Conformal transformations and the beginning of the Universe. Part III.

Pawel Nurowski

Centrum Fizyki Teoretycznej
Polska Akademia Nauk

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- A conformal transformation: $g \to \hat{g} = \frac{1}{\Omega^2} g$ results in the following transformation of the respective Ricci tensors:
$$\hat{R}_{\mu\nu} = \Omega^2 R_{\mu\nu} + 2\Omega \nabla_\nu \Omega_\mu + g_{\mu\nu} \left(\Omega \Box \Omega - 3g(\nabla\Omega, \nabla\Omega) \right)$$
- Contracting we get the following transformation of the Ricci scalars:
$$\hat{R} = \Omega^2 R + 6\Omega\Box\Omega - 12g(\nabla\Omega, \nabla\Omega).$$
- Interpreting $\hat{g}$ as **the physical metric of spacetime**, whose conformal compactified metric g has $\mathscr{I}$ where $\Omega \to 0$, we see that the **causal properties of** $\mathscr{I}$ are governed by the formula:
$$\hat{R} = -12g(\nabla\Omega, \nabla\Omega).$$
- Recall that the signature is $(-, +, +, +)$, so $\mathscr{I}$ **is spacelike** if $\nabla\Omega$ is timelike, i.e. **if the Ricci scalar** $\hat{R}$ of the physical metric is **positive**, $\hat{R} > 0$.

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Conformal transformations of spacetimes and Ricci

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- Using the Einstein equations satisfied by the metric $\hat{g}$,
$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} + \hat{\Lambda}\hat{g}_{\mu\nu} = \hat{T}_{\mu\nu}$$
or
$$\hat{R}_{\mu\nu} = \hat{T}_{\mu\nu} + (\hat{\Lambda} - \frac{1}{2}\hat{T})\hat{g}_{\mu\nu},$$
one can express this also in terms of the cosmological constant $\hat{\Lambda}$ of the physical spacetime and the trace of its energy momentum tensor $\hat{T}$:
$$4\hat{\Lambda} - \hat{T} > 0.$$
- This, in particular means that if close to $\mathscr{I}$ the trace of the energy momentum $\hat{T}$ vanishes, a **positive cosmological constant** $\hat{\Lambda}$ **makes** $\mathscr{I}$ **spacelike**.

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- In particular the Ricci scalar $R = \frac{6}{\Omega^2}$ **is positive.** This means that Einstein's model has **spacelike** $\mathscr{I}$.

- **Perfect cosmological principle** (Copernicus ?): The Universe is the same **everywhere**, **in every direction**, and **at every moment of time.**
- A **General Relativity model** of such Universe is **due to Einstein** (1917):
$$M = \mathbb{R} \times \mathbb{S}^3, \text{ and } g_{Einst} = -\mathrm{d}t^2 + \Omega^2 g_{\mathbb{S}^3},$$
with $g_{\mathbb{S}^3}$ is the standard metric on $\mathbb{S}^3$ of radius 1, and $\Omega = const$. This is the **Einstein's static Universe model.**
- The Ricci tensor for this spacetime is $Ricci = \frac{2}{\Omega^2} g_{\mathbb{S}^3}$.This satisfies Einstein's equations
$$Ricci - \frac{1}{2} Rg + \Lambda g = \mu u \otimes u,$$
with $u = -\mathrm{d}t$, and $\Lambda = \frac{1}{\Omega^2}$, $\mu = \frac{2}{\Omega^2}$.
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- **Note** that **all** FLRW metrics **are conformally flat!**

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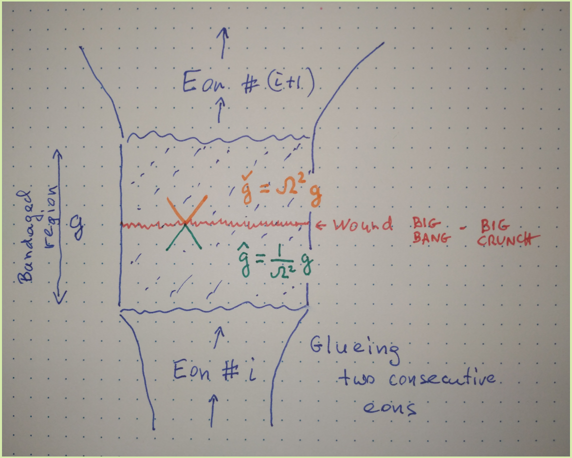
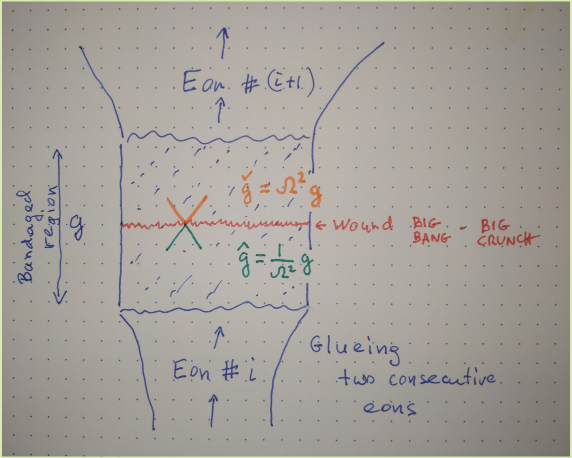

Eon #(i+1)
Bandaged region
g
$\check{g} = \Omega^2 g$
← Wound
BIG BANG - BIG CRUNCH
$\hat{g} = \frac{1}{\Omega^2} g$
Glueing two consecutive eons
Eon # i


Eon # (i+1)
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so it is **positive** if $\boxed{-1 \leq \check{w} < 1/3}$ (recall the energy conditions $-1 \leq \check{w} \leq 1$).

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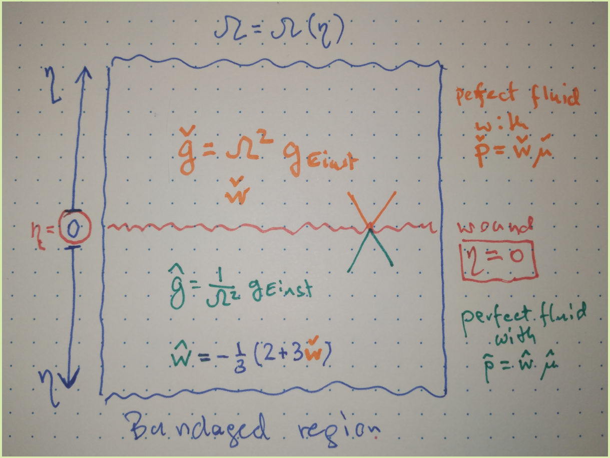

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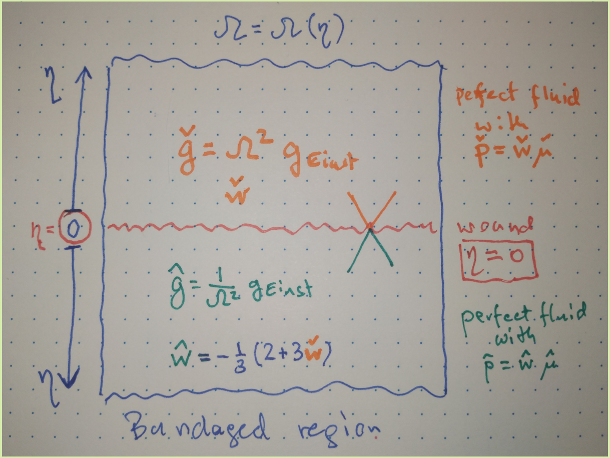
η
Bandaged region

Transformation $\check{g} \to \hat{g} \to \check{g} \to \hat{g} \to \ldots$ of fluids



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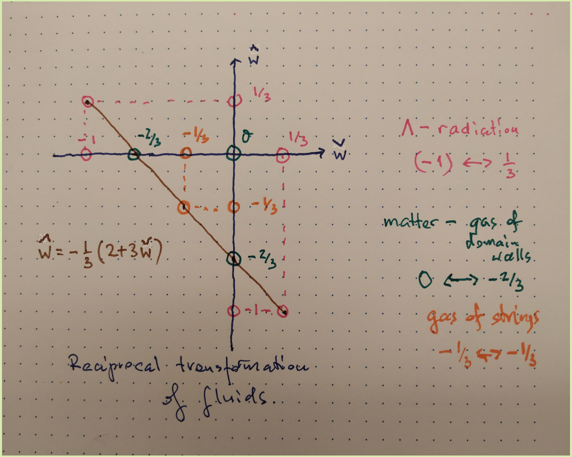

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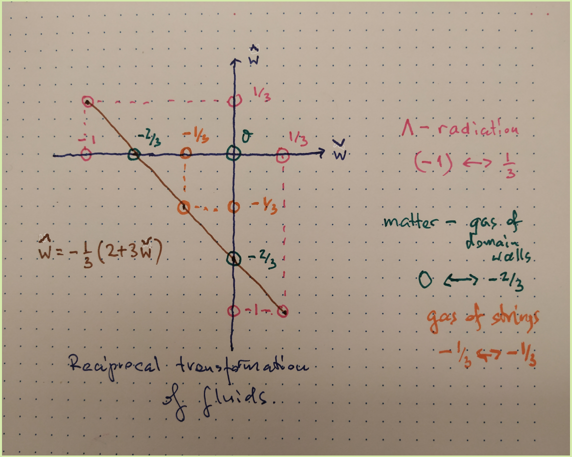

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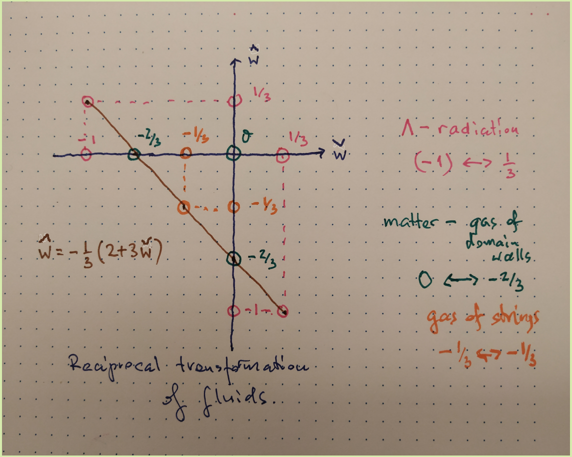

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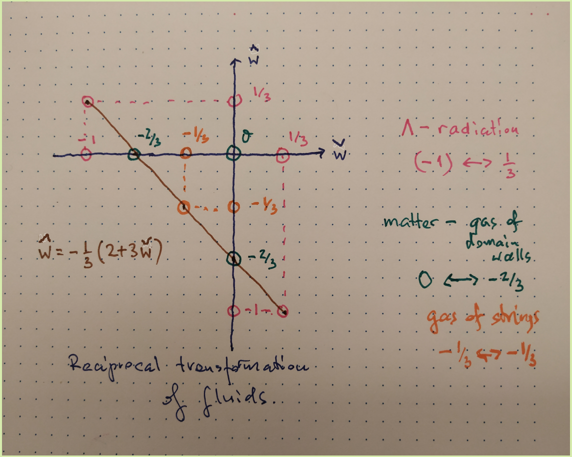

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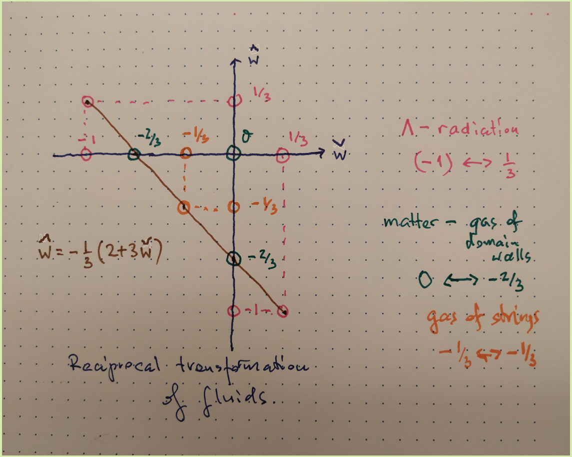

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THANK YOU!