

# Hydrodynamics and Elasticity 2025/2026

## Sheet 7

One of the problems will be handed in and marked. If sending solutions over e-mail, please address them to [Agnieszka.Makulska@fuw.edu.pl](mailto:Agnieszka.Makulska@fuw.edu.pl)

**Problem 1** Consider a model of a universe, which is a generalization of the model analyzed in class, in which, except for an ordinary matter, there is also dark energy of density  $\rho_0$ . We assume that this density is constant in time and homogeneous in space. In this case the formula for energy, derived in class will take the form

$$E = \frac{1}{2}\dot{a}^2 - G\frac{M + M_0}{a} = \frac{4\pi}{3}Ga^2(\rho_c - (\rho + \rho_0)),$$

where  $M_0 = (4/3)\pi\rho_0 a^3$  and  $\rho$  is the density of an ordinary matter (which obeys the continuity equation). Finally,  $\rho_c$  is a critical density, as defined in class.

(a) Find the equation describing the time evolution of the energy,  $\dot{E}$ , remembering that  $\rho_c$  is not constant. Next show that the condition of constancy of energy leads to the following dynamic equation

$$\dot{H} + H^2 = -\frac{4\pi}{3}G(\rho - 2\rho_0)$$

(b) Can such a universe be stationary (with time-independent scale factor)? Under which conditions this can happen? How would you interpret the situation in which  $2\rho_0 > \rho$ ?

(c) Next, consider a situation in which  $\rho_0 < \rho_c/3$ . Show that if, at a certain moment of time,  $2\rho_0 < \rho < \rho_c - \rho_0$ , then initially the expansion of the universe will decelerate with time ( $\ddot{a} < 0$ ), but at a certain moment the expansion speed will begin to increase. Recent observations seem to indicate that the cosmic expansion is in fact accelerating and that it may have been decelerating in the past.

**Problem 2** A spherical cavity with radius  $a$  is suddenly formed in an incompressible fluid of density  $\rho$  filling all space. Determine the time taken for the cavity to be filled with fluid. Assume that far from the cavity the pressure is equal to  $p_0$ . (Rayleigh, 1917)

**Problem 3** Show that the Euler equations for fluid motion are invariant under Galilean transformation.

*Rafał Błaszkiwicz, Maciej Lisicki & Piotr Szymczak*