

# Poincare-Einstein approach to Penrose's Conformal Cyclic Cosmology

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Conformal Geometry, Analysis, and Physics  
Seattle, June 14, 2022

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- The **Conformal Cyclic Cosmology**, or CCC, is a certain mathematical frame for **cosmology**. As **Roger Penrose** once told me, it emerged because he wanted to have some answer to a question which he was asked many times. This annoying question was: '**What was before the Big Bang?**'.
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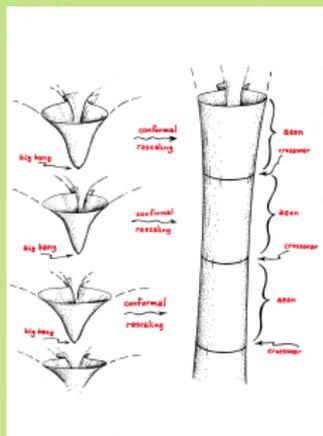
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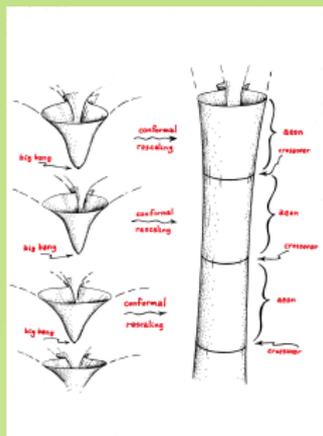
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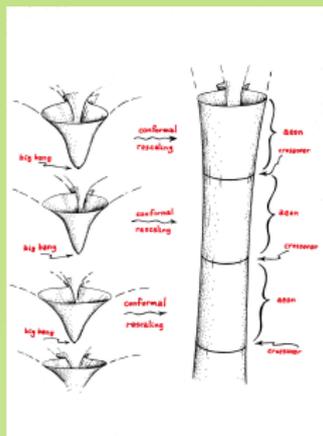
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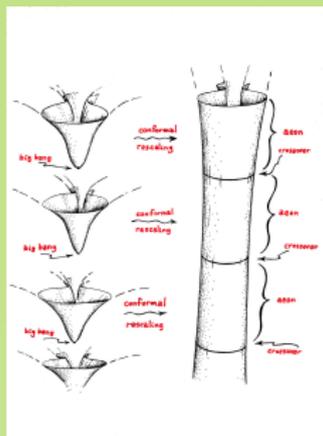
- The Universe consists of **eons**, each being a **time oriented** spacetime, i.e. a 4-dimensional **Lorentzian manifold**, whose **conformal compactifications** have **spacelike  $\mathcal{I}$ s**. The **Weyl tensor** of the 4-metric on each  $\mathcal{I}$  is zero.
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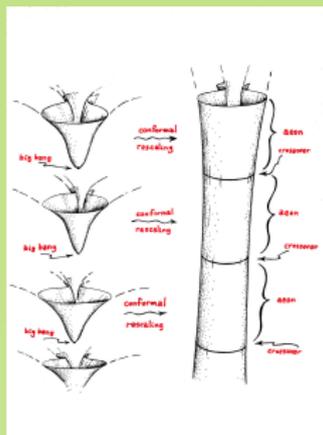
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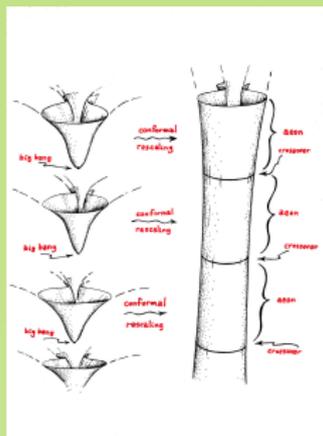
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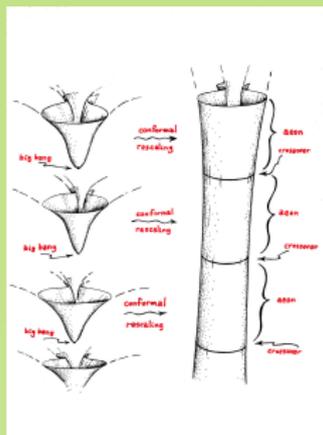
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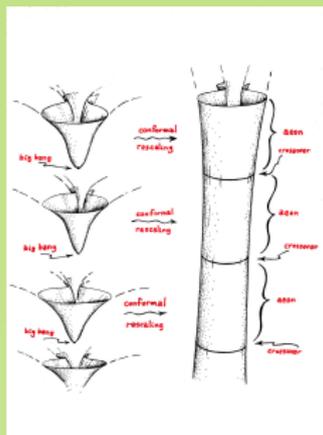
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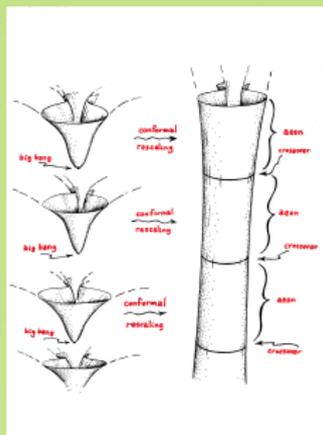
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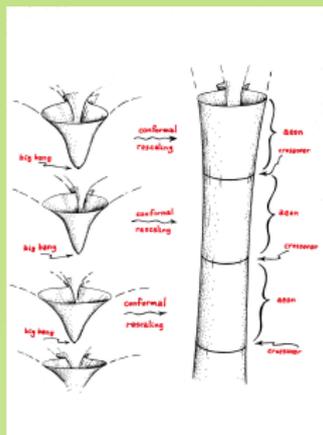
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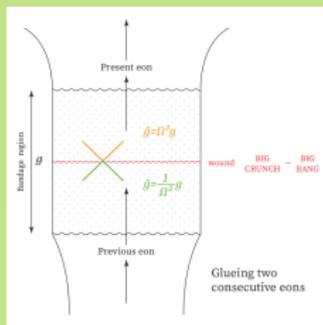
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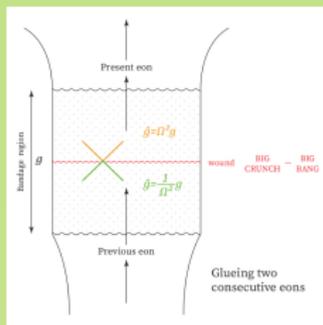
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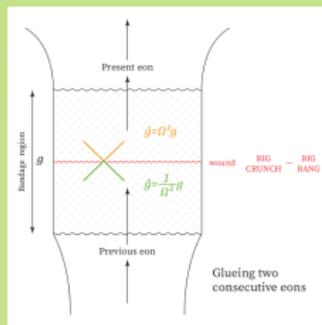
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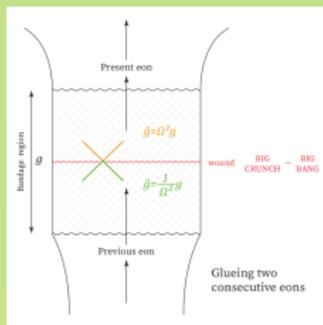
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  - a Lorentzian metric  $g$  which is regular everywhere,
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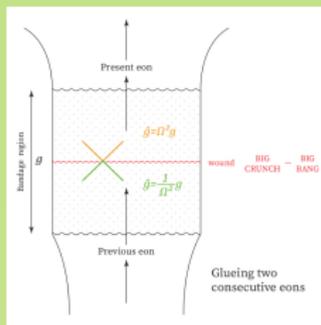
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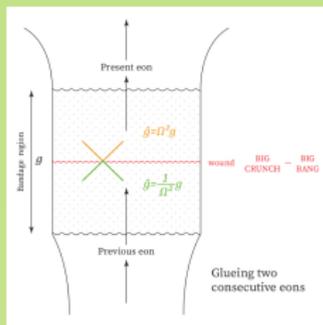
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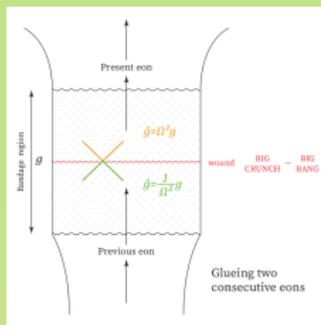
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- How to make this relation specific is debatable, but Penrose proposes that
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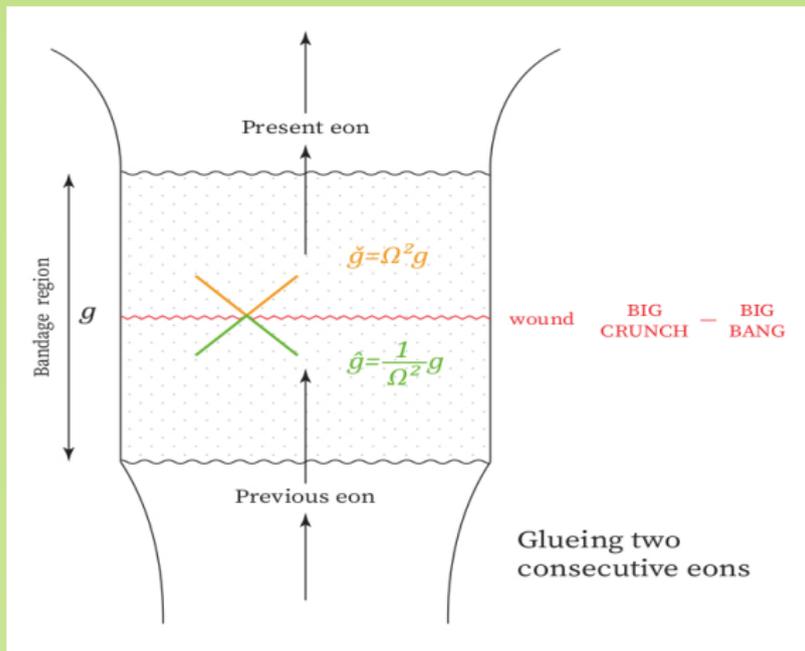
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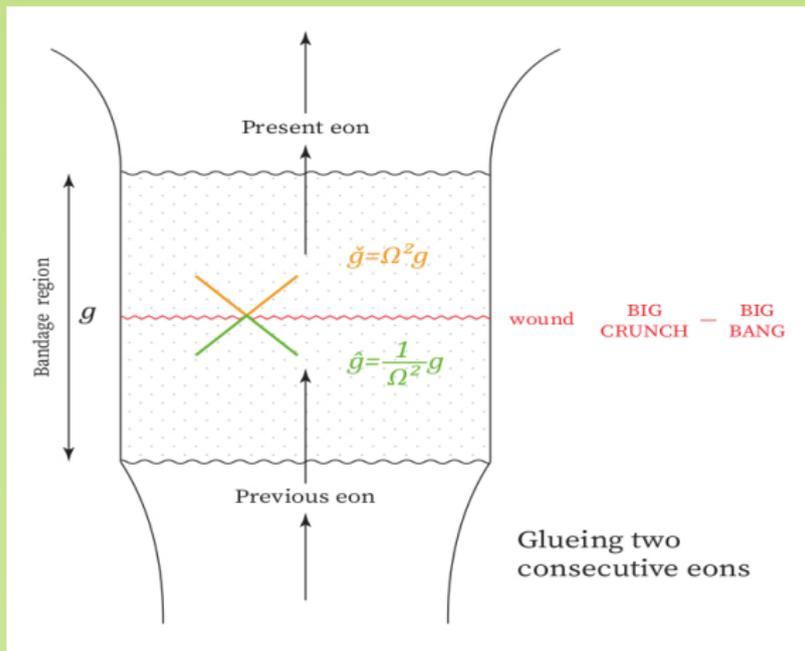
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# Penrose's Conformal Cyclic Cosmology



- **Question:** How to make a model of Penrose's bandage region of two eons?
- One needs a function  $\Omega$ , **vanishing on some spacelike hypersurface**, and a **regular** Lorentzian 4-metric  $g$  **conformally flat at this hypersurface**, such that if  $\hat{g} = \frac{1}{\Omega^2}g$  **satisfies Einstein equations** with some physically reasonable energy momentum tensor, **then**  $\check{g} = \Omega^2g$  **also satisfies Einstein equations** with possibly different, but still physically reasonable energy momentum tensor.
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## Einstein spaces which are mapped conformally on each other.

Von

H. W. Brinkmann in Cambridge (Mass., U. S. A.).

§ 1.

### Introduction.

It is well known that a Euclidean  $n$ -space, that is, a Riemann space whose squared line element is

$$ds^2 = dx_1^2 + dx_2^2 + \dots + dx_n^2$$

can, for  $n \geq 3$ , be mapped conformally on itself only by inversions and similarity transformations (Liouville's theorem). An analogous question arises when, instead of Euclidean space, we consider those spaces which satisfy Einstein's gravitational equations for "empty space" and which we call *Einstein spaces*. A difference arises, however, out of the fact that for  $n \geq 4$  two Einstein spaces need not be isometric, hence we set the problem as follows: When can an Einstein space be mapped conformally on some (possibly different) Einstein space and in how many ways can it be so mapped? The present paper is devoted to the solution of this problem. The related question: When can an Einstein space be mapped conformally on itself? will be answered also in the course of the investigation.

- In particular, **in dimension four in Lorentzian signature**, Brinkman **found all conformal classes** which contain at least **two different Ricci flat metrics**.
- **In bandage regions of CCC the problem is similar** to this of Brinkmann. Here, the **same function  $\Omega$**  should provide two scales:  $\Omega$  and  $-\frac{1}{\Omega}$ , for the 'healing' metric  $g$ , which make the corresponding **conformally related metrics  $\check{g} = \Omega^2 g$  and  $\hat{g} = \Omega^{-2} g$**  into **two solutions of the Einstein equations**. This seems to make the problem of finding possible candidates for bandage metrics **very restrictive**.
- The situation would be really bad if not the fact that now, the **two solutions  $\hat{g}$  and  $\check{g}$  of the Einstein equations** may have **different energy momentum tensors**: a **prescribed** energy momentum tensor **on the  $\hat{M}$  part**, and a **reasonable** energy momentum tensor **on the  $\check{M}$  part**.

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I will now outline how to effectively make a bandage region model. This will be done in a number of steps.

- **Step one:** How to create a metric  $\hat{g}$  of the past eon?
- **Step two:** How to pick up the scale  $\Omega$  once  $\hat{g}$  is determined?
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- **Step four:** How to interpret the physics content of the bandage region?

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- But I do it in a **Poincare-Einstein way** of **Charles Fefferman and Robin Graham!**
- I will briefly review the **Fefferman-Graham result specialized to the 4-dimensional Lorentzian situation**, now.
- Their result **shows how to uniquely associate an Einstein Lorentzian 4-metric  $\hat{g}$  on  $\hat{M}$  with a 3-dimensional Riemannian metric  $h_0$  on the conformal boundary  $\partial\hat{M} = \mathcal{I}$  of  $\hat{M}$ .**

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- Start with a **conformal class of Riemannian 3-metrics**  $[h_0]$  with a representative  $h_0$  on  $\mathcal{I}$ .
- Consider a symmetric rank 2 tensor  $h(t)$  on  $\mathcal{I}$  in terms of its **power series expansion**  $h(t) = \sum_{i=0}^{\infty} h_i t^i$ , and define a **Lorentzian 4-metric**

$$\hat{g} = \frac{-dt^2 + h(t)}{t^2}$$

on  $\hat{M} = ]-\epsilon, 0[ \times \mathcal{I}$  with  $t \in ]-\epsilon, 0[$ .

- Choose an **arbitrary** symmetric rank 2 tensor  $h$  on  $\mathcal{I}$  which is **trace-free** and **divergence-free** w.r.t.  $h_0$ .
- Then the **conditions**

$$\text{Ric}(\hat{g}) = 3\hat{g}$$

and  $\left( \text{the trace-free part of } h_3 \right) = h$

**uniquely determine**  $h(t)$ , and in turn  $\hat{g}$ , **up to infinite order at**  $t = 0$ .

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$$\text{Ric}(\hat{g}) = 3\hat{g}$$

and  $\left( \text{the trace-free part of } h_3 \right) = h$

**uniquely determine**  $h(t)$ , and in turn  $\hat{g}$ , **up to infinite order at**  $t = 0$ .

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## Step one: The past eon metric $\hat{g}$

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- Impose the **Einstein conditions**

$$\text{Ric}(\hat{g}) - \frac{1}{2}\hat{R}\hat{g} + \hat{\Lambda}\hat{g} = \hat{T}$$

on  $\hat{g}$ , where  $\hat{T}$  is the **energy momentum tensor** suitable **for the past eon**. If the energy momentum tensor  $\hat{T}$  in  $\hat{M}$  is not too wild, I **expect the similar uniqueness result** as in the FG case.

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## Steps two and three: The choice of $\Omega$ and Penrose's reciprocity

- When  $\hat{g}$  is determined according to the procedure I described, its manifold  $\hat{M}$  has a **natural spacelike boundary at  $t = 0$** : i.e. at  $\partial\hat{M} = \{0\} \times \mathcal{I}$ , where the metric  $\hat{g}$  blows up.
- It is therefore natural to **extend  $\hat{M}$  to  $M = ] - \epsilon, \epsilon[ \times \mathcal{I}$** , and to **define the Penrose scale function  $\Omega$  to be  $\Omega = t$** , where  $t \in ] - \epsilon, \epsilon[$ .
- Defining  $g = \Omega^2 \hat{g}$  **provides a metric regular for all  $t \in ] - \epsilon, \epsilon[$** , and after its **extension to positive  $t$ s** defines **the healing metric  $g = -dt^2 + h(t)$  in the entire bandage region  $M = ] - \epsilon, \epsilon[ \times \mathcal{I}$** .
- The **wound** of the bandage region **is placed at  $\Omega = 0$** .
- Now, the **Penrose's reciprocity hypothesis**, gives the **metric  $\check{g}$  in the present eon  $\check{M} = [0, \epsilon[ \times \mathcal{I}$**  as  $\check{g} = \Omega^2 g$ , or what is the same as  $\check{g} = t^2 \left( -dt^2 + h(t) \right)$ .

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## Step four: The physical content of the present eon

- The described procedure **defined the present eon manifold**  $\check{M} = [0, \epsilon[ \times \mathcal{I}$  and its metric  $\check{g} = t^2(-dt^2 + h(t))$ .
- Now all choices have been made, and the **physical content** of  $\check{M}$  should be **read off** from the **right hand side** of the **Einstein's equations**

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- So the **physics in the new eon** is **determined by a discrete flip** of  $-\frac{1}{t} \rightarrow t$  rather, than via a differential equation for  $\Omega$ .
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$$\check{R}ic(\check{g}) - \frac{1}{2}\check{R}\check{g} + \check{\Lambda}\check{g} = \check{T}.$$

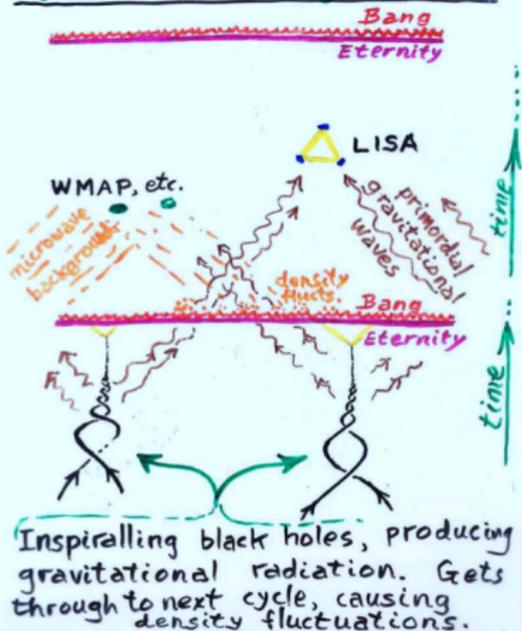
- So the **physics in the new eon** is **determined by a discrete flip** of  $-\frac{1}{t} \rightarrow t$  rather, than via a differential equation for  $\Omega$ .
- This flip is called **Penrose reciprocity hypothesis**.



Does this procedure give reasonable results?

### Observational Implications

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- Primordial density fluctuations



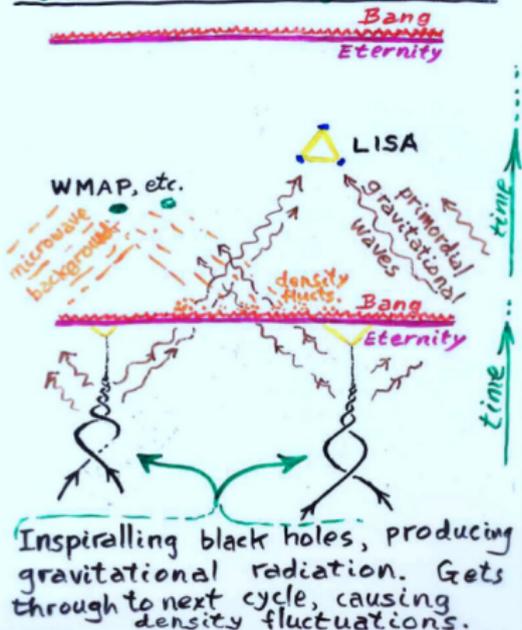
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## Conclusion?

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- As mentioned at the beginning I believe that **Penrose's CCC** is a perfect new subject that could (should?) interest people at this conference. Studies on CCC **need a broader team of conformal geometers, specialists on Poincare-Einstein/Fefferman-Graham expansions, analysts who could prove convergence** of various models, etc.
- **Robin** very quickly provided me an elegant proof of convergence of my computer-generated spherical wave power series solution. I think that many other mathematical problems that appear in CCC can be quickly and elegantly solved by mathematicians. I would be most happy if my lecture would inspire somebody at this audience to **look closer at mathematics and/or physics of CCC.**

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HAPPY BIRTHDAY ROBIN!

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