Maximising the excitement of a roller coaster ride
Problem B

Team 408

An analysis of what constitutes an exciting roller coaster ride was conducted. Obtained insight was then used to design a track of very high excitement value that is also safe and obeys the imposed restrictions. The track presented is composed of modular parts, each imagined to be a curve in 3D space with its coordinates given as functions of a single parameter p. A numerical simulation of a car undertaking the ride was also performed to validate whether the ride was safe and possible to complete under imposed restrictions.

Contents

1 Analysis of excitement

Whether a ride was exciting or not is of course a subjective matter and such "excitement" is by thus by no means a physical quantity. Therefore, the authors' personal experience with roller coasters was the main source of insight into the constituents of a fun ride. A set of properties was universally agreed on that make the ride memorable:

- ⋇ High speeds -
  the feeling of wind against one's face is among the most appealing things a roller coaster has to offer, even, or perhaps even more so, when the passenger is unable to catch a breath because of it(short lasting, hence not a safety concern);
- ⋇ High acceleration -
  being forced into or pulled from one's chair is another of most coveted roller coaster experiences;
- ⋇ High jerk -
  the feeling of being tossed around while actually safe is perhaps not enjoyable in all cases but can definitely prove exciting;
- ⋇ Sights -
  while not the main objective of a ride, seeing the surroundings from up high is definitely fun;
- ⋇ Change in perspective -
  it is not often that one gets to experience Earth (seemingly)crashing down on them or switching places with the sky - the roller coaster has full control over the alignment of passengers and an exciting one should make good use of it;
- ⋇ Weightlessness -
  another rare experience that a ride has the capacity to provide;
- ⋇ Variety -
  a ride comprised only of loops or only going up and down will not exactly be the most thrilling.

An exciting ride will incorporate all those elements while avoiding repetition. This alone gives some clues as to the design of the sought track.

2 Requirements

The problem comes with a constraint on the height our coaster will be able to reach - 30m. As this is the height onto which the coaster will be lifted and no other energy sources are allowed, we cannot possibly go any higher. This also means that the potential energy given by the lift will never be replenished during the ride and the track will have to be designed in such a way so that all hills can be climbed and all loops rode despite the losses of energy due to friction and air resistance.
The ride also has to be safe. That means there must be no way in which the passengers can come in contact with the track or its supports and the g-forces experienced by them, while high, as we are after excitement, must be tolerable by an average person. As evidenced by [4] humans have no trouble withstanding 10g acceleration for 5 - 10 seconds, which is more than necessary for our needs. This also refers to the "eyeballs-out" case, see [4], the one more taxing for the body. For "eyeballs-in" the human tolerance is even greater.

3 Technical specifications

We will analyse a single car, now used interchangeably with coaster, of mass(including passengers)

$$M = 400 \text{ kg}.$$

In our coaster we assume the standard wheels in use today contacting the rails from all sides, Fig.1, so that the rolling friction coefficient between the wheels and the rails is independent of the direction from which the wheels push on the railing. We take this coefficient μ to be constant along the rails and equal to

$$\mu = 0.015, \tag{1}$$

which is representative of modern roller coasters [3].
Our track consists of two rails separated by

$$2R_t = 0.6 \text{ m}.$$

When it comes to air resistance we assume drag to be the only significant factor and take the coaster car's drag coefficient to be about that of normal cars [1], which is

$$C_d \approx 0,3. \tag{2}$$

We approximate the car to have a circular cross-section with the radius of 1 m, which along with (2) and the density of air according to [2],

$$\rho \approx 1.25 \ \frac{\text{kg}}{\text{m}^3},$$

Figure 1: Coaster wheels, source: [3]

gives the aerodynamic force D of

$$D \approx 0.6\upsilon^2$$

with υ being the magnitude of the coaster's velocity at any given time.
As mass is constant, this translates into deceleration d due to drag

$$d = \beta\upsilon^2, \qquad \text{where} \tag{3}$$

$$\beta = 0.0015\ \frac{1}{\text{m}}. \tag{4}$$

We approximate the coaster by a cylinder of already mentioned radius $R = 1$ m. The middle point between the tracks is located the distance R from the axis of the cylinder. So the coaster's moment of inertia I about the axis parallel to the cylinder's and going through that point is

$$I = \frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2} = 600\ [\text{kg} \cdot \text{m}^2] \tag{5}$$

4 Motion along the track

The car's mechanical energy is not constant during the ride, as both friction and air drag are present. We decided to describe the forces acting on the coaster in terms of its instantaneous velocity as well as p by which the track is parametrised. If a section of the track is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix}, p \in [p_b, p_e]$$

then for the velocity vector $\vec{\upsilon}$ we have

$$\vec{\upsilon} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{dx}{dp}\frac{dp}{dt} \\ \frac{dy}{dp}\frac{dp}{dt} \\ \frac{dz}{dp}\frac{dp}{dt} \end{bmatrix} = \dot{p} \begin{bmatrix} \frac{dx}{dp} \\ \frac{dy}{dp} \\ \frac{dz}{dp} \end{bmatrix}.$$

Therefore the gravitational field acts on the coaster with power P_g,

$$P_g = m\vec{g} \cdot \vec{v} = -mg\dot{p}\frac{dz}{dp} = mv\frac{dv}{dt}$$

as we have chosen the xy plane of the Cartesian coordinate system to lie flat on the ground with the z axis pointing upward. g is the magnitude of gravitational acceleration commonly known to be around

$$g = 9,81\ \frac{\mathrm{m}}{\mathrm{s}^2}.$$

The rightmost side of the equation was obtained by differentiating the kinetic energy $\frac{mv^2}{2}$ with respect to time.
This gives us

$$\frac{dv}{dt} = -\frac{g\dot{p}}{v}\frac{dz}{dp}. \tag{6}$$

Combining (1), (2), (4) and (6) we get the following differential equation for v:

$$-\dot{v} = \frac{g\dot{p}}{v}\frac{dz}{dp} + \beta v^2 + \mu N(v,p), \tag{7}$$

where $N(v,p)$ is the magnitude of the contact force normal to the track.
To obtain values of v at every position of the track we solve this numerically for every part separately with the initial conditions obtained from the solution of (7) for the previous part. More on this in section 6.

5 The pieces

Here we will look more closely on the elements used in building the track.

5.1 *xz* arc

The xz arc of a circle of radius r centered at $\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$ is parametrised by p so that

$$\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix} = \begin{bmatrix} x_0 + r\cos(p) \\ y_0 \\ z_0 + r\sin(p) \end{bmatrix}, p \in [p_b, p_e],$$

where p is the angle(in radians) that the position vector $\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix}$ makes with the x axis in a coordinate system centered on $\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$. For such an arc we have

$$\frac{dz}{dp} = r\cos(p)$$

and

$$\frac{dp}{dt} = \frac{v}{r}, \tag{8}$$

which corresponds to the coaster moving along the arc in the direction of increasing p. We can also define xy and yz arcs with many analogies. In particular, (8) will be the same no matter the orientation.

5.2 *xy* arc

The parametrisation is

$$\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix} = \begin{bmatrix} x_0 + r\cos(p) \\ y_0 + r\sin(p) \\ z_0 \end{bmatrix}, p \in [p_b, p_e],$$

where p is the angle(in radians) that the position vector $\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix}$ makes with the x axis in a coordinate system centered on $\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$. Also here

$$\frac{dz}{dp} = 0.$$

5.3 *yz* arc

This one is parametrised by

$$\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 - r\cos(p) \\ z_0 + sin(p) \end{bmatrix}, p \in [p_b, p_e],$$

where p is the angle(in radians) that the position vector $\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix}$ makes with the negative y axis in a coordinate system centered on $\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$ and

$$\frac{dz}{dp} = r\cos(p).$$

5.4 loop

A loop is little more than an xz arc with $p \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$. In fact, the exact parametrisation for the first loop, after centering it at $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is

$$\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix} = \begin{bmatrix} R_1\cos(p) \\ \frac{1}{\pi}(\frac{1}{2}+p) \\ R_1\sin(p) \end{bmatrix}, p \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right].$$

This results in the loop's end being 1 m further along the y axis than the beginning so as not to collide with the supports along the ride.
Conversely, the second loop will be given by

$$\begin{bmatrix} x(p) \\ y(p) \\ z(p) \end{bmatrix} = \begin{bmatrix} R_1\cos(p) \\ \frac{-1}{\pi}(\frac{1}{2}+p) \\ R_1\sin(p) \end{bmatrix}, p \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right].$$

This way the y position of the track is the same before the loops and after both loops and the contact forces resulting from such a twist are small enough to neglect.

5.5 the ballistic curve

A ballistic curve was incorporated into the track's design. That is to provide a feeling of weightlessness for an extended period of time. It was calculated for a 45° ejection at 12.606 $\frac{\text{m}}{\text{s}}$. Aerodynamic forces were included in the calculation but friction was omitted. Partly because the contact(and frictional) forces resulting from the misalignment of predicted and actual trajectories will be small and partly because this will make for a softer landing as towards the end the car is already pressing down on the track. It is presented in Fig.2 and also obtainable from the enclosed algorithm.

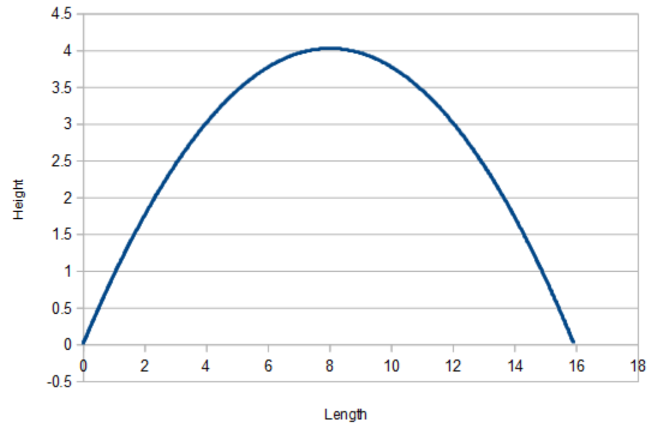

Figure 2: The ballistic curve of a car launched at 45°, both axes in meters

5.6 helix

We made use of helices with turn of π radians to turn the coaster upside down. As they are relatively short we assumed in our calculations that the contact forces on them are the same as they would be for straight rails. Two helices were used, both with turn of π, length of 10 meters and a constant rate of turn $\frac{d\theta}{dx}$ so

$$\frac{d\theta}{dx} = \frac{\pi}{10}.$$

Therefore a car traversing them would have an angular velocity ω of

$$\omega = \frac{d\theta}{dt} = \frac{d\theta}{dx}\frac{dx}{dt} = \upsilon\frac{d\theta}{dx}$$

and rotational kinetic energy Ω of

$$\Omega = \frac{I\omega^2}{2} = \frac{3MR^2}{4\upsilon^2}\left(\frac{d\theta}{dx}\right)^2 = 3\pi^2\upsilon^2$$

This energy is the result of contact force acting on the coaster, namely

$$\Omega = \int_{s_0}^{s_1} N(s)ds.$$

But at all times we have $T = \mu N$ where, again, T is the friction. We then have

$$\int_{s_0}^{s_1} T(s)ds = \int_{s_0}^{s_1} \mu N(s)ds = \mu\int_{s_0}^{s_1} N(s)ds = \mu\Omega.$$

This is the work done by friction when driving onto(or from) a helix.
we can therefore say that the kinetic energy taken from the car to make it spin is equal to $\Omega(1+\mu)$ The same process applies upon leaving the helix.

5.7 braking

The final stretch of the track is about 60 meters long. As evidenced by [5], modern brakes have no trouble stopping a coaster travelling at $\approx 9\frac{m}{s}$, which is about the speed resulting from our simulation.

6 Contact forces

Simple geometry shows that for xy arcs the contact force is

$$N = M\mu\sqrt{g^2 + \frac{v^4}{r^2}},$$

where r is the radius of the arc.
For both xz and yz arcs we have

$$N = M\mu\left(\frac{v^2}{r} - g\sin(p)\right),$$

where again r is the radius and p the angle measured from the horizontal.
For a straight track(and a helix) we have simply $N = M\mu g$.

7 The track

Fig.3 is the birds-eye view of our design. At least as seen through the lens of MS Paint. Here we will describe in detail the numbered sections. We will describe arcs by their orientation, radius r in meters and the range of parameter p.

1. arc $xz, r = 8, p \in [-\pi, \frac{-3\pi}{2}]$. This arc catches a 20 m vertical fall from the starting hill and its end is 2 m above ground level. The radius is large to keep acceleration below $10g$, as in this section coaster's velocity is the highest.
2. 10 m straight track in the x direction.
3. arc $xz, r = 8, p \in [\frac{-\pi}{2}, \frac{3\pi}{2}]$. This is the first loop.
4. 10 m straight track in the x direction.
5. 10 m helix in the x direction. This turns the passengers upside-down.
6. 10 m straight track in the x direction.
7. arc $xz, r = 10, p \in [\frac{-\pi}{2}, \frac{3\pi}{2}]$. This is the second loop and it's ridden upside down(or outwards).

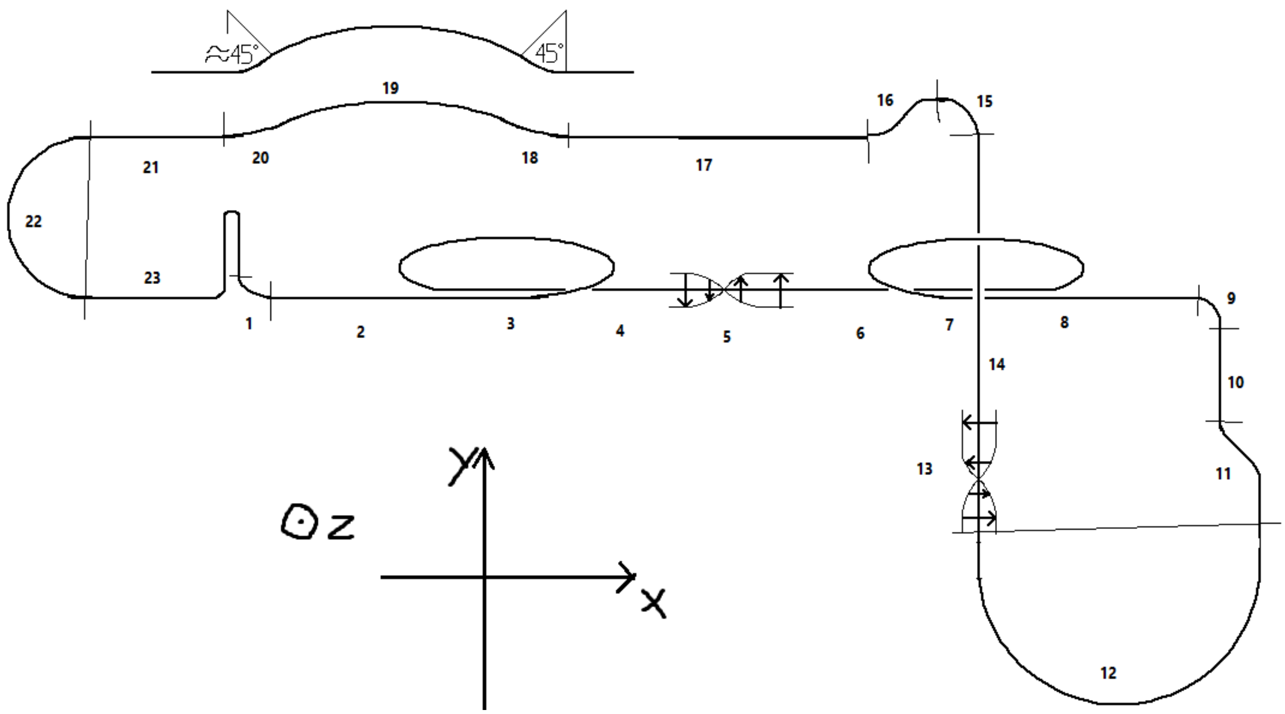

Figure 3: schematic of the track

8. 10 m straight track in the x direction.
9. arc $xy, r = 4, p \in [-\frac{\pi}{2}, 0]$. Here the parametrisation is inverted(the angle sweeps the arc clockwise), as will be the case with all xy arcs.
10. 10 m straight track in the $-y$ direction.
11. a set of yz arcs, as shown in Fig.4a
12. arc $xy, r = 10, p \in [0, \pi]$
13. 10 m helix in the y direction. This turns the back up again.
14. 19 m straight track in the y direction. This goes through the second loop.
15. arc $xy, r = 4, p \in [\frac{\pi}{2}, 0]$
16. a set of xz arcs, as shown in Fig.4b
17. 10 m straight track in the $-x$ direction.
18. a 45° $r = 4$ arc that launches the coaster into the ballistic curve.

19. the ballistic curve.
20. an almost 45° $r = 4$ arc, the exact value is obtainable from the simulation.
21. ≈49.5 m straight track in the $-x$ direction, the exact value is obtainable from the simulation.
22. arc $xy, r = 4.5, p \in [\frac{\pi}{2}, \frac{\pi}{2}]$
23. the braking line that also serves as the starting line.

(a) 4 yz arcs (b) 2 xz arcs

Figure 4: all radii in meters

8 Simulation

To ensure the non-lethality of our roller coaster, we have programmed a numerical simulation of a ride. It is written in C++ programming language and makes use of the following assumptions and principles:

- Simplification to the problem of one car per ride, greatly easening the complexity of the simulation.
- The modular structure of our roller coaster. It can be brought down to four constituent parts: the arcs, the turns, the helices and the straight lines, with two exceptional parts: the beginning freefall and the ballistic curve used in the coaster at the end of the ride.

⋇ The three forces acting upon a car during the ride, all well-known and described at every moment of the ride: its weight, friction with the coaster and the air drag.

Every step of the algorithm is based upon the idea: in time dt (equal to 0.000001s) the car moves by $v * dt$ and then changes its velocity v by $a * dt$ where a denotes acceleration derived from forces. The equations for a are as follows:

$$\frac{dv}{dt} = (g - \beta v^2) \qquad \text{for freefall,} \tag{9}$$

$$\frac{dv}{dt} = -g\cos(p) - \mu(v^2/r - g\sin(p) - \beta v^2) \qquad \text{for an arc, where } p \text{ is an angle,} \tag{10}$$

$$\frac{dv}{dt} = -\mu g - \beta v^2 \qquad \text{for a straight line,} \tag{11}$$

$$\frac{dv}{dt} = -\mu\sqrt{g^2 + v^4/r^2} - \beta v^2 \qquad \text{for a turn,} \tag{12}$$

The cases of the helices were more complex: although we assumed that due to the symmetry of the car's wheels the friction is independent of the car's angle (and no different from the case of a straight line), the energy lost to the rotary movement had to be considered. It led us to the following equations:

$$v = \sqrt{v^2 + \frac{3\omega^2(1+\mu)}{2}} \qquad \text{when entering the helix} \tag{13}$$

$$v = \sqrt{v^2 + \frac{3\omega^2(1-\mu)}{2}} \qquad \text{when exiting the helix} \tag{14}$$

We have also included a ballistic curve formed by the free-falling car. The numerical solution follows from the differential equations:

$$\frac{d^2x}{dt^2} = -\beta\frac{dx}{dt}^2 \tag{15}$$

$$\frac{d^2y}{dt^2} = -mg - \beta\frac{dy}{dt}^2 \tag{16}$$

We have then proceeded to analyse the ride using 3 and to write out the resulting velocity and time of the ride at the end of each component. We have also included the centripetal acceleration at the most stressing moments (that is, the beginnings and endings of turns and arcs) to ensure the non-lethal levels of g-force. The ride resulted in approximately 40 seconds of fun, with g-force not exceeding $6g$ and the final velocity of 8.5 m/s (before deceleration when finishing).

9 Conclusion

We have proposed an optimized solution to the problem of exciting roller coaster - one that utilizes elements such as arcs, freefalls and helices to maximise experienced acceleration, velocity and jerk, respectively. It includes the possible visual feelings, achieved with loops drive-throughs, or upside-down riding towards the ground. One of its advantages lies in its modular structure, as the roller coaster could be hypothetically easily rearranged should a better solution be found. Although we have no certainty of our solution to be the best, we believe there is no possibility of such, because everybody experiences excitement differently. It is also worth noting that while some parameters of our roller coaster could be increased (such as the maximum g-force acting upon a rider), we wanted to ensure the safety of the ride. Many of the ride's elements can be further optimised with the use of CAD tools, but the authors had no access to such. Perhaps the only perfect (although very difficult to implement) solution to the problem of maximising the excitement of a roller coaster would be to build everybody one, tailored to his needs.

References

[1] *Aerodynamic drag.* website. https://physics.info/drag/.

[2] *Air - Density, Specific Weight and Thermal Expansion Coefficient at Varying Temperature and Constant Pressures.* website. https://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html.

[3] *Coasters-101: Wheel design.* website. https://www.coaster101.com/2011/10/24/coasters-101-wheel-design.

[4] Y. Brent Creer et al. *Centrifuge study of pilot tolerance to acceleration and the effects of acceleration on pilot performance.* technical note D-337. NASA, 1960.

[5] Ann-Marie Pendrill, Magnus Karlsteen, and Henrik Rödjegård. "Stopping a roller coaster train". In: *Physics Education* 47.6 (2012), p. 728.

10 Appendix

Here is the code used for the simulation.

```
#include<iostream>
#include<cmath>
using namespace std;

double m=400, mi=0.015, beta=0.0015, g=9.81, dt=0.000001, v=0, t=0, dv, pi=4*atan(1), p0;

void freefall(double h, double hmin)
{
  double dh;
  while(h>=hmin)
    {
      //dt step
      dv=(g-beta*v*v)*dt;
      dh=-v*dt;
      h+=dh;
      v+=dv;
      t+=dt;
    }
  cout<<"Freefall:"<<endl;
  cout<<"v = "<<v<<endl;
  cout<<"t = "<<t<<endl;
}

void arc(double r, double p, double pmax, bool clockwise, int i)
{
  double dp;
  cout<<"Acceleration: "<<v*v/r<<endl;
  if(!clockwise)
    {
      while (p<=pmax)
{
  //dt step
  dv=(-g*cos(p) - mi*(v*v/r - g*sin(p)) - beta*v*v)*dt;
  dp=(v*dt)/r;
  p+=dp;
  v+=dv;
  t+=dt;
}
```

```
    }
  else
    {
      while (p>=pmax)
{
  //dt step
  dv=(-g*cos(p) - mi*(v*v/r - g*sin(p)) - beta*v*v)*dt;
  dp=-(v*dt)/r;
  p+=dp;
  v+=dv;
  t+=dt;
}
    }
  //When car is moving clockwise, the angle increases with time,
  //whilst when moving counter-clockwise - it decreases
  cout<<"Acceleration: "<<v*v/r<<endl;
  cout<<"Arc #"<<i<<":"<<endl;
  cout<<"v = "<<v<<endl;
  cout<<"t = "<<t<<endl;
  return;
}

void straightline(double xmax, bool write, int i)
{
  double dx;
  double x=0;
  while (x<xmax)
    {
      //dt step
      dv=(-mi*g-beta*v*v)*dt;
      dx=v*dt;
      x+=dx;
      v+=dv;
      t+=dt;
    }
  if (write)
    {
      cout<<"Straight line #"<<i<<":"<<endl;
      cout<<"v = "<<v<<endl;
      cout<<"t = "<<t<<endl;
    }
```

```
  return;
}

void helix(double rotations, double x, int i)
{
  double dx;
  double omega;
  double theta=(pi*rotations)/x;
  omega=v*theta;
  //Loss of energy to rotary motion upon entering the helix
  v=sqrt(v*v-3*omega*omega/2*(1+mi));
  //Approximation of the friction on the helix by a straight line of the same length
  straightline(x,false,0);
  omega=v*theta;
  //Partial recovery of energy from rotary motion upon exiting the helix
  v=sqrt(v*v+3*omega*omega/2*(1-mi));
  cout<<"Helix #"<<i<<":"<<endl;
  cout<<"v = "<<v<<endl;
  cout<<"t = "<<t<<endl;
  return;
}

void turn(double r, double pmax, int i)
{
  double p=0;
  double dp;
  cout<<"Acceleration: "<<v*v/r<<endl;
  while(p<=pmax)
    {
      //dt step
      dv=(-mi*sqrt(g*g + v*v*v*v/(r*r)) - beta*v*v)*dt;
      dp=(v*dt)/r;
      v+=dv;
      p+=dp;
      t+=dt;
    }
  cout<<"Turn #"<<i<<":"<<endl;
  cout<<"v = "<<v<<endl;
  cout<<"t = "<<t<<endl;
}
```

```
void ballisticcurve()
{
  double dv;
  double vx=v/sqrt(2);
  double vy=v/sqrt(2);
  double dvx;
  double dvy;
  double y=0;
  double x=0;
  while(y>=0)
    {
      //Numerical solution to the differential equation of the ballistic curve
      dvx=-beta*vx*vx*dt;
      dvy=(-g-beta*vy*vy)*dt;
      vx+=dvx;
      vy+=dvy;
      x+=vx*dt;
      y+=vy*dt;
      t+=dt;
      //cout<<x<<" "<<y<<endl; //This part was used for plotting the ballistic curve
    }
  cout<<"Ballistic curve:"<<endl;
  //The coordinates of the end of the curve
  cout<<"x = "<<x<<endl;
  cout<<"y = "<<y<<endl;
  //The car's velocity at the end of the curve
  v=sqrt(vx*vx+vy*vy);
  cout<<"v = "<<v<<endl;
  cout<<"t = "<<t<<endl;
  //The angle of the end of the curve
  p0=atan(vx/vy);
  cout<<"p0 = "<<p0<<endl;
}

int main()
{                                           //Notation from the drawing
  freefall(30,10);
  arc(8,pi,3*pi/2,false,1);                 //1
  straightline(5,true,1);                   //2
  arc(10,-pi/2,3*pi/2,false,2);             //3
  straightline(10,true,2);                  //4
```

```
  helix(1,10,1);                             //5
  straightline(10,true,3);                   //6
  arc(8,-pi/2,3*pi/2,false,3);               //7
  straightline(10,true,4);
  turn(10,pi/2,1);                           //8
  arc(6,-pi/2,0,false,4);                    //9
  arc(4,0,-pi/2,true,5);
  arc(2,-pi/2,-pi,true,6);
  arc(2,-pi,-pi/2,false,7);
  turn(10,pi,2);                             //10
  helix(1,10,2);                             //11
  straightline(19,true,5);
  turn(4,pi/2,3);                            //12
  arc(4,-pi/2,-pi,true,8);                   //14
  arc(4,-pi,-pi/2,false,9);
  straightline(10,true,6);                   //15
  arc(4,-pi/2,-pi/4,false,10);               //16
  ballisticcurve();                          //17
  arc(4,p0,-pi/2,true,11);                   //18
  straightline(83-(27,9+4*sqrt(2)),true,7); //19
  turn(4.5,pi,4);                            //20
  return 0;
}
```

Also, Looking at the problem from a more theoretical side, one may try to define some properties of a trajectory of the coaster, which best fits to what human understands as exciting, and then try to find a curve which has as many of these properties as possible. First of all, the only parameter of motion that human can actually feel is acceleration, which makes it coveted when designing a roller coaster. But acceleration itself is very common. We feel it almost every moment, since we experience the Earth's gravitational field. We expect the trajectory to cause the acceleration to differ the most from what we can feel normally. Secondly, in the Earth conditions, we want the coaster to move with the highest speed possible. Finally, a coaster which moves with unusual but constant acceleration will be simply boring, so we need it to change the acceleration over time, that is, to move with a jerk. To put everything together, let us define the *funity* of the curve. For a test particle moving along the curve s, the funity will be defined as follows:

$$F(s) = \int_{t_1}^{t_2} \left(\vec{v}^2 + \vec{j}^2 + (\vec{a} + g\hat{c})^2\right) dt$$

where t_1 and t_2 are the moments when particle was at the beginning and at the end of the curve respectively, $\vec{v}, \vec{a}, \vec{j}$ are the velocity, acceleration and jerk of the particle at a given moment, and the $\hat{c}$ vector requires more discussion. Since the organs responsible for feeling the acceleration (the inner ear) are located in the head, it is noteworthy that the test particle is precisely the head. On every point of its trajectory we attach the unit vector $\hat{c}$ perpendicular to the trajectory, which represents direction of the body. The acceleration a human would be feeling in his normal position due to Earth's gravitation is expressed as $-g\hat{c}$, where g is the value of standard gravity (the vector indicates from the head to the feet, and we feel the standard gravity as we would accelerate in the opposite direction). Hence $\vec{g} + \vec{a} + g\hat{c}$ represents the difference between the normal acceleration that human experiences and the summary acceleration felt due to move and Earth's gravity. Since the $\hat{c}$ vector is always perpendicular to the trajectory, we can express it by a single parameter α denoting an angle between $\vec{c}$ and the normal vector of TNB frame of reference, which itself can be expressed by the velocity and acceleration vector. Hence we end up with the functional of the form

$$\int F(t, \vec{v}(t), \vec{a}(t), \vec{j}(t), \alpha(t))dt$$

which may be solvable with the variation calculus.