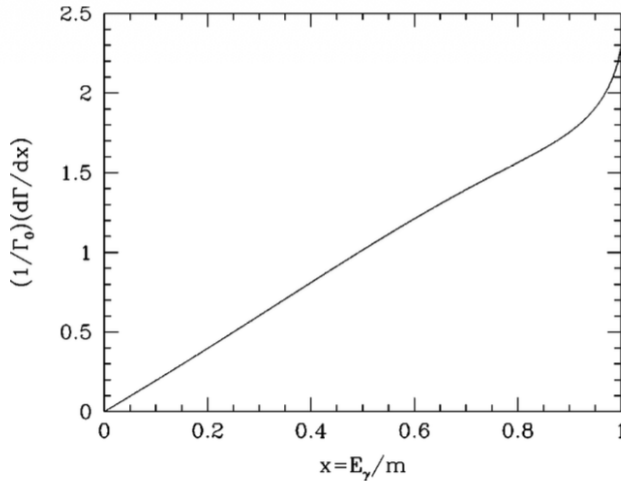
Ore+Powell,
Manohar+Ruiz-Femenia
Voloshin

para-Ps



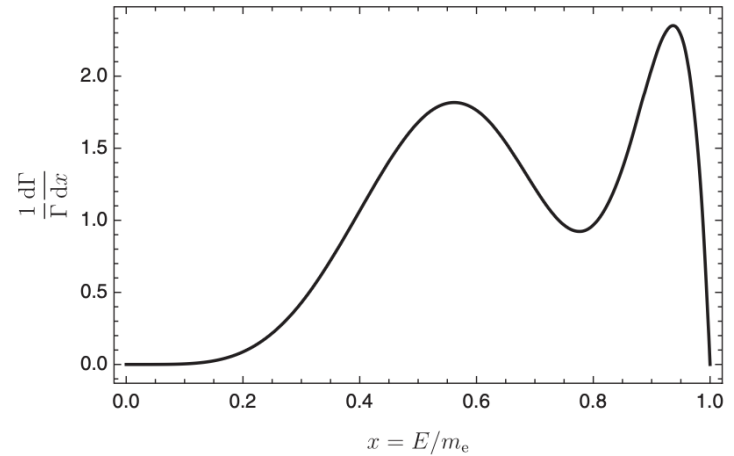
Pokraka+AC

ortho-Ps



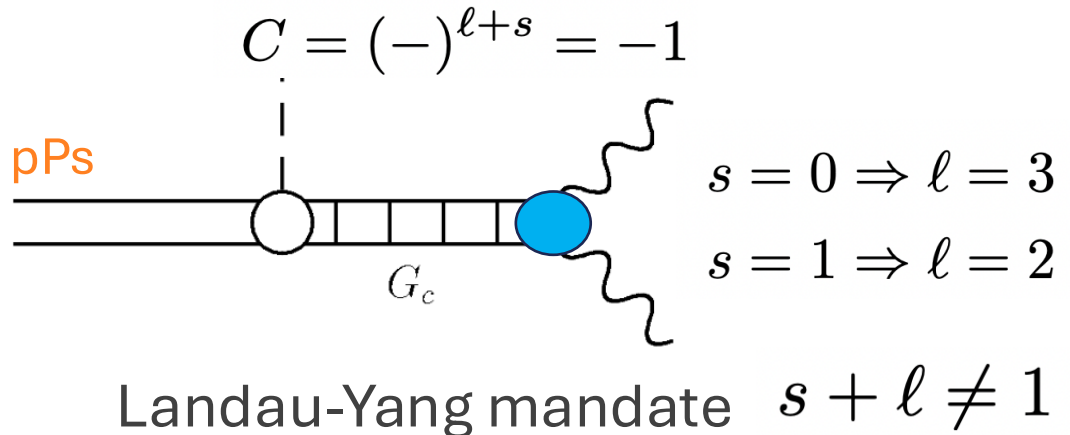
Ore+Powell,
Manohar+Ruiz-Femenia
Voloshin

para-Ps

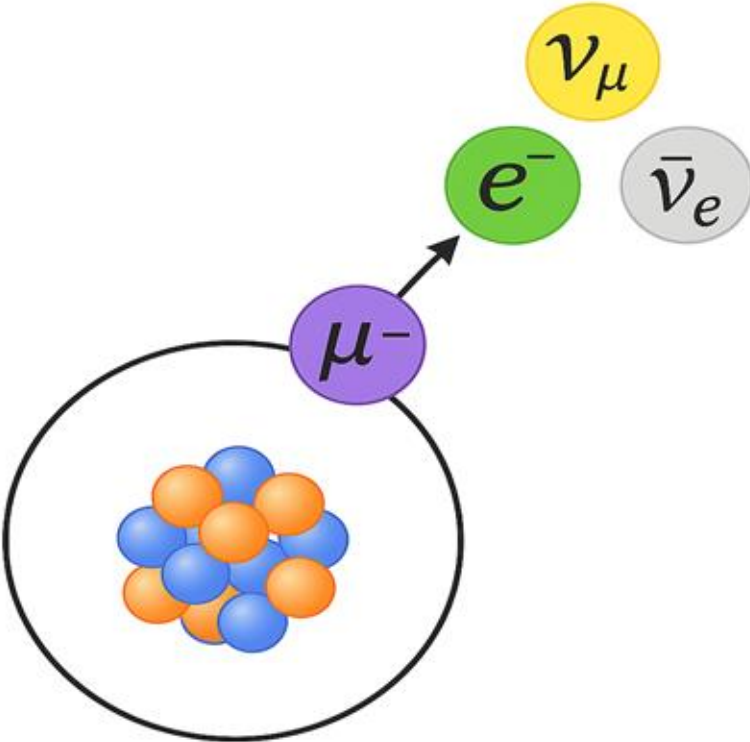


Pokraka+AC

For very soft photons,
binding effects
suppress the spectrum
even more

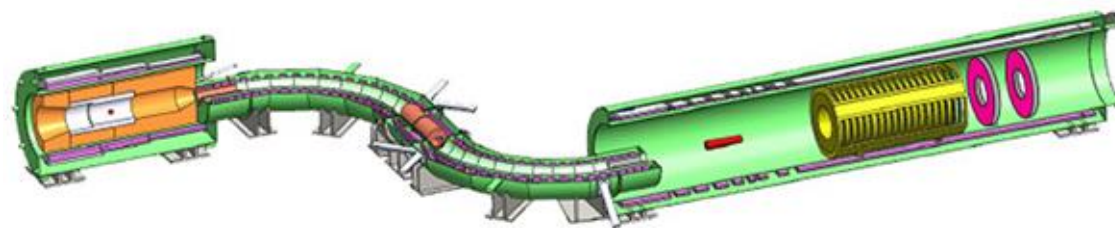


Muonic atoms

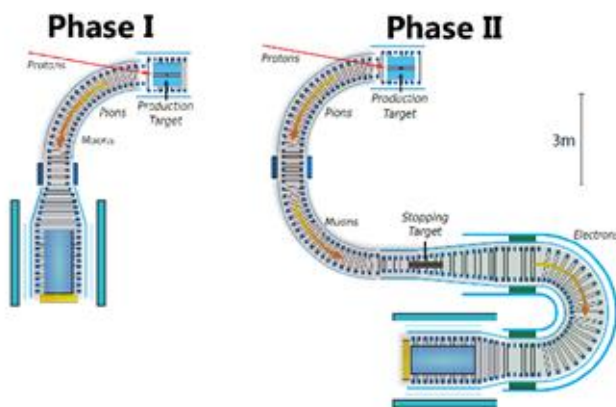


Bound muon decay: why do we care?

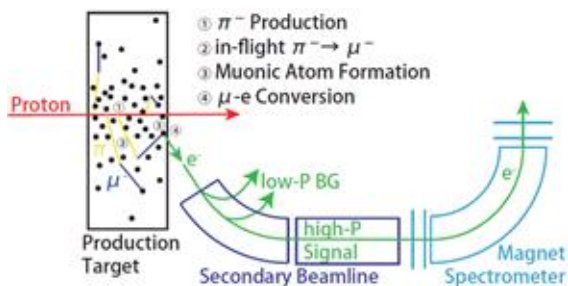
Muon-electron conversion searches: starting 2027 (hopefully)



Mu2e
Fermilab

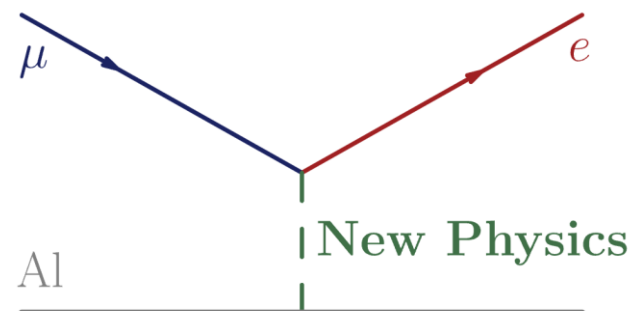


COMET
J-PARC

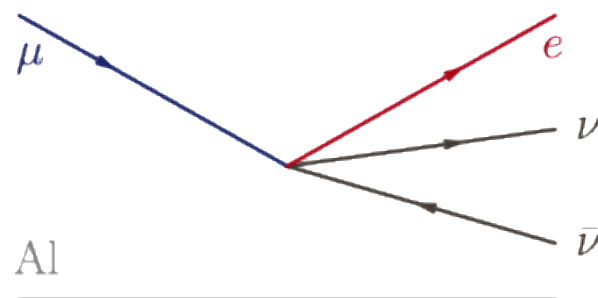


DeeMe
J-PARC

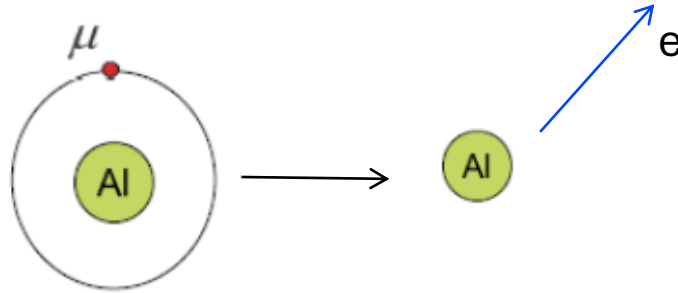
searching for:



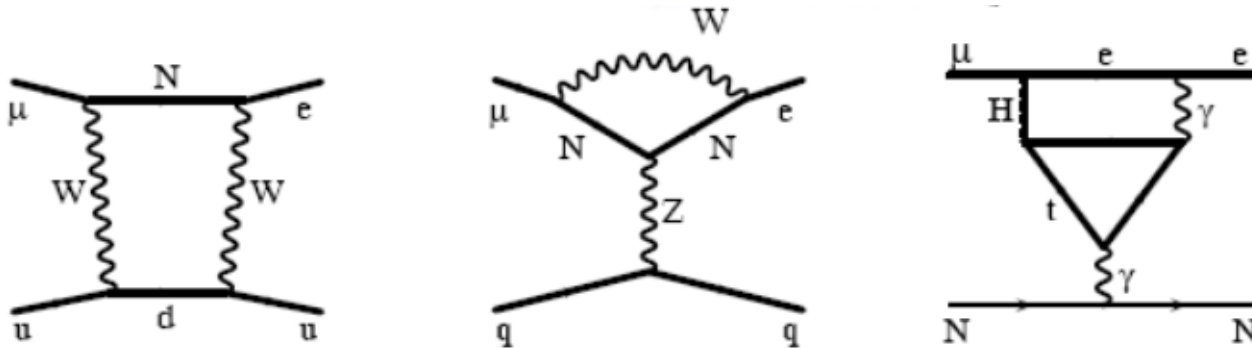
background:



A really rare process: muon-electron conversion



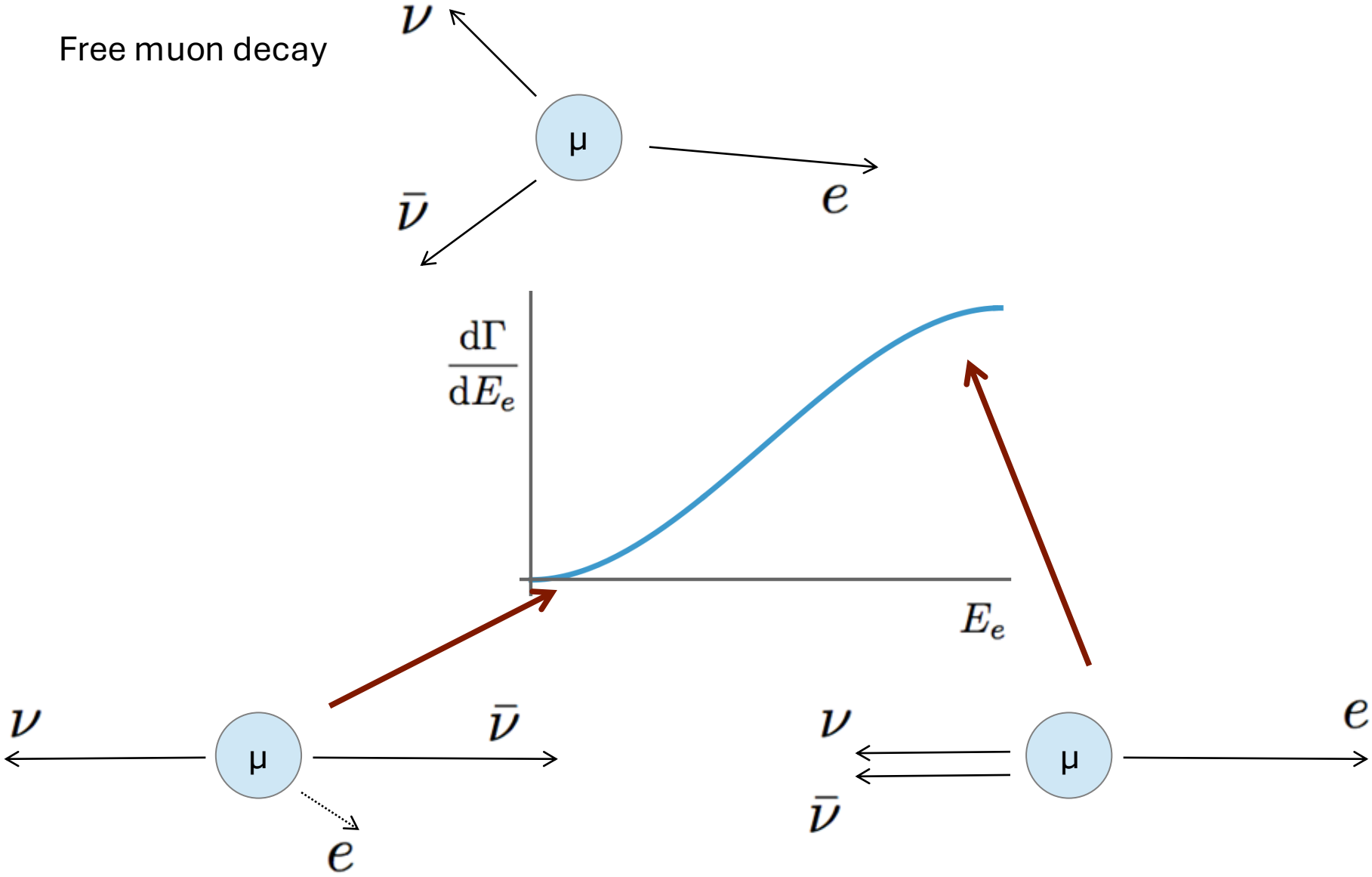
Variety of mechanisms:



Background: high energy electrons from the bound-muon decay.

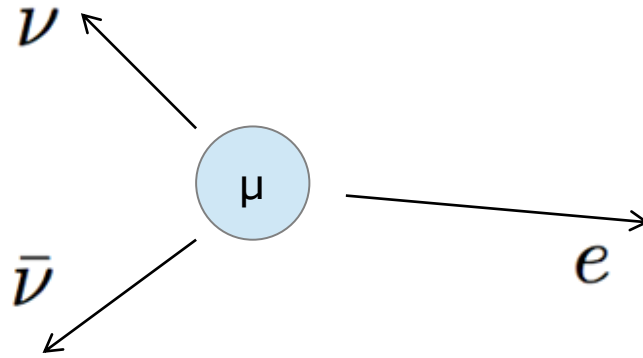
Muon decay: electron energy spectrum

Free muon decay

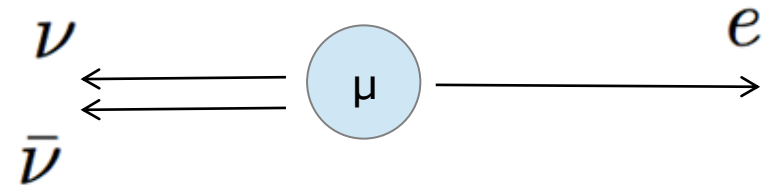
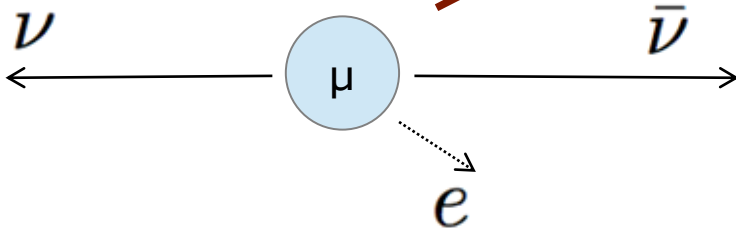
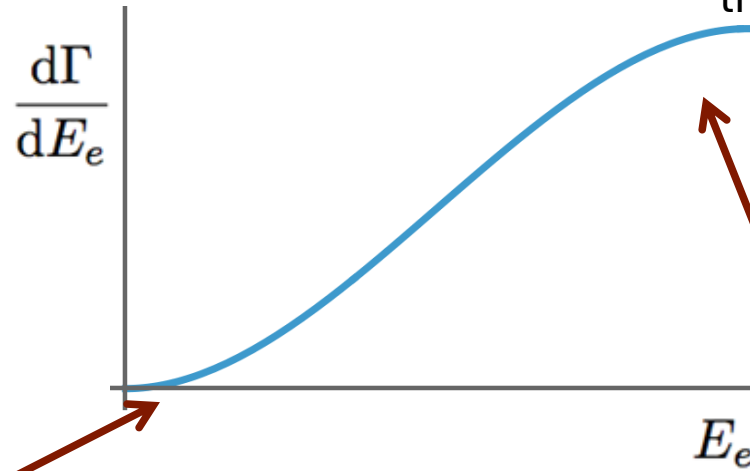


Muon decay: electron energy spectrum

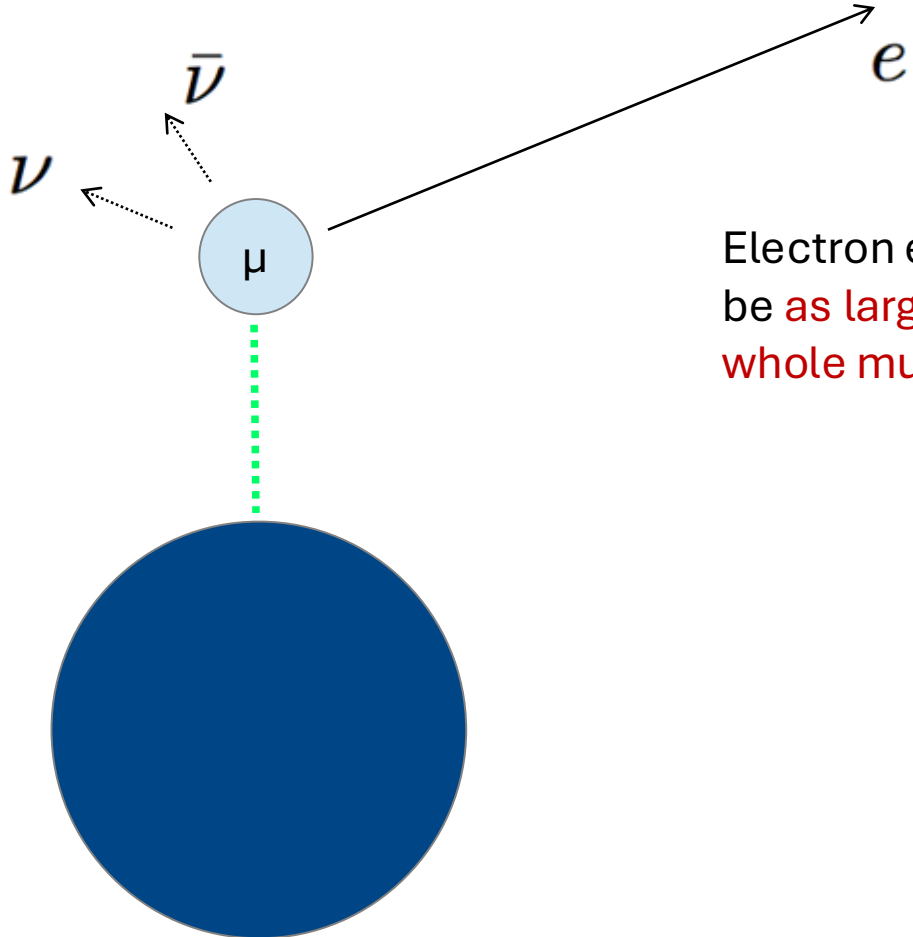
Free muon decay



Maximum electron energy:
half the muon mass;
the other half: neutrinos.

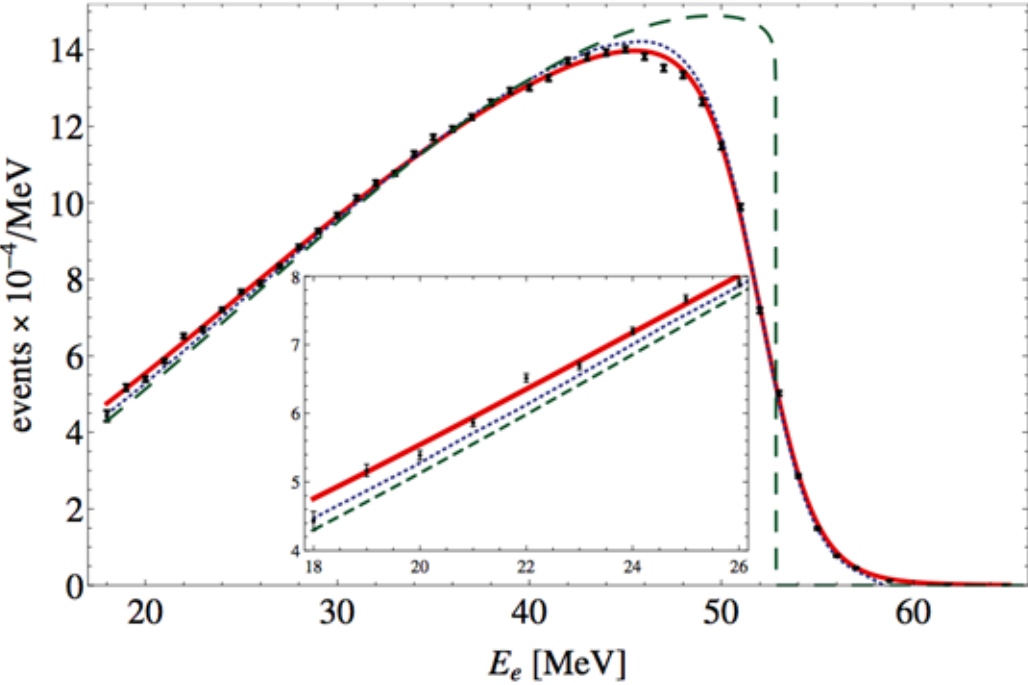


Electron spectrum in a mu-decay near nucleus

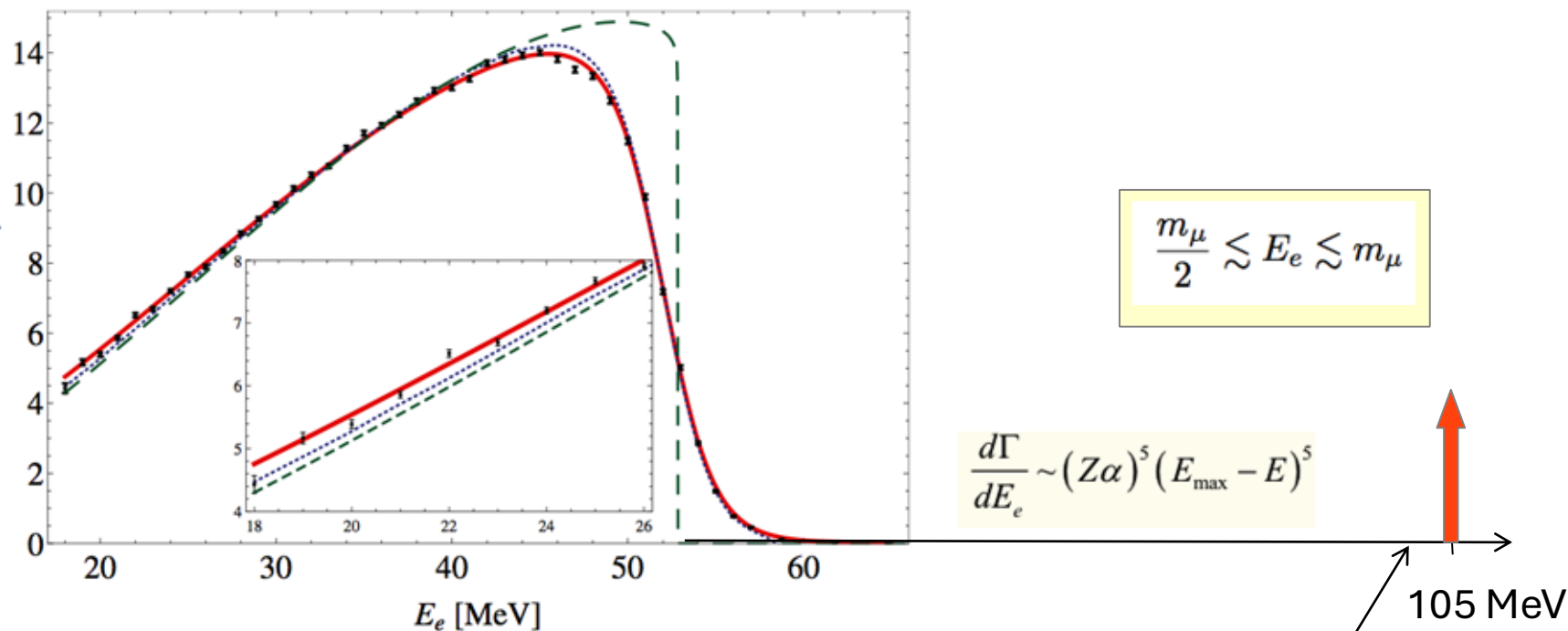


Electron energy can be as large as the whole muon rest energy

Bound muon decay has been measured: TWIST/TRIUMF

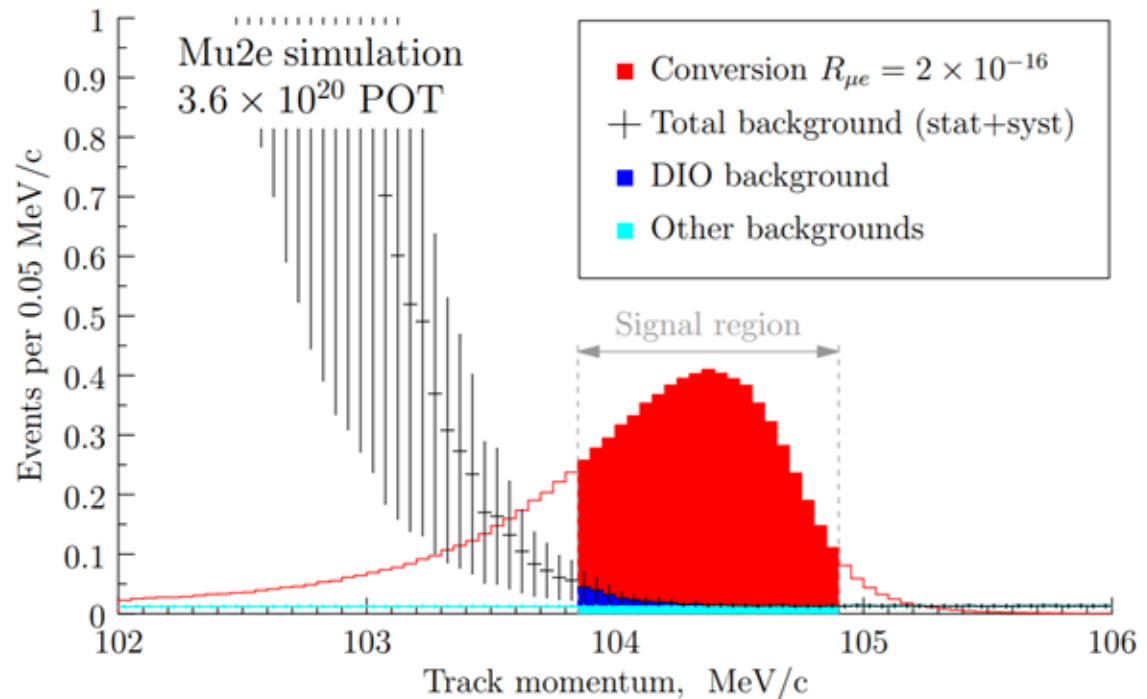
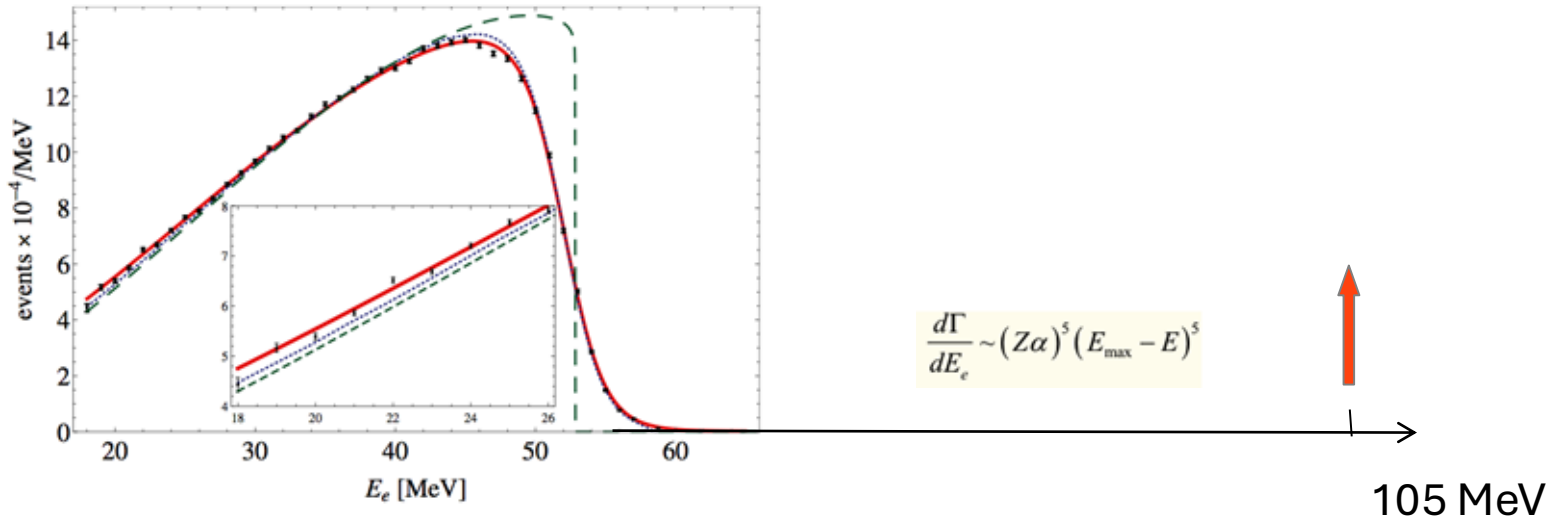


What about the upper half of the spectrum?

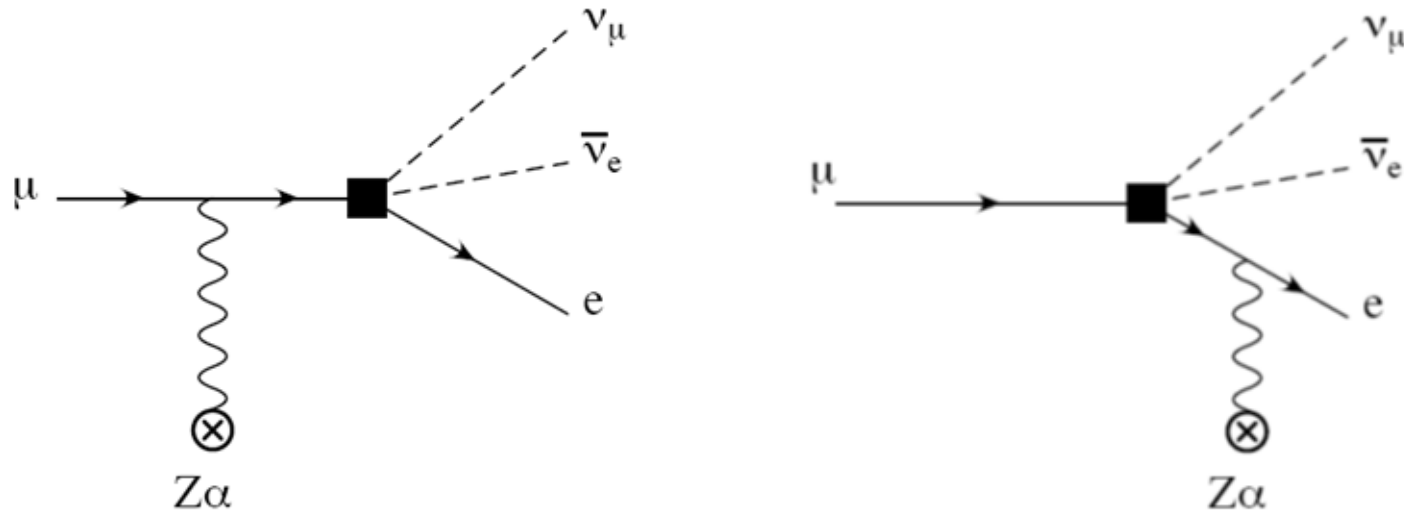


It is the main background for the expected conversion signal

What about the upper half of the spectrum?



Origin of the $(Z\alpha)^5$ and $(E_{\max} - E)^5$ suppressions



Neutrinos get no energy;

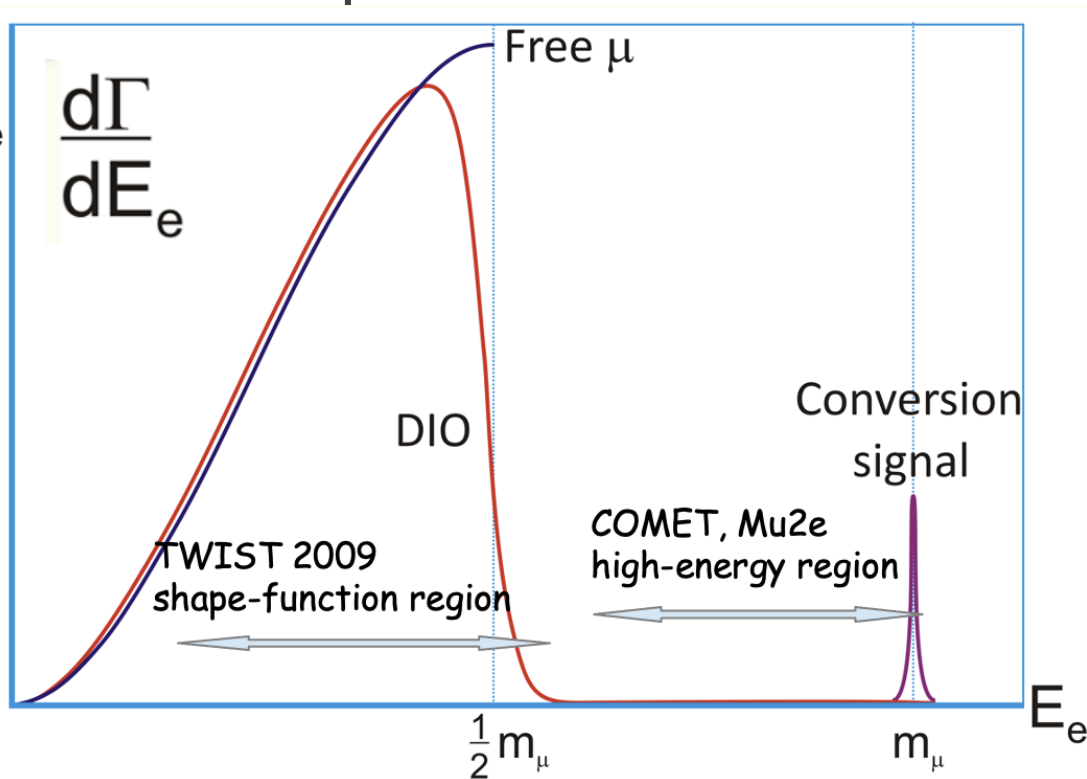
The nucleus balances electron's momentum, takes no energy.

Near the end point:

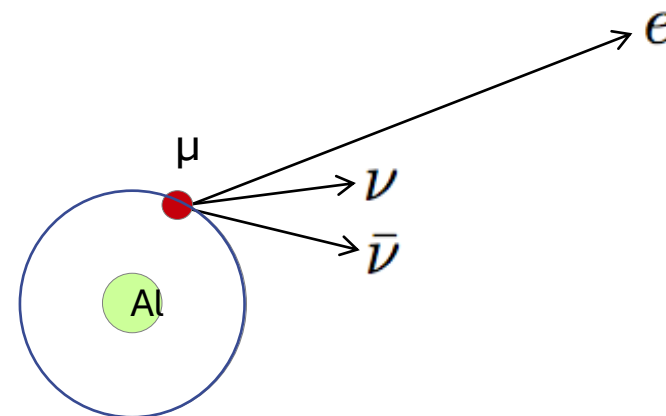
$$\begin{aligned} \frac{d\Gamma}{dE_e} &\sim |\psi(0)|^2 (Z\alpha)^2 \frac{d^3\nu_e}{\nu_e} \frac{d^3\nu_\mu}{\nu_\mu} \delta(E_{\max} - E_e - \nu_e - \nu_\mu) \text{Tr} \dots \psi_e \dots \psi_\mu \\ &\sim (Z\alpha)^5 (E_{\max} - E_e)^5 \end{aligned}$$

Bound muons: total decay-in-orbit rate

Binding effects distort the electron spectrum

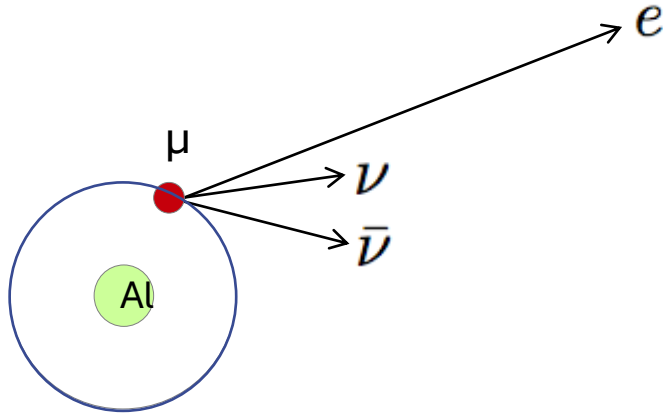


but largely cancel in the total rate



$$\frac{\Gamma}{\Gamma_0} = 1 - \frac{(\alpha Z)^2}{2} + \mathcal{O}(\alpha Z)^3.$$

How to find the total decay rate



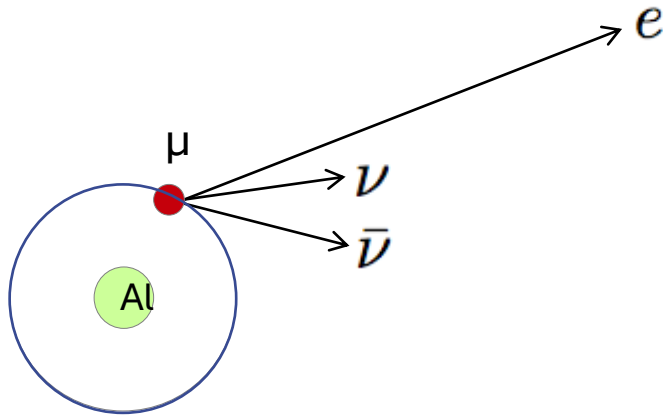
$$|\mathcal{M}_{(Z\mu^-) \rightarrow [Ze^-]A}|^2 = \frac{1}{2} \sum_{\mu_\mu} \sum_{\kappa_e \mu_e} \frac{g^2}{2} J^\alpha(\vec{k}) J^{\dagger\beta}(\vec{k}) \times \left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_A^2} \right),$$

$$\Gamma((Z\mu^-) \rightarrow [Ze^-]A) = \frac{1}{64\pi^5} \int \frac{d^3\vec{p}_e}{p_e E_e} \frac{d^3\vec{k}}{k_0} \delta(E_\mu - E_e - k_0) |\mathcal{M}_{(Z\mu^-) \rightarrow [Ze^-]A}|^2$$

$$\Psi_{\mu_\mu}(\vec{r}) = \frac{1}{r} \begin{pmatrix} G(r) \Omega_{-1\mu_\mu}(\hat{r}) \\ iF(r) \Omega_{1\mu_\mu}(\hat{r}) \end{pmatrix} \quad \Psi_{E_e \kappa_e \mu_e}(\vec{r}) = \frac{1}{r} \begin{pmatrix} g_{E_e \kappa_e}(r) \Omega_{\kappa_e \mu_e}(\hat{r}) \\ i f_{E_e \kappa_e}(r) \Omega_{-\kappa_e \mu_e}(\hat{r}) \end{pmatrix}$$

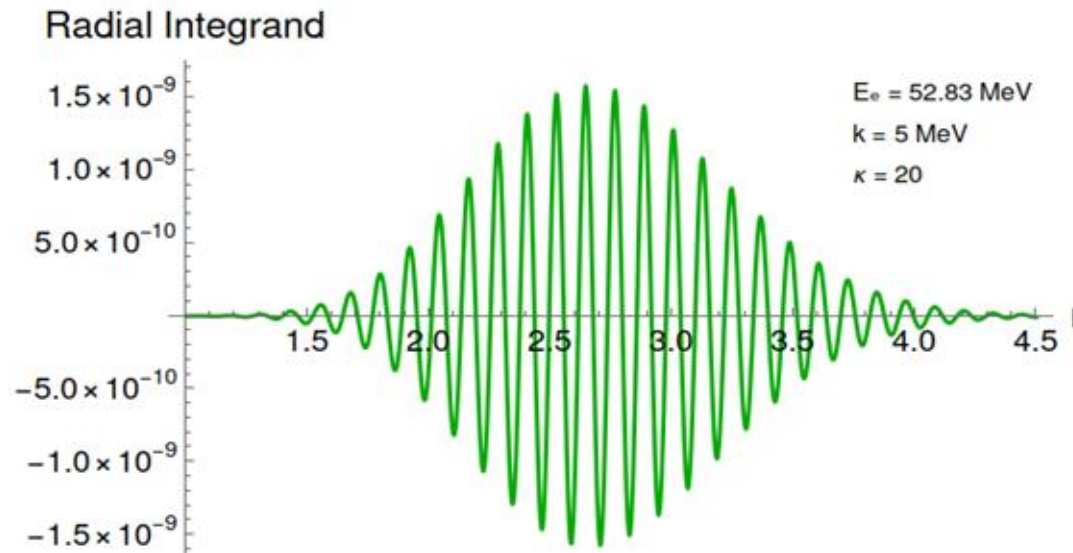
$$\frac{\Gamma}{\Gamma_0} = \sum_{L \kappa_e} \frac{8}{m_\mu^5} (2j_e + 1) \int_0^{E_\mu} dE_e \int_0^{E_\mu - E_e} k^2 dk \left\{ [(E_\mu - E_e)^2 - k^2] \left(\frac{|S_{\kappa_e L}^0|^2}{L(L+1)} + \frac{|S_{\kappa_e L}^{-1}|^2}{L(2L+1)} + \frac{|S_{\kappa_e L}^{+1}|^2}{(L+1)(2L+1)} \right) + k^2 \left(|S_{\kappa_e L}|^2 + \frac{|S_{\kappa_e L}^{+1} + S_{\kappa_e L}^{-1}|^2}{(2L+1)^2} \right) + 2(E_\mu - E_e) k \operatorname{Im} \left[\frac{S_{\kappa_e L} (S_{\kappa_e L}^{-1*} + S_{\kappa_e L}^{+1*})}{2L+1} \right] \right\} \equiv \sum_{\kappa_e} \frac{\Gamma_{\kappa_e}}{\Gamma_0}$$

How to find the total decay rate



$$|\mathcal{M}_{(Z\mu^-) \rightarrow [Ze^-]A}|^2 = \frac{1}{2} \sum_{\mu_\mu} \sum_{\kappa_e \mu_e} \frac{g^2}{2} J^\alpha(\vec{k}) J^{\dagger\beta}(\vec{k}) \times \left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_A^2} \right),$$

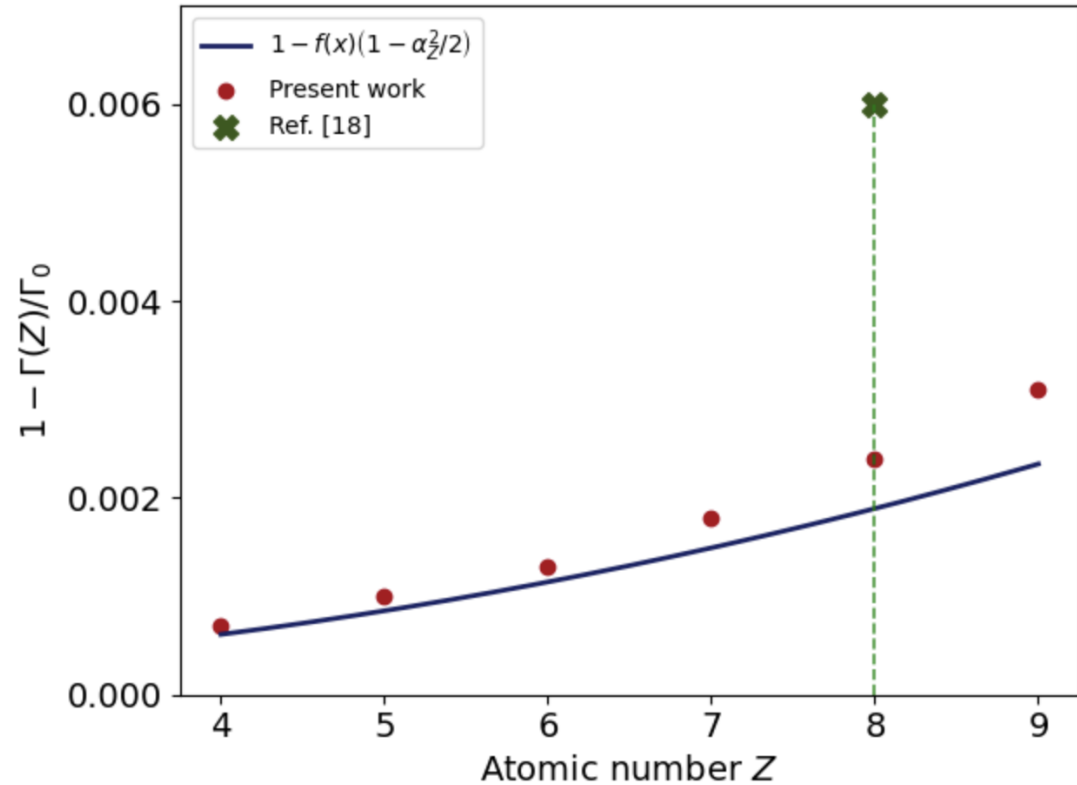
$$\Gamma((Z\mu^-) \rightarrow [Ze^-]A) = \frac{1}{64\pi^5} \int \frac{d^3\vec{p}_e}{p_e E_e} \frac{d^3\vec{k}}{k_0} \delta(E_\mu - E_e - k_0) |\mathcal{M}_{(Z\mu^-) \rightarrow [Ze^-]A}|^2$$



Bound muons: total decay-in-orbit rate

Numerical calculation pioneered by Watanabe, Fukui, Ohtsubo, Morita 1987-1993.

However, puzzling discrepancy for a light oxygen nucleus $Z=8$.



$$|\mathcal{M}_{(Z\mu^-) \rightarrow [Ze^-]A}|^2 = \frac{1}{2} \sum_{\mu_\mu} \sum_{\kappa_e \mu_e} \frac{g^2}{2} J^\alpha(\vec{k}) J^{\dagger\beta}(\vec{k}) \times \left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_A^2} \right),$$

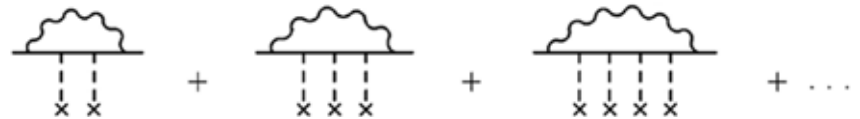
sum over electron angular momenta: numerically challenging

Bound decay rate as an atomic-like observable

Example 1: Lamb shift for an electron in the hydrogen ground state

$$E \simeq m - \frac{m(Z\alpha)^2}{2}$$

Dirac eq $\longrightarrow m\sqrt{1 - (Z\alpha)^2}$



$$+ \frac{\alpha}{\pi} \left[(Z\alpha)^4 (\ln Z\alpha + c) + \dots \right]$$

Bethe 1947
Feynman
Schwinger...

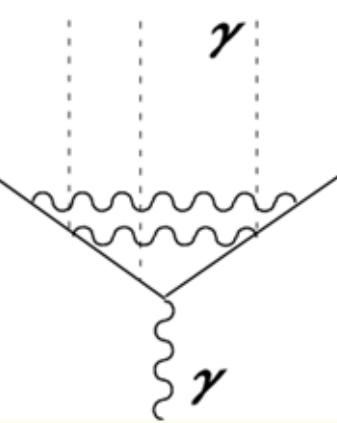
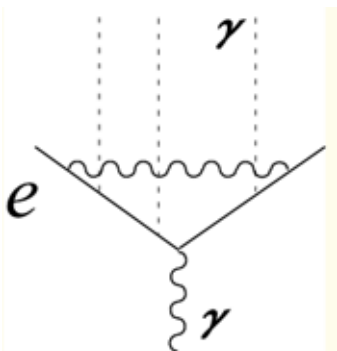
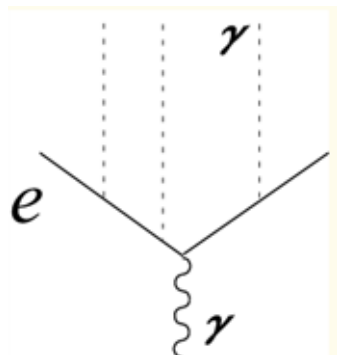
$$+ \left(\frac{\alpha}{\pi}\right)^2 \dots$$

Appelquist+Brodsky
Barbieri, Magnaco, Remiddi 1970

$$+ \left(\frac{\alpha}{\pi}\right)^3 \dots$$

Melnikov + van Ritbergen 1999
Pachucki, Pachucki et al.

Example 2: Bound-electron g-2



$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots$$

$$+ \frac{\alpha}{\pi} \left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 (a_{41} \ln Z\alpha + a_{40}) + \dots \right]$$

$$+ \left(\frac{\alpha}{\pi}\right)^2 \left[-0.65.. \left(1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \dots \right]$$

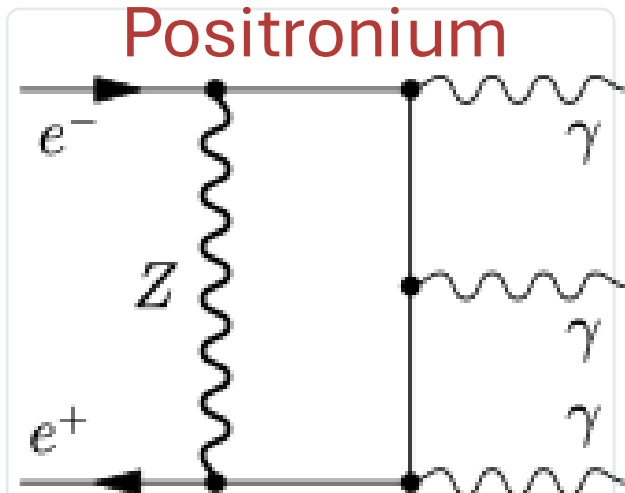
two-loop corrections

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

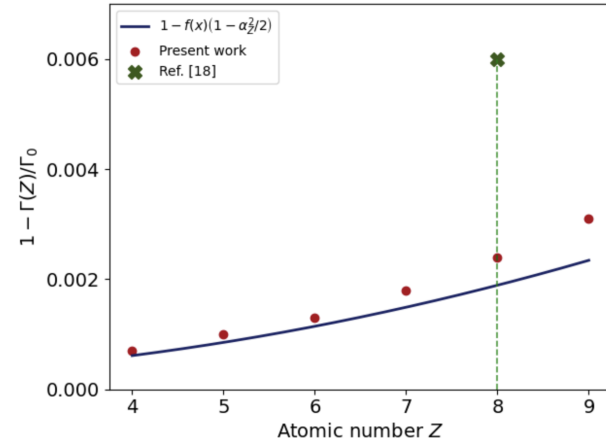
Pachucki,
AC
Jentschura,
Yerokhin

Radiative Corrections in Bound States: summary



C-forbidden channel; full SM rate now complete, 47 orders of magnitude smaller than estimated

Muonic atoms



Bound muon lifetime: old puzzle of oxygen discrepancy resolved. Next step: improve the analytical prediction