Simple models in Penrose's Conformal Cyclic Cosmology

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What is *CCC*?

- *CCC* or *C*onformal *C*yclic *C*osmology is a proposal for a Cosmology Hypothesis which answers the question 'What was before the Big Bang?'.
- The scheme of **Penrose**'s *CCC* is as follows:[1]

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- In a bandage region, the **three metrics** g, $\check{g}$ and $\hat{g}$, are **conformally related** on their overlaping domains.
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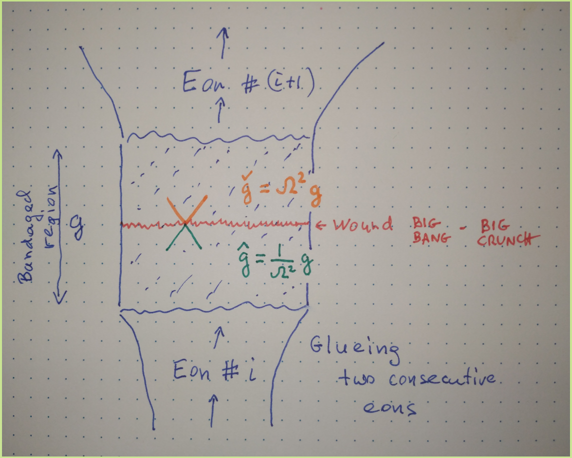
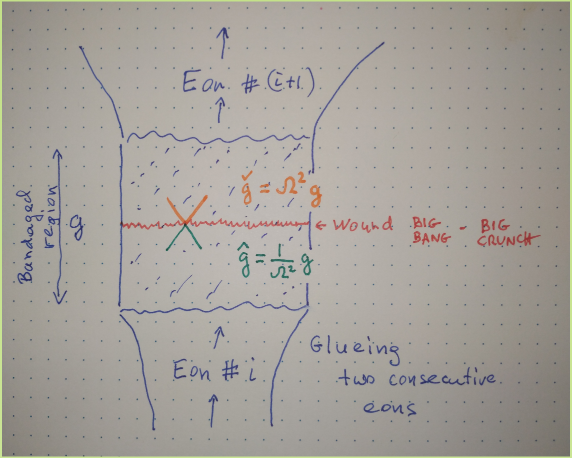
Penrose's Conformal Cyclic Cosmology




Eon # (i+1)
$\check{g} = \Omega^2 g$
← Wound
BIG BANG - BIG CRUNCH
$\hat{g} = \frac{1}{\Omega^2} g$
Bandaged region
g
Glueing two consecutive eons
Eon # i

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- One needs a function Ω, **vanishing on some spacelike hypersurface**, and a **regular** Lorentzian 4-metric g, such that **if** $\check{g} = \Omega^2 g$ **satisfies Einstein equations** with some physically reasonable energy momentum tensor, **then** $\hat{g} = \frac{1}{\Omega^2} g$ **also satisfies Einstein equations** with possibly different, but still physically reasonable energy momentum tensor.

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$$g_{test} = -\mathrm{d}t^2 + \Omega^2(t) r_0^2 \left(\mathrm{d}\chi^2 + \sin^2\chi(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \right).$$
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i.e. $g_{test} = \Omega^2(\eta) g_{Einst}$.
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$$Ric - \frac{1}{2} R g_{test} = (\mu + p) u \otimes u + p g_{test}$$
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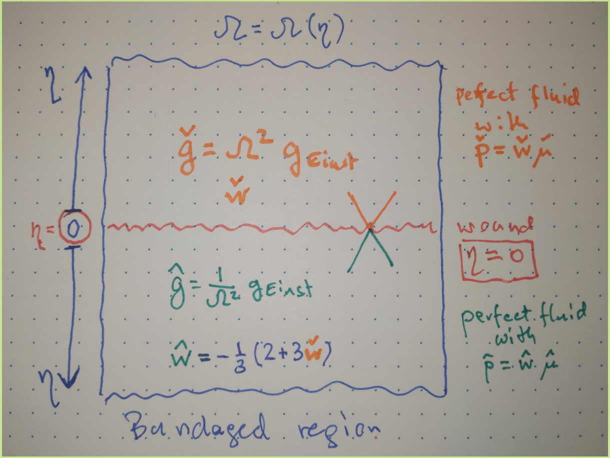
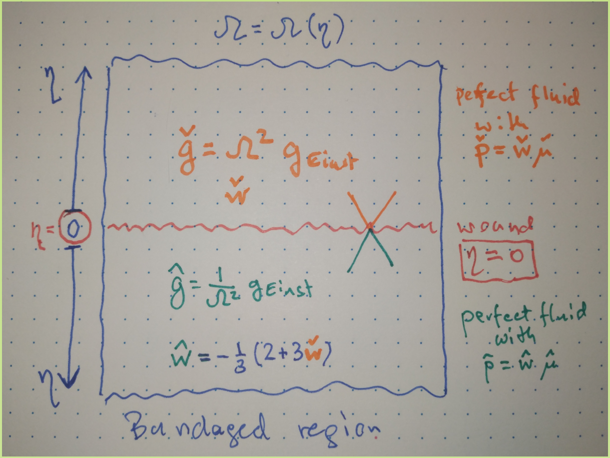
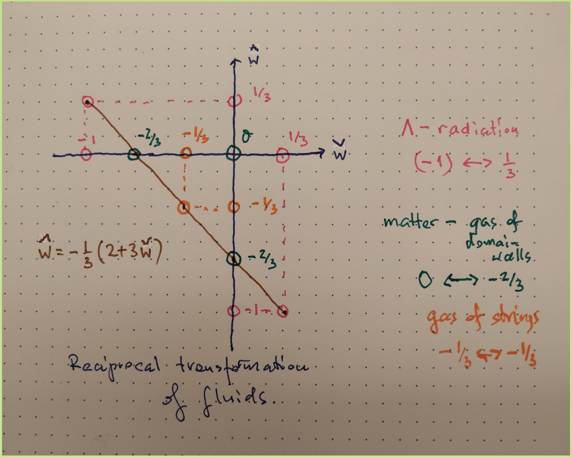
Transformation $\check{g} \to \hat{g} \to \check{g} \to \hat{g} \to \ldots$ of fluids

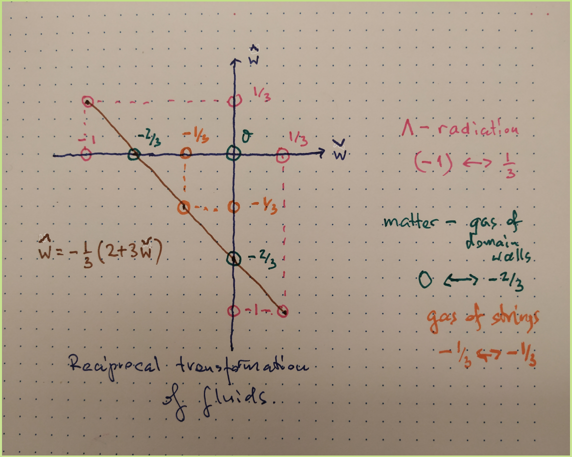



Ω = Ω(η)
η
ǧ = Ω² g_Einst
w̌
perfect fluid
with
p̌ = w̌ μ̌
η = 0
wound
η = 0
ĝ = 1/Ω² g_Einst
ŵ = -1/3 (2+3w̌)
perfect fluid
with
p̂ = ŵ μ̂
η
Bandaged region

Transformation $\breve{g} \to \hat{g} \to \breve{g} \to \hat{g} \to \dots$ of fluids



Suspiscious points: $\breve{w} = -1, 1/3$ (cosmological constant - radiation), since the scalar curvature $R = 0$, when $\breve{w} = 1/3$; and $\breve{w} = -1/3$ (gas of strings), when $\Omega \neq 0$ on $\mathscr{I}$.



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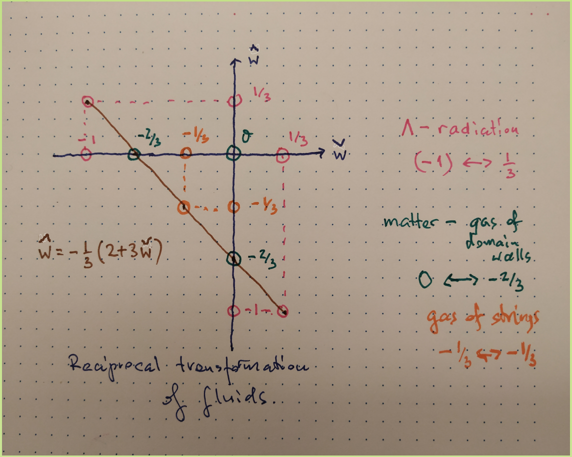

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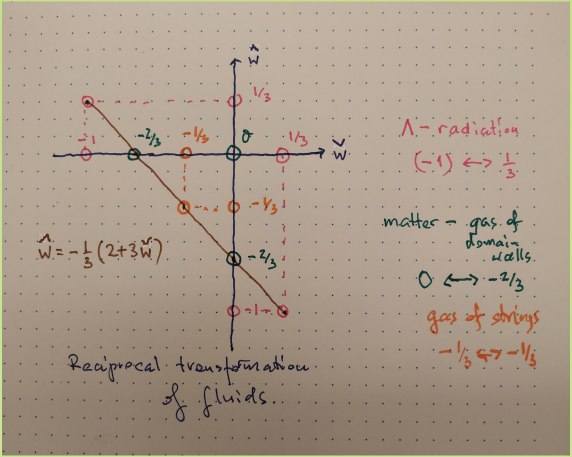

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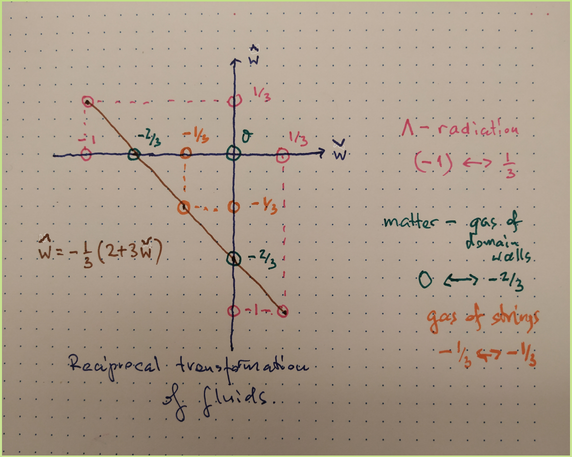

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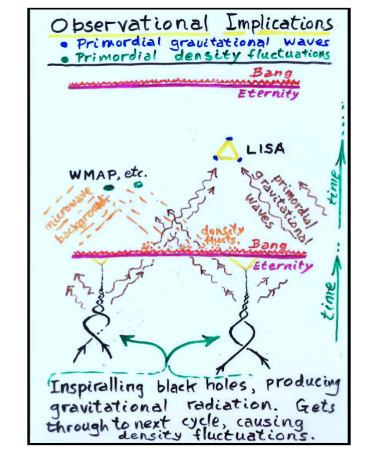
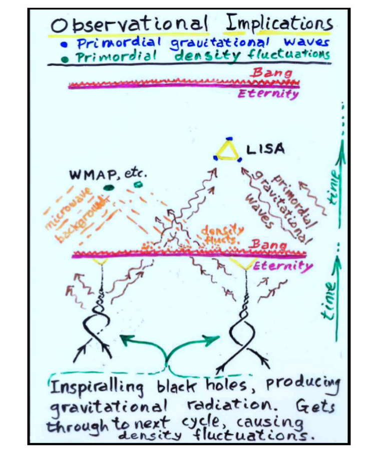
Motivation for the next model (picture by R. Penrose)




Observational Implications
• Primordial gravitational waves
• Primordial density fluctuations
Bang
Eternity
LISA
WMAP, etc.
microwave background
primordial gravitational waves
time
density flucts.
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Eternity
time
Inspiralling black holes, producing gravitational radiation. Gets through to next cycle, causing density fluctuations.

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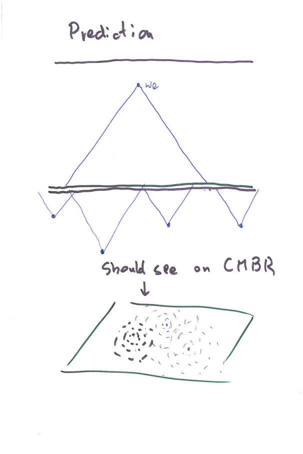

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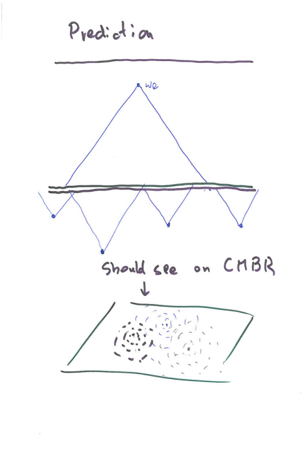

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- Applying the reciprocal hypothesis, $\hat{g} \to \check{g} = t^4 \hat{g}$, I show that the **new eon** $(\check{M}, \check{g})$ created from the one containing a single spherical wave, **is filled** at its initial state **with three types of radiation**: (i) the **damped spherical wave** which continues its life from the previous eon, (ii) the **ingoing spherical wave** obtained as a result of a colision of the wave from the past eon with the Bang hypersurface and (iii) **randomly scattered waves** that could be interpreted as perfect fluid with the energy density $\check{\rho}$ and the isotropic pressure $\check{p}$ such that $\check{p} = \frac{1}{3}\check{\rho}$. The metric $\check{g}$ solves the Einstein's equations **without cosmological constant** and with the energy-momentum tensor

$$\check{T}^{ij} = \check{\Phi} K^i K^j + \check{\Psi} L^i L^j + (\check{\rho} + \check{p}) \check{u}^i \check{u}^j + \check{p} \check{g}^{ij},$$

in which $\check{u}^i \check{u}^j \check{g}_{ij} = -1$, $\check{g}_{ij} L^i L^j = 0$ and $L^i K^j \check{g}_{ij} = -2$.

- Applying the reciprocal hypothesis, $\hat{g} \to \check{g} = t^4\hat{g}$, I show that the **new eon** $(\check{M}, \check{g})$ created from the one containing a single spherical wave, **is filled** at its initial state **with three types of radiation**: (i) the **damped spherical wave** which continues its life from the previous eon, (ii) the **ingoing spherical wave** obtained as a result of a colision of the wave from the past eon with the Bang hypersurface and (iii) **randomly scattered waves** that could be interpreted as perfect fluid with the energy density $\check{\rho}$ and the isotropic pressure $\check{p}$ such that $\check{p} = \frac{1}{3}\check{\rho}$. The metric $\check{g}$ solves the Einstein's equations **without cosmological constant** and with the energy-momentum tensor

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Possible generalizations

- I start with a conformal class $[h_0]$ represented by the flat 3-dimensional metric

$$h_0 = \frac{2r^2 \mathrm{d}z\mathrm{d}\bar{z}}{(1+\frac{z\bar{z}}{2})^2} + \mathrm{d}r^2.$$

- Then I make Poincar'e anstaz by considering a 1-paramater family of 3-d metrics h_t. This will be a *spherically symmetric* family

$$h_t = \frac{2r^2\left(1+\nu(t,r)\right)\mathrm{d}z\mathrm{d}\bar{z}}{(1+\frac{z\bar{z}}{2})^2} + \left(1+\mu(t,r)\right)\mathrm{d}r^2,$$

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- This satisfies $h_{t=0} = h_0$ and because of the analyticity assumption we have $\nu(t,r) = \sum_{i=1}^{\infty} a_i(r)t^i$ and $\mu(t,r) = \sum_{i=1}^{\infty} b_i(r)t^i$, with a set of differentiable functions $a_i = a_i(r)$ and $b_i = b_i(r)$ depending on the r variable only.

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- This leads to the following ansatz for the Poincaré-type metric $\hat{g}$ for the past eon $\hat{M}$:

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  Our (pre)past eon manifold $\hat{M}$ is parameterized by $t > 0$, $r > 0$ and $z \in \mathbb{C} \cup \{\infty\}$.

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$$K = \partial_t + \Big(1+\sum_{i=1}^{\infty} b_i(r)t^i\Big)^{-\frac{1}{2}}\partial_r.$$

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$$t^{-2}\left(-\mathrm{d}t^2 + \frac{2r^2\left(1+\sum_{i=1}^{\infty} a_i(r)t^i\right)\mathrm{d}z\mathrm{d}\bar{z}}{(1+\frac{z\bar{z}}{2})^2} + \left(1+\sum_{i=1}^{\infty} b_i(r)t^i\right)\mathrm{d}r^2\right)$$

satisfies Einstein's equations

$$\hat{E}_{ij} := \hat{R}_{ij} - \hat{\Lambda}\hat{g}_{ij} - \hat{\Phi}\hat{K}_i\hat{K}_j = 0$$

with

$$K = K^i\partial_i = \partial_t + \left(1+\sum_{i=1}^{\infty} b_i(r)t^i\right)^{-\frac{1}{2}}\partial_r, \qquad \hat{K}_i = \hat{g}_{ij}K^j,$$

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then we have:

- The coefficients $a_1(r)$, $a_2(r)$ $b_1(r)$ and $b_2(r)$ identically vanish, $a_1(r) = a_2(r) = b_1(r) = b_2(r) = 0$, and the power series expansion of h_t starts at the t^3 terms, $h_t = t^3\chi(r) + \mathcal{O}(t^4)$.
- **The metric** $\hat{g}$, or what is the same, the power series expansions $\nu(t,r) = \sum_{i=1}^{\infty} a_i(r)t^i$ and $\mu(t,r) = \sum_{i=1}^{\infty} b_i(r)t^i$, are **totally determined up to infinite order** by an arbitrary differentiable function $f = f(r)$.
- More precisely, the Einstein equations $\hat{E}_{ij} = \mathcal{O}(t^{k+1})$ solved up to an order k, together with an arbitrary differentiable function $f = f(r)$, uniquely determine $\nu(t,r)$ and $\mu(t,r)$ up to an order $(k+2)$.

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- In the lowest order the solution reads:

$$\nu = \frac{f}{r^3} t^3 + \mathcal{O}(t^4) \quad \text{and} \quad \mu = -\frac{2f}{r^3} t^3 + \mathcal{O}(t^4);$$

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- With the use of computers we calculated this solution up to the order $k = 10$, finding explicitly $\nu = \sum_{k=3}^{10} a_k t^k$ and $\mu = \sum_{k=3}^{10} b_k t^k$. The formulas are compact enough up to $k = 8$ and up to the order $k = 8$ they read:

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\begin{aligned}
\nu(t,r) =& f\,\frac{t^3}{r^3} - \frac{3}{4} f'\,\frac{t^4}{r^4} + \frac{1}{10}\left(-2rf' + 3r^2 f''\right)\frac{t^5}{r^5} + \\
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& \frac{r}{280}\left(-24f' - 105ff' + 24rf'' - 12r^2 f^{(3)} + 5r^3 f^{(4)}\right)\frac{t^7}{r^7} - \\
& \frac{r}{960}\left(60f' + 288ff' - 150rf'^2 - 60rf'' - 216rff'' + 30r^2 f^{(3)} - 10r^3 f^{(4)} + 3r^4 f^{(5)}\right)\frac{t^8}{r^8} + \\
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Possible generalizations

- For a solution up to this order we find that:

$$\begin{aligned}\hat{\Phi} =\, & 3r^3 f' \tfrac{t^6}{r^6} + 3r^3 (f' - rf'') \tfrac{t^7}{r^7} + \tfrac{3r^3}{2} (2f' - 2rf'' + r^2 f^{(3)}) \tfrac{t^8}{r^8} + \\ & \tfrac{r^3}{2} (6f' + 6ff' - 6rf'' + 3r^2 f^{(3)} - r^3 f^{(4)}) \tfrac{t^9}{r^9} + \\ & \tfrac{r^3}{8} (24f' + 66ff' - 12rf'^2 - 24rf'' - 30rff'' + 12r^2 f^{(3)} - 4r^3 f^{(4)} + r^4 f^{(5)}) \tfrac{t^{10}}{r^{10}} + \\ & \tfrac{r^3}{40} (120f' + 522ff' - 177rf'^2 - 120rf'' - 378rff'' + 93r^2 f' f'' + 60r^2 f^{(3)} + 90r^2 ff^{(3)} - 20r^3 f^{(4)} + 5r^4 f^{(5)} \\ & \mathcal{O}(\left(\tfrac{t}{r}\right)^{12}),\end{aligned}$$

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- I have no patience to type the Weyl tensor components up to high order. It is enough to say that that up to the 4th order in t, modulo a **nonzero constant** tensor $C^i{}_{jkl}$, it is equal to:

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$$\begin{aligned}\hat{\Phi} =\;& 3r^3 f' \frac{t^6}{r^6} + 3r^3 (f' - rf'') \frac{t^7}{r^7} + \frac{3r^3}{2} (2f' - 2rf'' + r^2 f^{(3)}) \frac{t^8}{r^8} + \\ & \frac{r^3}{2} (6f' + 6ff' - 6rf'' + 3r^2 f^{(3)} - r^3 f^{(4)}) \frac{t^9}{r^9} + \\ & \frac{r^3}{8} (24f' + 66ff' - 12rf'^2 - 24rf'' - 30rff'' + 12r^2 f^{(3)} - 4r^3 f^{(4)} + r^4 f^{(5)}) \frac{t^{10}}{r^{10}} + \\ & \frac{r^3}{40} (120f' + 522ff' - 177rf'^2 - 120rf'' - 378rff'' + 93r^2 f' f'' + 60r^2 f^{(3)} + 90r^2 ff^{(3)} - 20r^3 f^{(4)} + 5r^4 f^{(5)} \\ & \mathcal{O}\left(\left(\frac{t}{r}\right)^{12}\right),\end{aligned}$$

$$\hat{\Lambda} = 3 + \mathcal{O}(t^9).$$

- I have no patience to type the Weyl tensor components up to high order. It is enough to say that that up to the 4th order in t, modulo a **nonzero constant** tensor $C^i{}_{jkl}$, it is equal to:

$$W^i{}_{jkl} = \left(\frac{f}{r^2}\frac{t}{r} - \frac{f'}{r}\frac{t^2}{r^2} + \frac{f''}{2}\frac{t^3}{r^3}\right) C^i{}_{jkl} + \mathcal{O}\left(\left(\frac{t}{r}\right)^4\right).$$

- Of course, for the positivity of the energy density $\hat{\Phi}$ close to the surface $\mathscr{I}^+$ of $\hat{M}$ we need

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The Poincaré-type metric $\hat{g}$ can be interpreted as the ending stage of the evolution of the past eon in Penrose's CCC. The eon has a positive cosmological constant $\hat{\Lambda} \simeq 3$, which is filled with a spherically symmetric pure radiation moving along the null congruence generated by the vector field K.

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Possible generalizations

For the solutions $\nu(t,r)$, $\mu(t,r)$ of the past eon's Einstein's equations, which were given in terms of the power series expansions as $\nu(t,r) = \sum_{i=3}^{k+2} a_i(r)t^i + \mathcal{O}(t^{k+3})$ and $\mu(t,r) = \sum_{i=3}^{k+2} b_i(r)t^i + \mathcal{O}(t^{k+3})$ in Theorem 1, the formulae for the power series expansions of the energy densities $\check{\Phi}$ $\check{\Psi}$, $\check{\rho}$ and the pressure $\check{p}$ are as follows:

$$\begin{aligned}\check{\Phi} = &- \frac{9f}{r^3}t^{-3} + \frac{9f'}{r^3}t^{-2} + \frac{1}{2r^4}(8f' - 11rf'')t + \frac{3}{4r^5}(5f' - 5rf'' + 3r^2f^{(3)}) + \\ &\frac{9}{40r^6}(16f' + 5ff' - 16rf'' + 8r^2f^{(3)} - 3r^3f^{(4)})t + \\ &\frac{1}{120r^7}(420f' + 1068ff' - 30rf'^2 - 420rf'' - 384rff'' + 210r^2f^{(3)} - 70r^3f^{(4)} + 19r^4f^{(5)})t^2 + \\ &\cdots + \mathcal{O}(t^{k-3}),\end{aligned}$$

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For the solutions $\nu(t,r)$, $\mu(t,r)$ of the past eon's Einstein's equations, which were given in terms of the power series expansions as $\nu(t,r) = \sum_{i=3}^{k+2} a_i(r)t^i + \mathcal{O}(t^{k+3})$ and $\mu(t,r) = \sum_{i=3}^{k+2} b_i(r)t^i + \mathcal{O}(t^{k+3})$ in Theorem 1, the formulae for the power series expansions of the energy densities $\check{\Phi}$ $\check{\Psi}$, $\check{\rho}$ and the pressure $\check{p}$ are as follows:

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$$\check{\Psi} = -\frac{9f}{r^3}t^{-3} + \frac{6f'}{r^3}t^{-2} + \frac{1}{2r^4}(2f' - 5rf'')t^{-1} + \frac{3}{4r^5}(f' - rf'' + r^2 f^{(3)}) + \frac{1}{40r^6}(24f' - 75ff' - 24rf'' + 12r^2 f^{(3)} - 7r^3 f^{(4)})t + \frac{1}{60r^7}(30f' + 39ff' + 75rf'^2 - 30rf'' + 33rff'' + 15r^2 f^{(3)} - 5r^3 f^{(4)} + 2r^4 f^{(5)})t^2 + \cdots + \mathcal{O}(t^{k-3}),$$

$$\begin{aligned}\check{\rho} =& 3t^{-4} + \frac{18f}{r^3}t^{-1} - \frac{18f'}{r^3} + \frac{-6f' + 9rf''}{r^4}t - \frac{3}{4r^6}(9f^2 + 3rf' - 3r^2f'' + 2r^3f^{(3)})t^2 + \\ & \frac{3}{20r^6}(-24f' + 105ff' + 24rf'' - 12r^2f^{(3)} + 5r^3f^{(4)})t^3 - \\ & \frac{1}{20r^7}(60f' + 96ff' + 120rf'^2 - 60rf'' + 72rff'' + 30r^2f^{(3)} - 10r^3f^{(4)} + 3r^4f^{(5)})t^4 + \\ & \cdots + \mathcal{O}(t^{k-1}),\end{aligned}$$

$$\begin{aligned}\check{p} =& t^{-4} + \frac{6f}{r^3}t^{-1} + \frac{1}{r^4}(2f' - rf'')t + \frac{1}{2r^6}(18f^2 + 3rf' - 3r^2f'' + r^3f^{(3)})t^2 - \\ & \frac{3}{20r^6}(-8f' + 45ff' + 8rf'' - 4r^2f^{(3)} + r^3f^{(4)})t^3 + \\ & \frac{1}{30r^7}(30f' + 57ff' + 45rf'^2 - 30rf'' + 39rff'' + 15r^2f^{(3)} - 5r^3f^{(4)} + r^4f^{(5)})t^4 + \\ & \cdots + \mathcal{O}(t^{k-1}).\end{aligned}$$

In these formulas all the *doted* terms are explicitly determined in terms of f and its derivatives (I was lazy, and typed only the terms adapted to the choice $k = 6$ in Theorem 1).

Possible generalizations

$$\begin{aligned}\check{\rho} =& 3t^{-4} + \frac{18f}{r^3}t^{-1} - \frac{18f'}{r^3} + \frac{-6f' + 9rf''}{r^4}t - \frac{3}{4r^6}\left(9f^2 + 3rf' - 3r^2f'' + 2r^3f^{(3)}\right)t^2 + \\ & \frac{3}{20r^6}\left(-24f' + 105ff' + 24rf'' - 12r^2f^{(3)} + 5r^3f^{(4)}\right)t^3 - \\ & \frac{1}{20r^7}\left(60f' + 96ff' + 120rf'^2 - 60rf'' + 72rff'' + 30r^2f^{(3)} - 10r^3f^{(4)} + 3r^4f^{(5)}\right)t^4 + \\ & \cdots + \mathcal{O}(t^{k-1}),\end{aligned}$$

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Remarks.

- Note that since in $\check{M}$ the time $t < 0$, the requirement that the energy densities are positive near the Big Bang hypersurface $t = 0$ implies that $f > 0$ in addition to $f' > 0$, the requirement we got from the past eon. Note also that $f > 0$ and $f' > 0$ are the only conditions needed for the positivity of energy densities, as the leading term in $\check{\rho}$ is $\check{\rho} \simeq 3t^{-4}$, and is positive regardless of the sign of t.
- Remarkably the leading terms in $\check{\rho}$ and $\check{p}$, i.e. the terms with negative powers in t, are proportional to each other with the numerical factor *three*. We have

$$\check{p} = \frac{1}{3}\check{\rho} + \mathcal{O}(t^0).$$

- This means that immediately after the Bang, apart from the matter content of the two spherical ingoing and outgoing waves in the new eon, there is also a scattered *radiation* there, described by the perfect fluid with $\check{p} = \frac{1}{3}\check{\rho}$.

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Possible generalizations

- This solution to the three metrics in Penrose-Tod's bandage region has the following apealing physical property: Immediately after the Bang, the spherical wave from the previous eon not only produces two spherical waves - one is obvious: it is the still expanding but damped wave that survived the Bang; the other is less obvious, becuase although it is still sphereical it **focuses** - but there is also there a third ingredient: it is a randomly scattered *radiation* described by the perfect fluid with $\check{p} = \frac{1}{3}\check{\rho}$.
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THANK YOU!