Dynamics of vortices
role of internal modes and false vacuum

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contents

- **vortices** topology and motivation

- Abelian Higgs vortices and **internal modes**

  BPS limit and **geodesic dynamics**

  $\rightarrow$ **spectral flow** - flow of the mode structure

  $\rightarrow$ collisions of **excited vortices** and **break down of the geodesic dynamics**

  non-BPS limit

  $\rightarrow$ vortex-antivortex **annihilation**

  $\rightarrow$ **chaos** from **Feshbach resonances**

- global vortices and **role of false vacuum**

  ϕ^4 potential

  $\rightarrow$ **triviality** of vortex-antivortex collisions

  dimension-six operator $\rightarrow$ ϕ^6 potential

  $\rightarrow$ **non-trivial** vortex-antivortex collisions

  $\rightarrow$ importance of **unbroken false vacuum**

  $\rightarrow$ **chaos** from **oscillon**

vortices and topology

vortices

- **planar truncation of (cosmic) strings** in (3+1) dim Vilenkin, Shellard; Hindmarsh, Kibble...

  dynamics of vortices $\Rightarrow$ dynamics of cosmic strings Blanco-Pillado, Hindmarsh....

  understanding of dynamics - fundamental for computation of observables

  $\rightarrow$ abundance of axions (dark matter)

  $\rightarrow$ emission of GW

  $\rightarrow$ primordial BH, relic defects....

- complex scalar field in (2+1) dimensions $\phi(\vec{x}, t) \in \mathbb{C}$

  unbroken (trivial) vacuum

$$\lim_{|\vec{x}| \to \infty} \phi(\vec{x}) = \phi_\infty = 0$$

  broken (nontrivial) vacuum

$$\lim_{|\vec{x}| \to \infty} \phi(\vec{x}) = \phi_\infty \neq 0, \ |\phi_\infty| = v = \text{const.}$$

- topology in the broken vacuum **arbitrary phase**

$$\phi_\infty : \partial \mathbb{R}^2 = \mathbb{S}^1 \ni \theta \to \phi(\theta) = ve^{i\alpha(\theta)} \in \mathbb{S}^1$$

  single-valued if $\alpha(\theta) = n\theta$

  $n \in \pi_1(S^1)$ - degree of the map = **winding number** = topological charge

$$j_\mu = \frac{i}{2}\left(\overline{\phi} D_\mu \phi - \phi \overline{D_\mu \phi}\right), \quad n = \frac{1}{2\pi} \int d^2x \, j^0$$

Abelian Higgs vortices and role of the internal modes

AH vortices Nielsen, Olesen (73)

- $U(1)$ **gauged complex scalar** in (2+1) dimensions $\phi(\vec{x}, t)$, $A_\mu(\vec{x}, t)$

$$S[\phi, A] = \int d^3x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \overline{D_\mu \phi}\, D^\mu \phi - \frac{\lambda}{8} (\bar{\phi}\phi - 1)^2 \right]$$

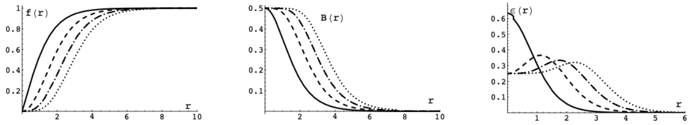
finiteness of the static energy (temporal gauge $A_0 = 0$)

$$\lim_{r\to\infty} \phi(r,\theta) = e^{in\theta}, \quad \lim_{r\to\infty} (A_1(r,\theta), A_2(r,\theta)) = \left(-ie^{-in\theta}\partial_1 e^{in\theta}, -ie^{-in\theta}\partial_2 e^{in\theta}\right)$$

n - again the winding number = magnetic flux $\quad n = \frac{1}{2\pi}\int_{\mathbb{R}^2} d^2x F_{12}$

- **vortex** - a solution with non-zero winding number

static vortices (radial gauge $A_r = 0$) $\qquad \phi(r,\theta) = f_n(r)e^{in\theta}, \quad rA_\theta(r,\theta) = n\beta_n(r)$



- coupling constant λ and interaction Speight (97)
  $\lambda < 1$ vortices **attract** $\to$ unique multi-vortex solution **exist**
  $\lambda > 1$ vortices **repeal** $\to$ multi-vortex solution **do not exist**
  $\lambda = 1$ **no force** between static vortices = **BPS limit** $\to$ infinitely many multi-vortices

BPS limit $\lambda = 1$ Manton (82); Samols (92)

- force balance = **attractive scalar** and **repulsive gauge** forces cancel each other

  $\rightarrow$ BPS multi-vortex solution with constituent **identical** 1-vortices at any position $z_i \in \mathbb{C}$

$$\phi_v(z_1,..,z_n;\vec{x}), \quad \vec{A}_v(z_1,..,z_n;\vec{x}), \quad \rightarrow \quad P_n = (z-z_1)...(z-z_n)$$

  $\rightarrow \mathcal{M}(z_1.,,z_n)$ moduli space of energetically equivalent $E = \pi n$

  $\rightarrow$ to change one solution into another costs arbitrary (zero) energy $\rightarrow$ kinetic motion

  $\rightarrow 2n$ zero modes

  two vortices: $P(z_1,z_2) = (z-z_1)(z-z_2) = z^2 - (z_1+z_2)z + z_1 z_2$

  center of mass $\rightarrow P_{CM}(z) = z^2 + w, \quad w = z_1 z_2,$

  $\dim \mathcal{M}_{CM}(w) = 2$

  $(y \rightarrow -y)$ sym $\rightarrow w \in \mathbb{R} \Rightarrow z_{1,2} = \pm d$ or $z_{1,2} = \pm id, \quad d \in \mathbb{R}_+$

  $\dim \tilde{\mathcal{M}}_{CM}(w) = 1$ **head-on** collision $\Rightarrow 90^\circ$ scattering (movie)

- **geodesic dynamics** = lowest energy dynamics

  passing through energetically equivalent BPS solutions

  promote the moduli $\vec{z}$ to time dependent variables $\vec{z}(t)$

  potential-less dynamics with velocity depended force $\rightarrow$ moduli space metric g

$$L[\vec{z}(t)] = \int d^2x \mathcal{L}(\phi_v(\vec{z};\vec{x}), A_\mu^v \phi(\vec{z};\vec{x})) = \frac{1}{2} g_{ij} \dot{z}^i \dot{z}^j$$

massive modes $\lambda = 1$

- geodesic dynamics = excitation only zero modes
- BPS vortices support **massive bound modes**
  linear perturbation of the static solution

$$\phi(\vec{x}, t) = \phi_V(\vec{x}) + \epsilon\eta(\vec{x})e^{i\omega t}, \quad A_k(\vec{x}, t), = A_k^V(\vec{x}) + \epsilon a_k(\vec{x}, t)e^{i\omega t}$$

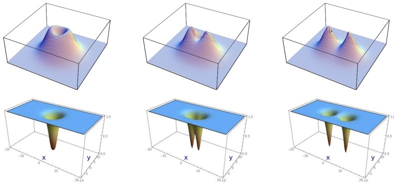
Schrodinger-like problem

$$\left(-\nabla^2 + U_{eff}(\phi_V(\vec{x}), A_k^V(\vec{x}))\right)\Psi = \omega^2\Psi, \qquad \Psi = (a_1, a_2, \phi_1, \phi_2)^T$$

with **mass threshold** $m = 1$ (mass of small fluctuations in the vacuum)

$\rightarrow n = 1$ vortex has one massive mode $\omega_1 = 0.777$

$\rightarrow n = 2$ vort**ices** - only relative distance matters (solution and U_{eff})
Alonso, Manton, Mateos Guilarte, JHEP (24)

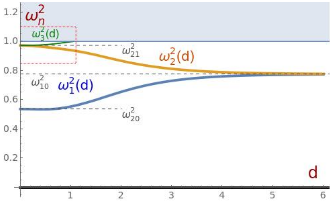


Schrodinger problem depends on the position on the moduli space $\rightarrow$ spectral flow

spectral flow $n = 2$ sector Alonso, Manton, Mateos Guilarte JHEP (24)

- **flow of the modes**

  depends only on the mutual distance $2d$



- **flow of the modes** on the head-on geodesic

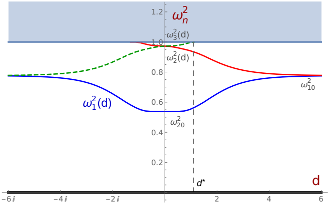
  correct coordinate $d \in \mathbb{R}_+ \cup i\mathbb{R}_-$

  interpolation between

  $\rightarrow$ two modes of the separated 1-vortices $|1\rangle_{sh} \pm |2\rangle_{sh}$

  $\rightarrow$ three modes of the radially symmetric $n = 2$ vortex

  $\rightarrow$ one mode **hits the mass threshold**



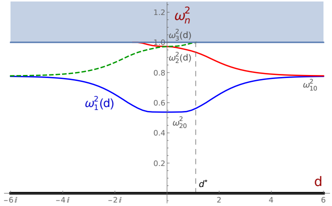
- **potential energy** due to the excited mode $\rightarrow$ **mode generated forces**
  Alonso, Manton, Mateos Guilarte, AW, PRD (24)

$$E_{mod} = \frac{1}{2}\omega(d)C^2$$

dynamics of the excited $n = 2$ vortex

Krush, Rees, Winyard (24) PRD (24)

Alonso, Manton, Mateos Guilarte, AW, PRD (24)



- lower mode

  $\rightarrow$ excitation: in-phase superposition of the shape modes on 1-vortices $|1\rangle_{sh} + |2\rangle_{sh}$

  $\rightarrow$ **attractive** force

  soliton-soliton scattering with **attractive force** and **bound state**....

  multi-bounce windows and chaotic (fractal) structure $\rightarrow$ kink-antikink in ϕ^4

  resonant energy transfer: zero (kinetic) mode + massive (vibrational) mode

- higher mode

  $\rightarrow$ excitation: out-of-phase superposition of the shape modes on 1-vortices $|1\rangle_{sh} - |2\rangle_{sh}$

  $\rightarrow$ mode crossing

  $\rightarrow$ $d \in \mathbb{R}$: **repulsive** force for incoming vortices (on x-axis)

  $d \in i\mathbb{R}$: **attractive** force for vortices after passing the on-top configuration (on y-axis)

  possible scenarios:

  180° scattering with 0,1,2 bounce

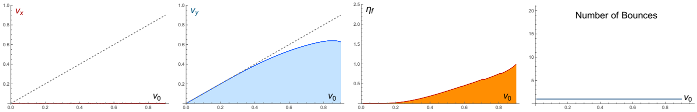
  90° scattering with 1 bounce

  no chaotic multi-bounce structure

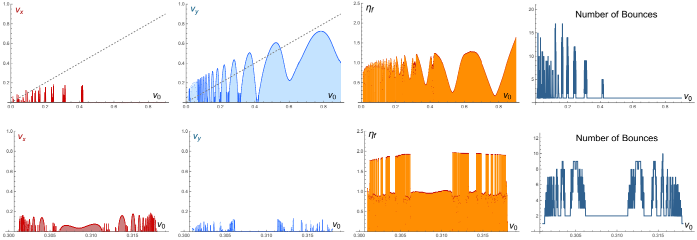
  the mode enters the mass threshold.

dynamics with the lower mode excited $n = 2$ Alonso, Gonzalez-Perra, AW (26)

- unexcited collision $\eta_{in} = 0$



- excited vortices $\eta_{in} = 1.5$



$\rightarrow$ chaotic (fractal) structure

$\rightarrow$ multi-bounce windows

$\rightarrow$ resonant energy transfer kinetic $\leftrightarrow$ internal d.o.f.

$\rightarrow$ excitation of the mode from kinetic motion

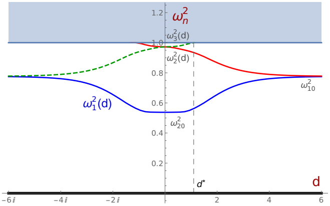
$\rightarrow$ **break down of the geodesic motion** (movie) Alonso, Bachmaier, AW (26)

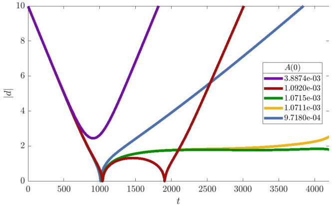
dynamics with the upper mode excited $n = 2$ Alonso, Mateos Guilarte, Rees, AW, PRD (26)

- **spectral wall** Adam, Oles, Romanczukiewicz, AW, PRL (19)

  $\rightarrow$ long-lived stationary state at the BPS configuration (separation) for which the excited mode hits the mass threshold

$v_{in} = 0.01$





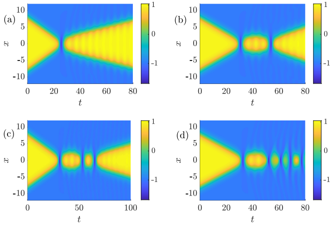
$\rightarrow$ **break down of the geodesic motion** (movie)

vortex-antivortex collision
a strongly non-BPS process

DW-antiDW collisions

- **DW in ϕ^4**

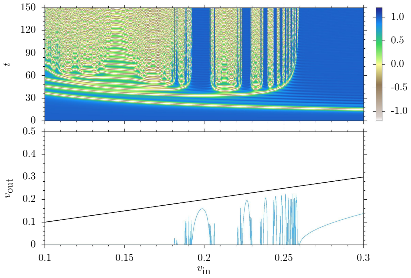
  scenarios
  $\rightarrow$ back-scattering sometimes with a **few bounces**
  $\rightarrow$ annihilation through formation of an **oscillon**



- **chaotic, self-similar** pattern in the final state formation

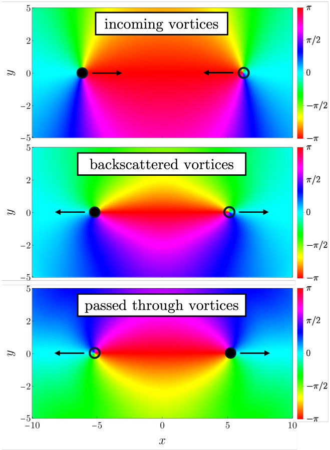
  resonant energy transfer

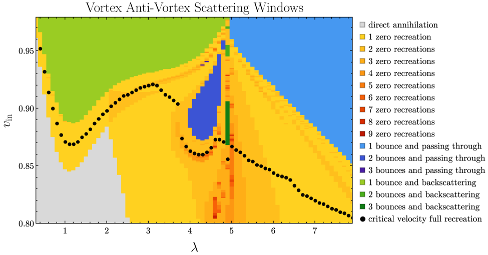
  the bound mode of the kink
  Sygiyama (79;) Cambel, Schonfeld, Wingate (83)
  Manton, Oles, Romanczukiewicz, AW, PRL (21)

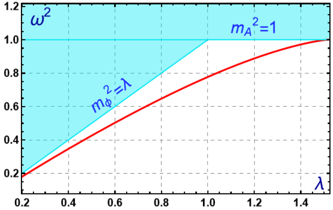


vortex-antivortex collisions for various λ Bachmaier, AW, PLB (26)

- scenarios (movie)
  $\rightarrow$ **re-creation** of the vortices - high velocity
  back-scattering
  passing-through
  $\rightarrow$ **annihilation** of the vortices - smaller velocity
  $\rightarrow$ **bounces**?



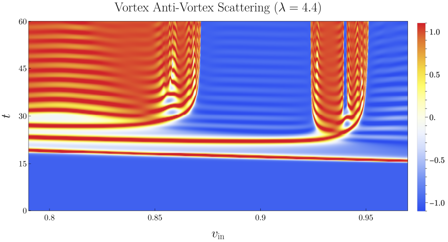


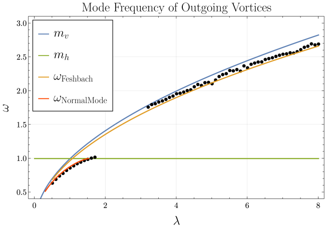


- fractal multi-bounce structure due to the **not due to the bound mode**

Feshbach resonances

- fractal multi-bounce structure due to the **due to the Feshbach resonance**
  $\longrightarrow$ a bound mode in the scalar channel (truncated Schrodinger problem)
  $\longrightarrow$ with frequency above the mass threshold in the gauge channel
  $\longrightarrow$ a quasi-normal mode





global vortices and role of the false vacuum

global vortices $\lambda \to \infty$

- **complex scalar** in (2+1) dimensions $\phi(\vec{x}, t)$,
  the gauge field decouples

$$S[\phi] = \int d^3x \left[\frac{1}{2} \overline{\partial_\mu \phi} \, \partial^\mu \phi - \frac{\lambda}{8} (\bar{\phi} \, \phi - 2)^2 \right]$$

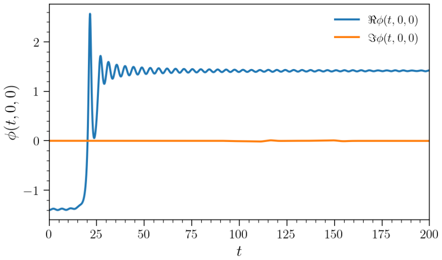
  two real component of the field $\phi = fe^{i\chi}$

- $\phi(r, \theta) = f(r)e^{in\theta}$ global vortices with topological charge n
  $\to$ infinite energy due to the logarithmic divergency at $r \to \infty$
- spectral structure
  - broken vacuum $f = \sqrt{2}$ while $\chi \in \mathbb{S}^1$
  - massive f-channel - mass threshold $m_f = 2$
  - massless χ-channel = Goldstone mode $m_\chi = 0$

  $\to$ **infinitely many bound modes** in the f-channel
  $\to$ only scattering (massless) states in the χ-channel

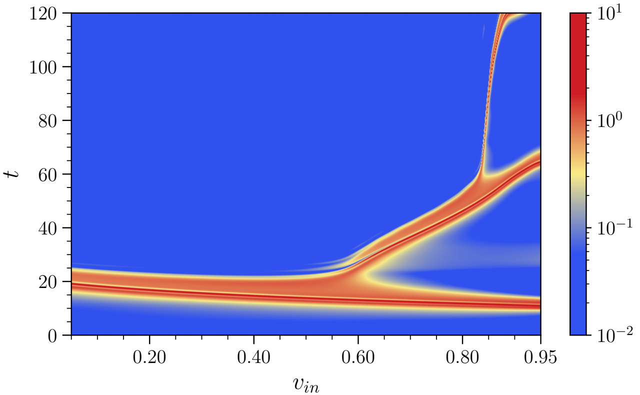
no mass threshold in the full linear perturbation $\Rightarrow$ **Feshbach resonances**
any localized (top. trivial) perturbation disperse and decay to the vacuum $\to$ Goldstone waves

no oscillons



vortex-antivortex collision Canillas, Gonzalez-Parra, Miguelez-Caballero, AW, (26)

- initial state: vortex-antivortex pair at distance $2d = 20$ boosted towards each other v_{in}
  energy density at the origin:



- trivial collisions
  → **direct annihilation**
  → a few bounces at very high v_{in}

global vortices in ϕ^6

- ϕ^6 potential

$$V_6 = \frac{\lambda}{8}(\bar{\phi}\,\phi - 2)^2 |\phi|^2$$

two vacua

$\rightarrow$ **unbroken** $\phi = 0$

$\rightarrow$ **broken** $|\phi| = \sqrt{2}$ as in the ϕ^4 model

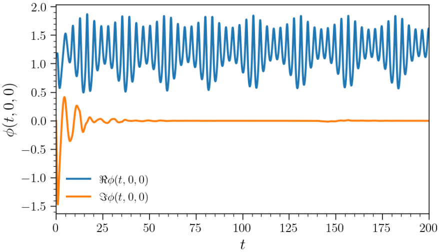
broken vacuum $f = \sqrt{2}$ while $\chi \in \mathbb{S}^1$

massive f-channel - mass threshold $m_f = 2$ (Feshbach)

massless χ-channel = Goldstone mode $m_\chi = 0$

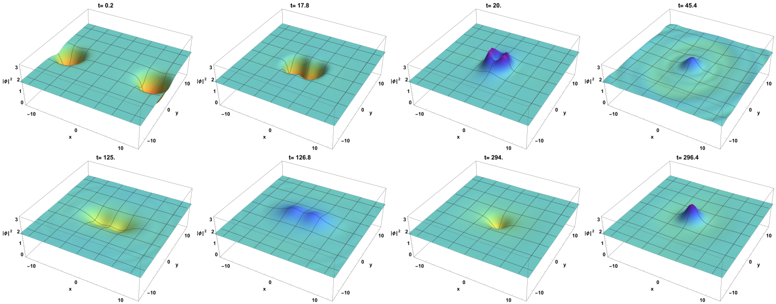
no common mass threshold $\Rightarrow$ no oscillons in the broken vacuum?

there are oscillons!



vortex-antivortex collision in ϕ^6

- initial state: vortex-antivortex pair at distance $2d = 20$ boosted towards each other $v_{in} = 0$



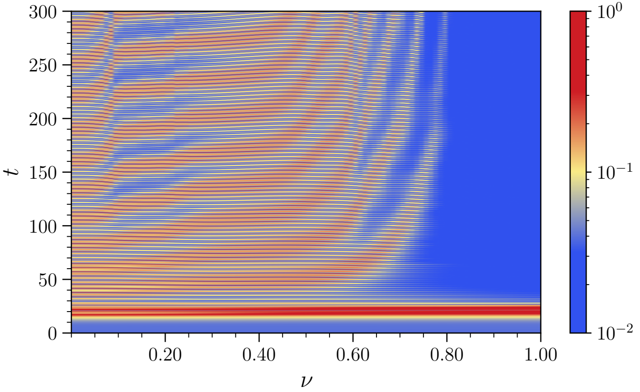
- formation of a long-lived localized structure = **oscillon** (movie)

false vacuum

- inclusion of the dimension-six operator (low energy limit of a new sector **SMEFT**)

$$V(|\phi|) = \frac{1}{8(1+\nu^2)}\left(\nu^2 + |\phi|^2\right)\left(|\phi|^2 - 2\right)^2$$

false vacuum for $\nu < 1$



- **remote false vacuum** responsible for the existence of the **oscillon** in the **true broken vacuum** where no oscillons are expected
- **false vacuum** determines the complexity of the vortex-antivortex collisions
- **changes the medium- long-time scale energy budget in the evolution of the cosmic strings**

summary

- **internal modes** crucial for understanding dynamics of vortices

  local vortices in the BPS limit
  $\longrightarrow$ **mode-generated force** due to massive discrete normal modes
  $\longrightarrow$ **chaos** & **multi-bounces**
  $\longrightarrow$ **spectral wall** due to the transition of the mode to the continuum

  $\longrightarrow$ **break down of the geodesic dynamics**

  non-BPS processes $\longrightarrow$ vortex-antivortex **annihilation**
  $\longrightarrow$ **chaos** & **multi-bounces**
  $\longrightarrow$ from **Feshbach resonances**

- **false vacuum**
  remote modification of the potential significantly change the dynamics

  global vortex model
  $\longrightarrow$ **oscillon** in the broken vacuum due to the remote false vacuum
  $\longrightarrow$ **oscillon survives quantization** Evslin, Romanczukiewicz, Slawinksa, AW (26)
  dimension-six operator and **SMEFT**

  vortex-antivortex collisions
  $\longrightarrow$ **chaos** from **oscillon**

- **impact on cosmic strings evolution and appearance of oscillons**