Hydrodynamics and Elasticity 2025/2026

Sheet 10

The general form of differential operators in curvilinear coordinates can be found here:
https://www.fuw.edu.pl/~mklis/hydro2025/operatory.pdf

Problem 1 The pressure drop Δp along a pipe segment of length L, required to transport a mass flux $\rho_0 Q$ in Poiseuille flow, is completely analogous to the voltage drop during the flow of electric current through a conductor. By analogy with Ohm's law, the hydrodynamic resistance of the pipe R is defined as

$$R = \frac{\Delta p}{\rho_0 Q}.$$

Show that the resistance defined in this way is additive when pipes are connected in series, and that the inverses of the resistances are additive when the pipes are connected in parallel.

Problem 2 Find the flow profile for a fluid of constant viscosity μ flowing in an annular space between two concentric pipes of radii R_1 and $R_2 < R_1$ under a constant pressure gradient $G = |\partial p/\partial z|$

Problem 3 *Couette flow.* Consider a stationary flow of a fluid between two coaxial cylinders. The inner cylinder of radius a rotates with angular velocity Ω, while the outer cylinder of radius b is at rest. Show that, for this flow symmetry, the Navier–Stokes equation takes the form

$$-\rho_0 \boldsymbol{e}_r \frac{v_\phi^2}{r} = -\nabla p + \mu \boldsymbol{e}_\phi \frac{d}{dr}\left(\frac{1}{r}\frac{d(r v_\phi)}{dr}\right).$$

Find the general form of the velocity in this flow and, using appropriate boundary conditions, determine the velocity field $\boldsymbol{v}$.

Rafał Błaszkiewicz, Maciej Lisicki & Piotr Szymczak