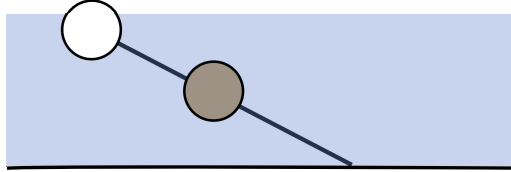


# Hydrodynamics and Elasticity 2025/2026

## Sheet 6

One of the problems will be handed in and marked.

**Problem 1** Two spheres of equal size, one light and the other heavy, are attached to a thin rod, with the heavy sphere fixed at the middle of the rod and the light sphere at one of its ends. After immersion in not very deep water, the free end of the rod rests on the bottom. The rod itself is inclined, and only a part of the light sphere protrudes above the water, such that the ratio of the volume of the emerged part to the volume of the entire sphere is equal to  $n$ . Will this system float or sink when it is placed in deep water? The masses of the light sphere and of the rod should be regarded as negligibly small.



**Problem 2** Two identical homogeneous spheres of mass  $M$  and radius  $a$  are situated a distance  $D \gg a$  apart in a barotropic fluid. Due to their field of gravity, the fluid will be denser near the spheres. There is no other gravitational field present, the fluid density is  $\rho_0$ , and the pressure is  $p_0$  in the absence of the spheres. One may assume that the pressure corrections due to the spheres are small everywhere in comparison with  $p_0$ . (a) Show that the spheres will repel each other and calculate the magnitude of the force to leading order in  $a/D$ . (b) Compare the result with the gravitational attraction between the spheres. (c) Under which conditions will the total force between the spheres vanish?

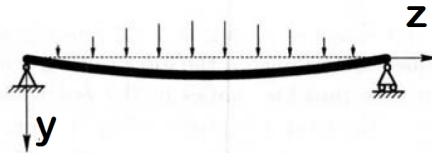
**Problem 3 Roof collapse problem:** A simply supported<sup>†</sup> beam of length  $l$  and flexural rigidity  $EI$  has, in the absence of load, a small initial (natural, stress-free) deflection

$$w_0(x) = \eta_0 \sin \frac{\pi x}{l}, \quad 0 \leq x \leq l,$$

with  $\eta_0 > 0$  small. When rainwater collects in the trough formed by the deflected beam, it exerts a transverse distributed load proportional to the total deflection,  $w(x)$

$$q(x) = \alpha w(x),$$

where  $\alpha > 0$  is a constant. Let  $w(x) = w_0(x) + \eta(x)$ , where  $\eta(x)$  is the additional deflection relative to the natural shape, and assume that only  $\eta$  produces bending moments. Find the limiting value of  $\alpha$  above which the beam becomes unstable under the water load, and explain briefly what this means physically for a slightly sagging roof during heavy rain.



<sup>†</sup> A beam is referred to as simply supported if both its ends are hinged, and one of the hinges can freely slide in the axial direction.