Statistical Physics B

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1. **Exercise 1:** (Ex 1.7 Pathria) Energy of extreme relativistic particle in a cubic box of volume $V = L^3$ reads:
$$\epsilon = \frac{hc}{2L}(n_x^2 + n_y^2 + n_z^2)^{\frac{1}{2}}$$
Study the thermodynamic properties of N such particles in a box of given volume. Calculate the ratio C_p/C_V

   Hint 1 : *If you are ask to study thermodynamics its assumed that the number $N \gg 1$. Stirling formula:*
$$\log(N!) \approx N\log(N) - N$$
   Hint 2 : *The total energy will be given by:*
$$E = \sum_{i=1}^{N}\epsilon_i = \frac{hc}{2L}\sum_{i=1}^{N}|\vec{k}_i|$$
   *You can calculate the number of microstates by integration over spatial and momenta coordinates with this constraint on momenta. However to extract the main thermodynamic properties you only need to argue that **(explain why)**:*
$$\Omega \sim (V^{\frac{1}{3}}E)^{3N}$$

2. **Exercise 2:** (Ex 1.12 Pathria) Entropy of mixing of two gases equals:
$$\Delta S = k\Big[(N_1+N_2)\log\left(\frac{V_1+V_2}{N_1+N_2}\right) - N_1\log\left(\frac{V_1}{N_1}\right) - N_2\log\left(\frac{V_2}{N_2}\right)\Big]$$
Show that $\Delta S \leq 0$. For which values of (N_1, V_1, N_2, V_2) does the equality hold?

3. **Exercise 3:** (Ex 2.2 Pathria simplified) Show that the volume element is invariant under transformation from Cartesian to polar coordinates in 2 dimensions.

   Hint 1: *Using the definition of generalized momenta $p_i = \frac{\partial\mathcal{L}}{\partial\dot{q}_i}$ calculate: $p_r(x, y, p_x, p_y)$ and $p_\phi(x, y, p_x, p_y)$*