Hydrodynamics and Elasticity
Test II
16 December 2025

Each problem is worth 10 points. Please sign every sheet of paper used for your solutions.

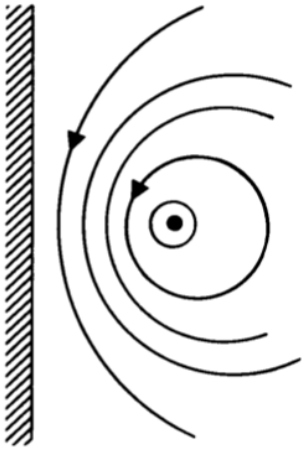
1. A simple model of a tornado is the two-dimensional Rankine vortex with the following velocity field

$$v_\theta = \begin{cases} \dfrac{\kappa r}{a^2} & \text{for } r < a, \\ \dfrac{\kappa}{r} & \text{for } r > a, \end{cases}$$
$$v_r = 0,$$

where a is the radius of the vortex core.
(a) Find the total vorticity $\Gamma(a) = \int \boldsymbol{\omega} \cdot \mathrm{d}\boldsymbol{S}$, for such a vortex as a function of a.
(b) Consider an inviscid and incompressible fluid of density ρ. Assuming the pressure far away from the core to be p_0, find the pressure distribution $p(r, \theta)$ in the whole space. What is the relation between the pressure p_0 at infinity and the pressure in the middle of the vortex?

2. Acceleration can be measured using a curved pipe shaped like a segment of a circle (see drawing). Inside such a water filled pipe there is an air bubble. Relate the position of the bubble to the horizontal acceleration of the pipe. The gravitational acceleration is g.

3. An inviscid and incompressible fluid occupies the region $x \geqslant 0$ bounded by a planar rigid wall at $x = 0$. At a point $(d, 0)$ there is a linear vortex of circulation Γ.
(a) Assuming that the vortex is fixed, find the complex potential which describes this situation. Find the streamfunction and describe the corresponding streamlines.
(b) Assume that the vortex is released and that it moves at the local ambient flow at its centre, find the velocity at which the vortex will move and describe its trajectory.



Good luck! *Rafał Błaszkiewicz, Maciej Lisicki & Piotr Szymczak*