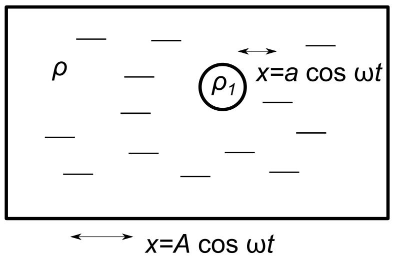
Hydrodynamics and Elasticity 2025/2026

Sheet 9

One of the problems will be handed in and marked. If sending solutions over e-mail, please address them to Agnieszka.Makulska@fuw.edu.pl

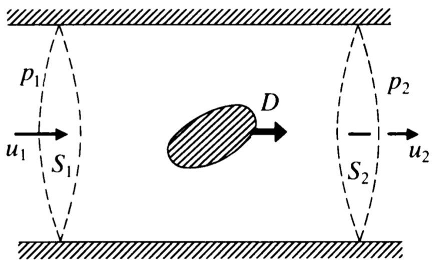
From problems 1A and 1B choose the one that has not been done in the tutorials of your group.

Problem 1A A large container filled with an ideal, incompressible fluid of density ρ is performing sinusoidal oscillations of amplitude A under the action of an external force. Inside the container there is a small bubble of density ρ_1 (see the figure). What is the amplitude of the motion of the bubble? Neglect any gravitational effects.



Problem 1B Consider an array of N identical point vortices, each with circulation Γ, in a 2D incompressible flow of an ideal fluid, equally spaced around a circle of radius a. Show that such a configuration will rotate with a constant angular speed Ω. Find the value of Ω. What will happen if we increase the number of vortices $N \to \infty$ while simultaneously decreasing their strength $\Gamma \to 0$ in such a way that $\Gamma N = C = \text{const}$? Assume that the centre of each vortex moves with a velocity due to all the other vortices at its location.

Problem 2: D'Alembert's paradox Consider the steady flow of an ideal fluid around a 3D body which is placed in a long straight channel of uniform cross-section (see below). The body experiences the drag force D in the downstream direction.



(a) By integrating the Euler equation over an arbitrary fixed region V enclosed by a surface S, show that

$$-\int_S p\boldsymbol{n}\mathrm{d}S = \int_S \rho\boldsymbol{u}(\boldsymbol{u}\cdot\boldsymbol{n})\mathrm{d}S.$$

(b) Apply the obtained equation to the region in the figure. Find the net force in the downstream direction and equate it to the downstream component of the flux of momentum to find

$$D = \int_{S_1}(p_1 + \rho u_1^2)\mathrm{d}S - \int_{S_2}(p_2 + \rho u_2^2)\mathrm{d}S.$$

(c) Now check the assumptions and apply the Bernoulli streamline theorem to a streamline that runs along the channel walls to deduce that $D = 0$.

Problem 3 Show that the problem of irrotational flow around a cylinder of radius R can be reduced to solving the two-dimensional Laplace equation for the velocity potential, which in cylindrical coordinates takes the form

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0$$

subject to the conditions

$$\Phi \sim U r \cos\theta, \quad r \to \infty$$

and

$$\frac{\partial \Phi}{\partial r} = 0 \quad \text{at} \quad r = R,$$

and then solve this equation using the method of separation of variables for the case of vanishing circulation of the velocity field around the cylinder.

Rafal Błaszkiewicz, Maciej Lisicki & Piotr Szymczak