The Beltrami – de Sitter model

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Outline

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1. Part I: How does de Sitter space appeared?
2. Part II: Beltrami and de Sitter spaces in 2 dimensions
3. Part III: Relation to the split real form of the simple exceptional Lie group G_2
4. Part IV: Generalization to higher dimensions

Motto

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or sand to people in the desert —
but you can sell refrigerators to people in the desert,
and sand to people in the Arctic.

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$$g = c^2\mathrm{d}t^2 - R^2\cosh(\tfrac{ct}{R})(\mathrm{d}\chi^2 + \sin^2\chi(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)),$$
then this new **non-flat** metric also **satisfies Einstein's modified equations**, $G_{\mu\nu} = \Lambda g_{\mu\nu}$. But now **without matter**!.
- After 1929 **Edwin Hubble's** discovery of the **recession of galaxies**, when it became clear that the **Universe is expanding**, Einstein realized that his static model of the universe was **wrong** and that, consequently, the introduction of the cosmological constant Λ into his equations had been unnecessary. He therefore returned to the **original equations**, $G_{\mu\nu} = T_{\mu\nu}$, which have remained the canonical Einstein equations **almost** up to the present day.
- But the World remained with a byproduct of this story which is **de Sitter metric**
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which went to oblivion in physics for many years.

From Einstein Universe to de Sitter space

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- In this interpretation, the **early** Universe obeys the original Einstein equations $G_{\mu\nu} = T_{\mu\nu}$, but with the energy momentum tensor of an **exotic matter**: a perfect fluid with the equation of state $\rho = -p$. This matter is called the **inflaton field**. It is **the inflaton field** that **is the source of de Sitter space**.
- Later, when other forms of matter are created, the energy–momentum tensor of the inflaton field changes to reflect the new matter content. Throughout this evolution, the Universe continues to obey the original Einstein equations. No cosmological constant is required.

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- A new era began at the end of the 1990s, when it was discovered that **astronomical data** on cosmological distances **are consistent with** the Friedmann – Lemaître – Robertson – Walker **Big Bang cosmology** only if the Universe **has a positive cosmological constant**.
- As a result, the constant $\Lambda > 0$ reappeared in the **Einstein equations** $G_{\mu\nu} = \Lambda g_{\mu\nu} + T_{\mu\nu}$, and **these** are now regarded as **the correct equations** of General Relativity.
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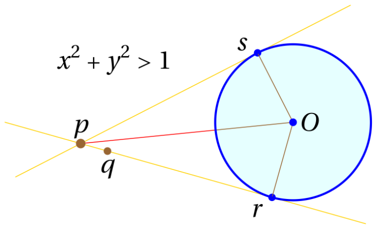
Outline

Natural Lorentz metric on the complement of a disk in 2D



- Remove the unit disk $B^2 = \{(x, y) \mid x^2 + y^2 \leq 1\}$ from the Euclidean (x, y) plane.
- Define a **conformal structure** on the resulting manifold $M^2 = \mathbb{R}^2 \setminus B^2$ by declaring that the **light cones** in M^2 are the cones whose tips lie in M^2 and whose generators are pairs of straight lines passing through the tip and tangent to the disk B^2.
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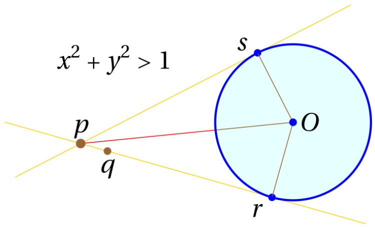

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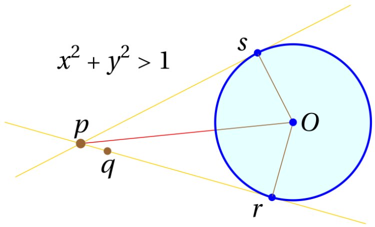

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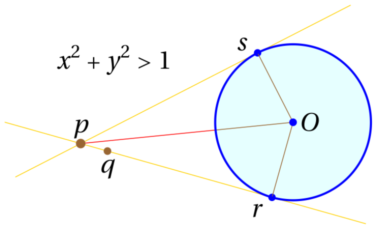

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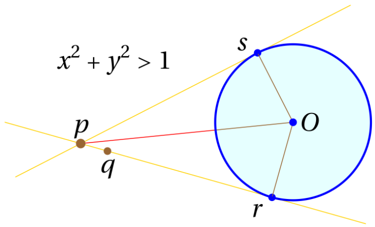

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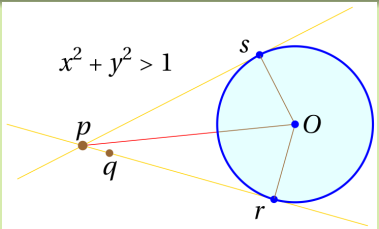

- We consider an infinitesimal vector $\overrightarrow{pq} = (\mathrm{d}x, \mathrm{d}y)$, associated with the pair of points $p = (x, y)$ and $q = (x + \mathrm{d}x, y + \mathrm{d}y)$, which we want to lie on a line tangent to the light cone with tip at p.
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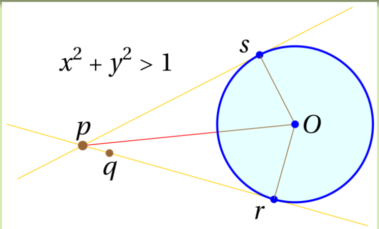

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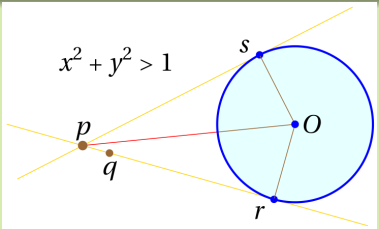

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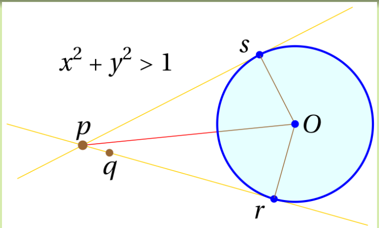

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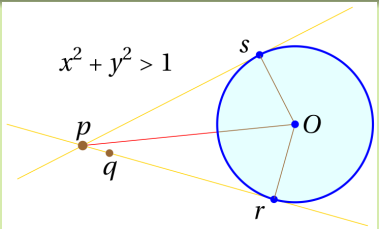

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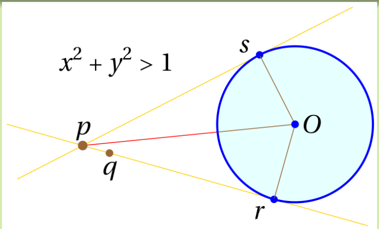

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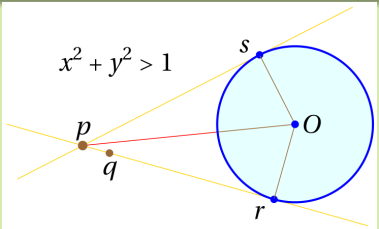

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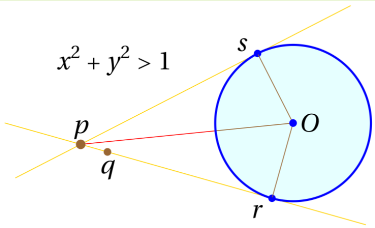

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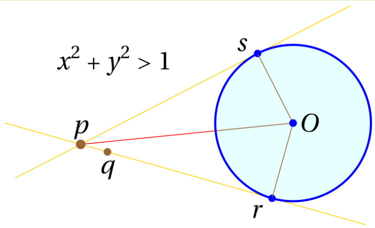

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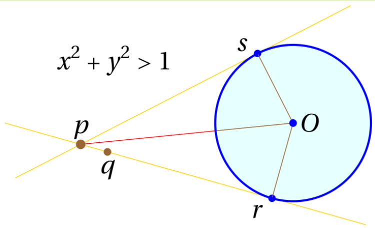

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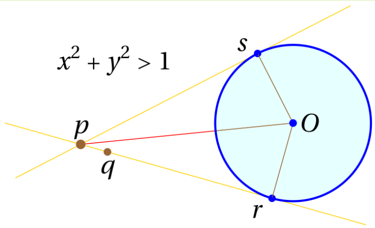

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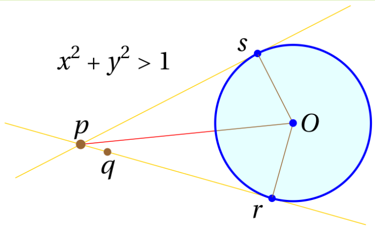

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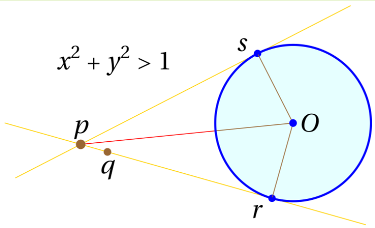

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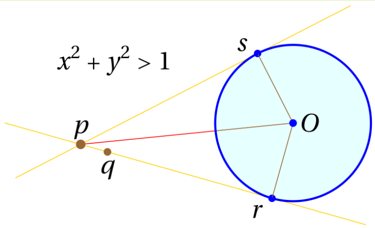

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which everywhere in $M^2 = \mathbb{R}^2 \setminus B^2$ has **Lorentzian signature.**

- This is a 2-dimensional **de Sitter metric.**
- Note also that this metric, although singular at the circle $x^2 + y^2 = 1$, is also well defined inside the disk B^2. Inside the disk, the metric is still Einstein, but it has **Riemannian signature.** This is a well know **Beltrami metric** in the disk, realizing the **Klein model** of non-Euclidean geometry. In particular, the **geodesics** in this metric connecting any two points in the disk are **chords** of the disk passing through these points.
- In this way we have a **Riemannian–Lorentzian hybrid** of the Beltrami–de Sitter metric $\mathrm{d}s^2$ on the entire plane $\mathbb{R}^2$, except the singular circle $x^2 + y^2 = 1$. We call the plane $\mathbb{R}^2$ equipped with the Einstein metric $\mathrm{d}s^2$ the **Beltrami–de Sitter model**.

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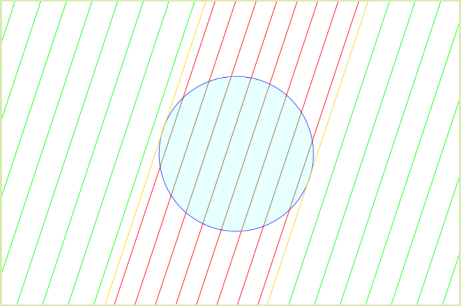

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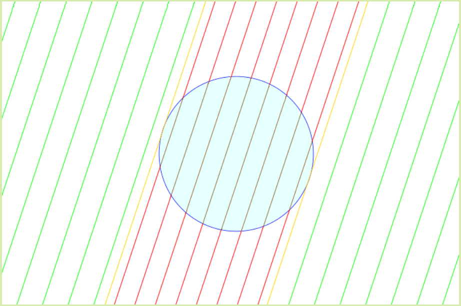

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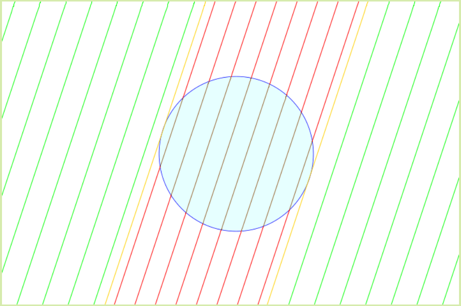

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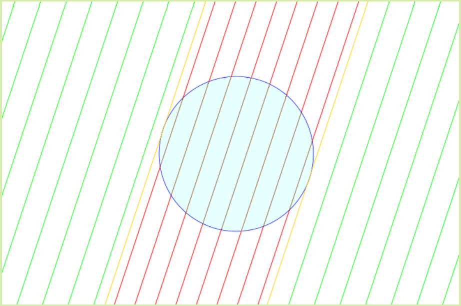

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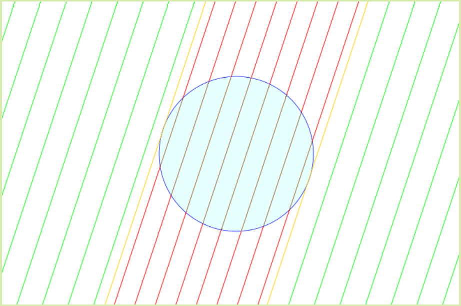

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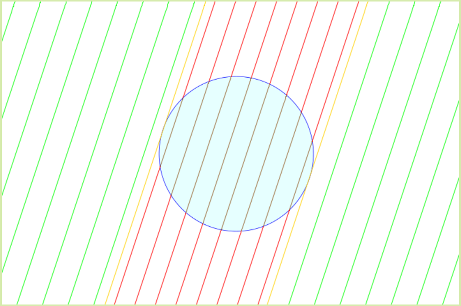

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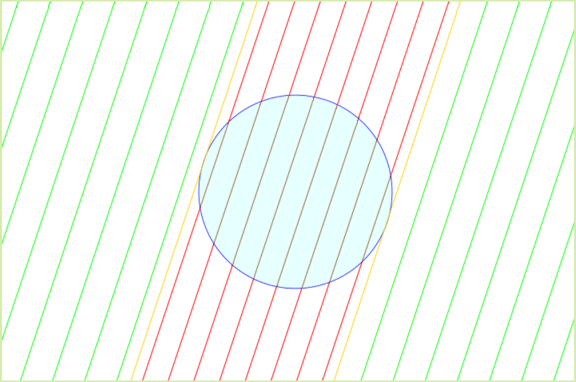

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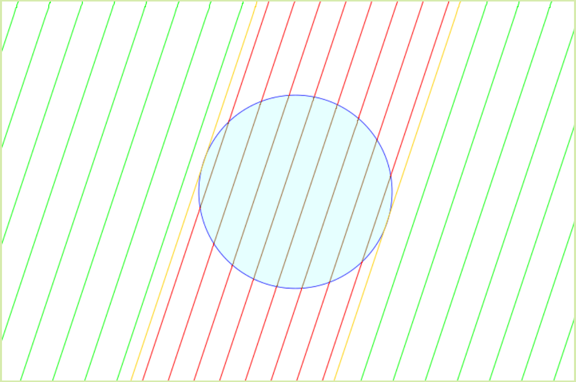

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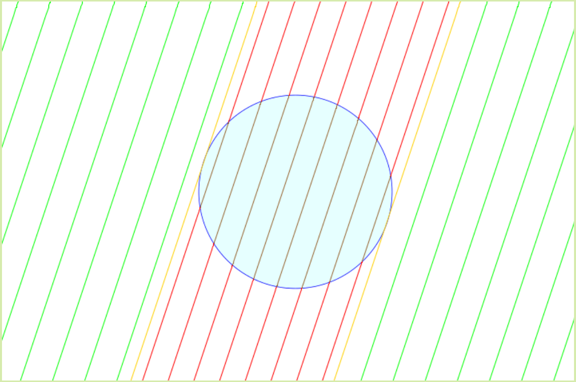

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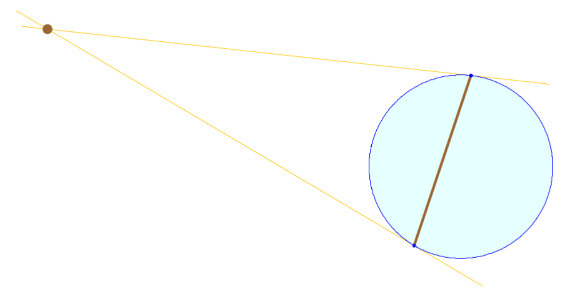
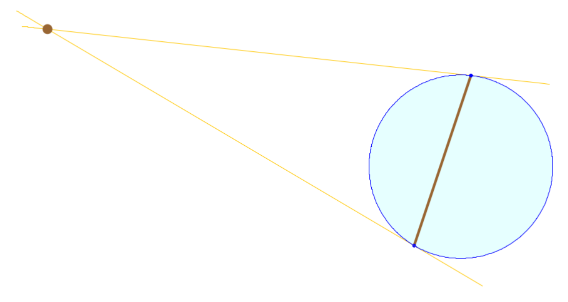
Points – lines duality



- Each point p outside the disk, **uniquely defines a Riemannian geodesic** c_p inside the disk.
- Given a point p outside the disk, consider its **light cone**, i.e. the two tangent lines to the disk outgoing from this point.
- The points of tangency of these lines define a **chord** c_p of the disk, i.e. a **geodesic** in the disk.
- But **also** every **point inside** the disk defines a **straight line outside** the disk!



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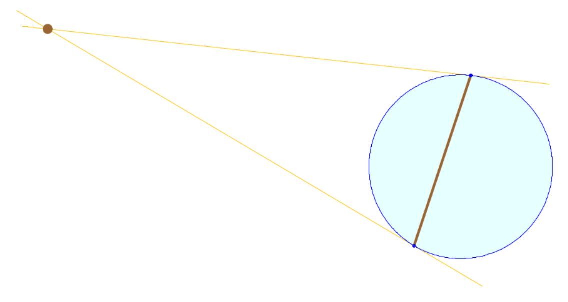

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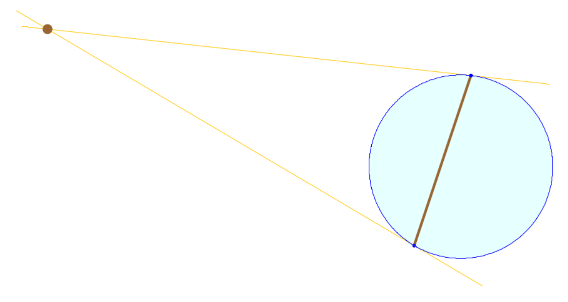

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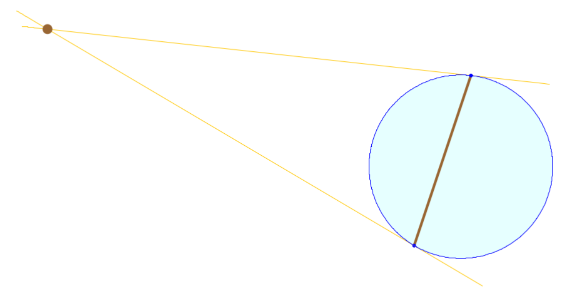

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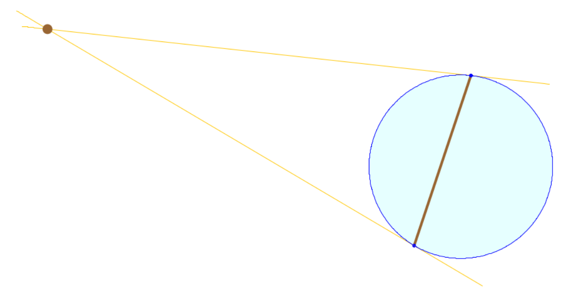

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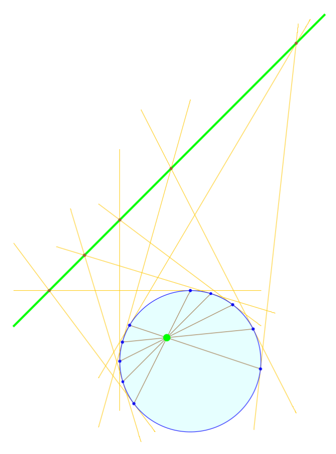

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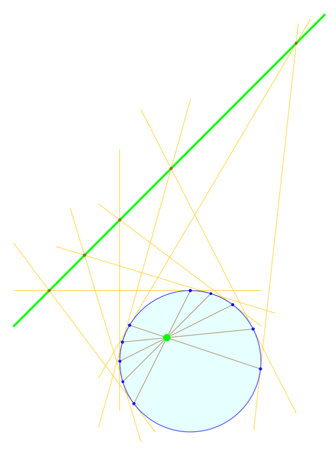

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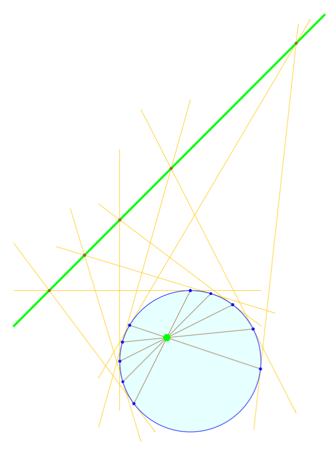

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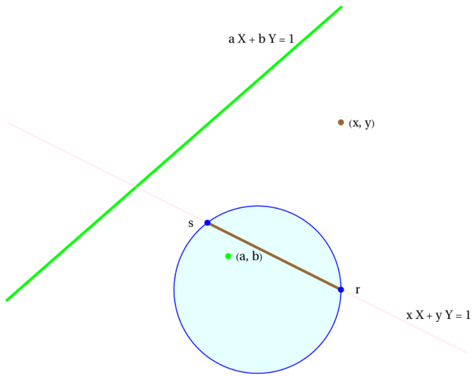

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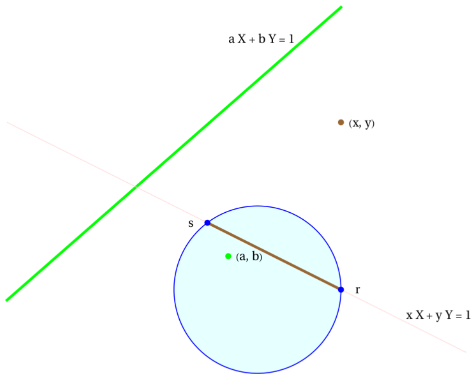

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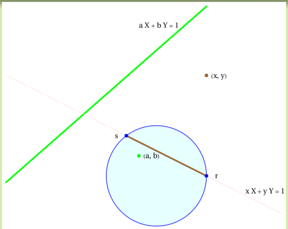

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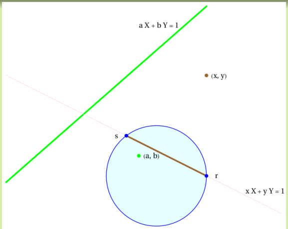

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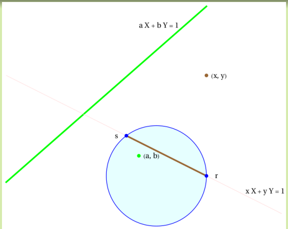

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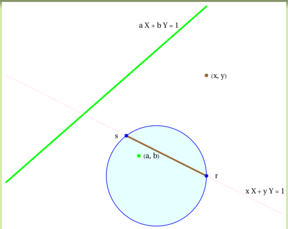

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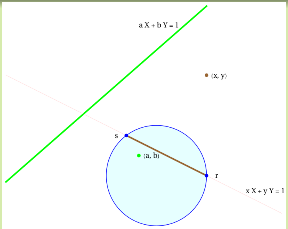

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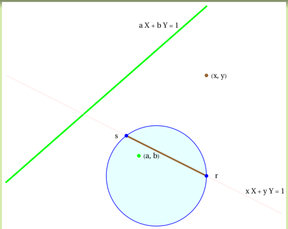

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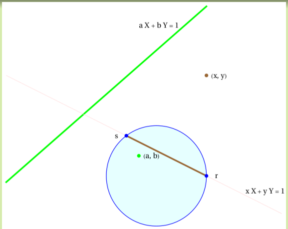

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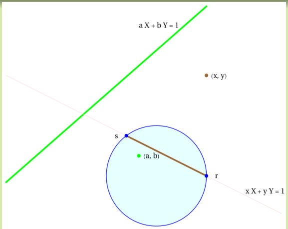

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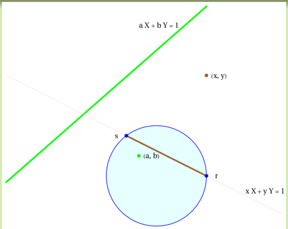

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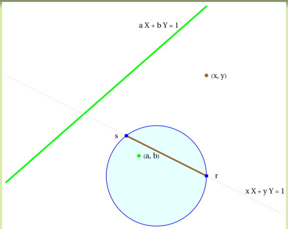

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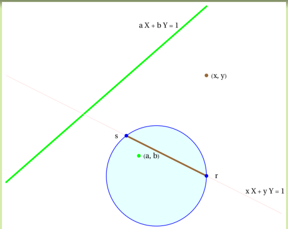

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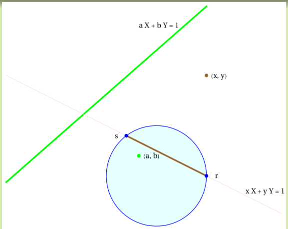

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- I now consider the space $T = M^2 \times B^2$ of all pairs $\xi = (p, P)$ such that $p \in M^2$ and $P \in B^2$. This space is 4-dimensional. Its points ξ can be parameterized by
$\xi = (x, y, a, b) \quad \text{with} \quad x^2 + y^2 > 1 \quad \text{and} \quad a^2 + b^2 < 1.$
- I call T the **correspondence space** for the 2-dimensional Beltrami–de Sitter model.
- We now show that the **correspondence space** T of the Beltrami – de Sitter model is naturally equipped with a **conformal structure**.
- Indeed...

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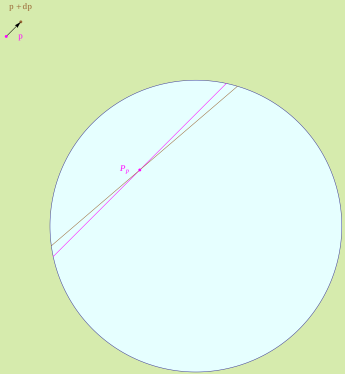
Ghosts



- If at a given time t point $p = p(t) \in M^2$ moves from p to $p + \mathrm{d}p$, its corresponding chord $c_p \subset B^2$ moves to a new chord $c_{p+\mathrm{d}p}$ in B^2, which is the chord of the infinitesimally shifted point $p + \mathrm{d}p$.
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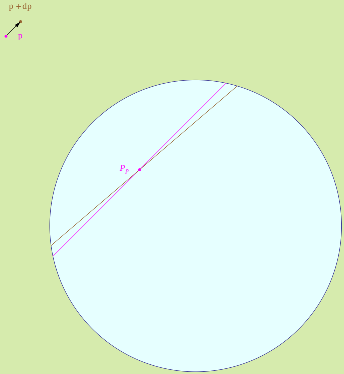

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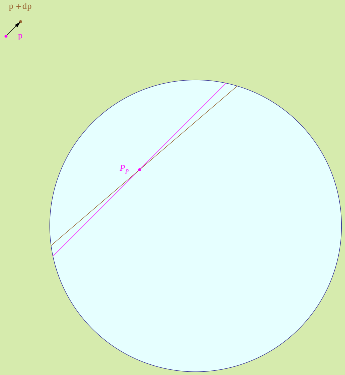

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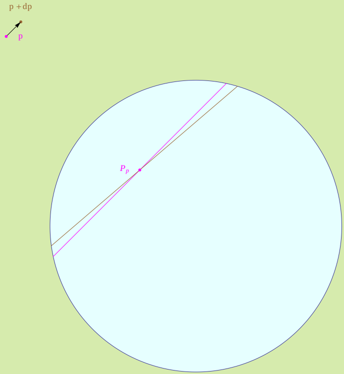

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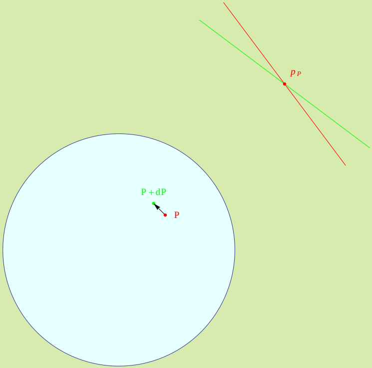

- Likewise, if at a given time t point $P = P(t) \in B^2$ moves from P to $P + \mathrm{d}P$, its corresponding spacelike line $\ell_P \subset M^2$ moves to a new line $\ell_{P+\mathrm{d}P}$ in M^2, which is the line of the infinitesimally shifted point $P + \mathrm{d}P$.
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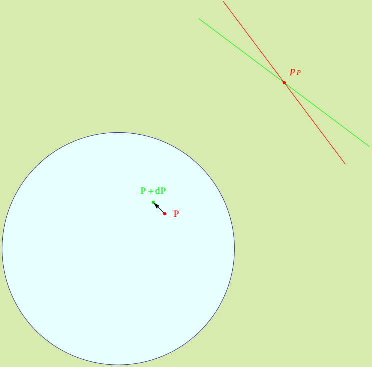

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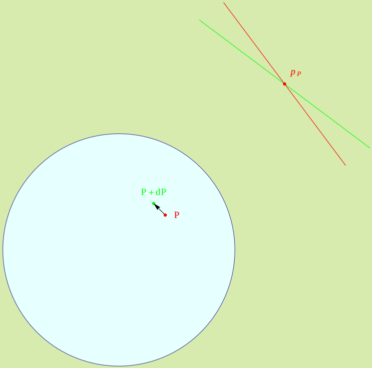

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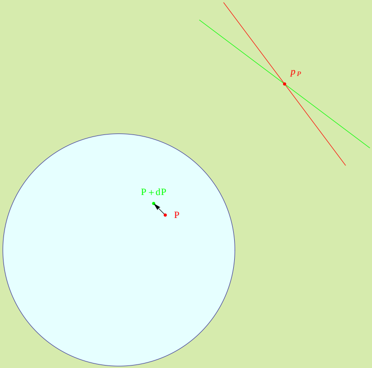

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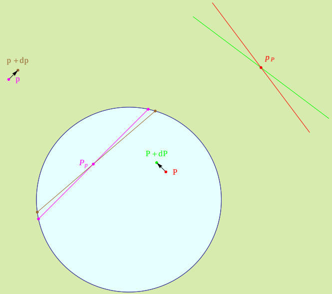

- Explicitly, if we have a **curve** $p(t) = (x(t), y(t)) \subset M^2$, i.e. a movement of a point $p(t)$ in the de Sitter space, its corresponding **ghost curve** in B^2 is given by
$$P_p(t) = \frac{1}{x\dot{y} - y\dot{x}}(\dot{y}, -\dot{x}).$$
- Similarly, if we have a **curve** $P(t) = (a(t), b(t)) \subset B^2$, i.e. a movement of a point $P(t)$ in the Beltrami space, its corresponding **ghost curve** in M^2 is given by
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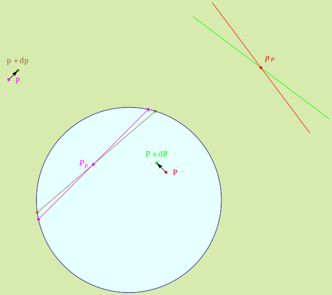

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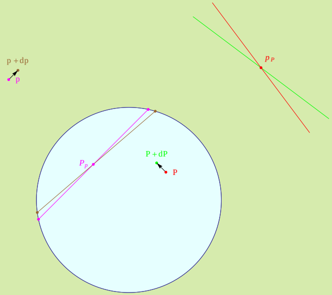

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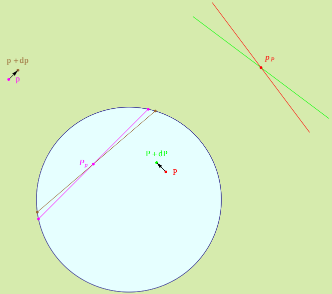

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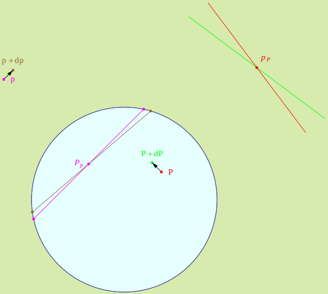

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Dancing pairs

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- We say that a pair (p, P) **dances** if the point p moves in M^2 in the direction of the image p_P in M^2 of the point $P \in B^2$, and if the point P moves in B^2 in the direction of the image P_p in B^2 of the point $p \in M^2$.
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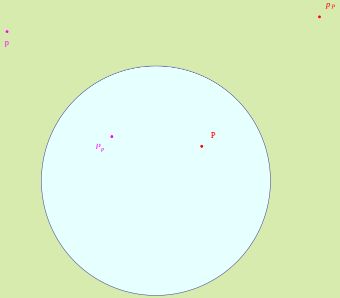

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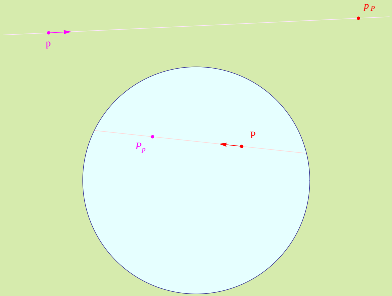

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- The metric g is **locally isometric** to the split-signature **cousin** of the **Fubini-Study metric** from the Riemannian signature.

de Sitter metric on the complement of the disk in the plane naturally conformally extends to the Lorentzian–Riemannian hybrid of the **Beltrami–de Sitter model**, and this, in turn, naturally defines a **4-dimensional split-signature dancing metric** on the model's correspondence space; this metric is self–dual and Einstein, and it possesses at least the $\mathfrak{sl}(3,\mathbb{R})$ **algebra** of **(conformal) Killing symmetries**.

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Outline

Glossary on vector distributions

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- A **rank** $k(\leq d)$ **(vector) distribution** $\mathcal{D}$ on an d-dimensional manifold M is a **smooth map** $M \ni p \to D_p \subset \mathrm{T}_pM$, where D_p is a k-dimensional vector subspace of T_pM.
- Given k vector fields $(X_1, X_2, \dots, X_k)$ on M such that $X_1 \wedge X_2 \wedge \dots \wedge X_k \neq 0$ at each point of M, a rank k distribution $\mathcal{D}$ can be written as $\mathcal{D} = \mathrm{Span}_{\mathcal{F}(M)}(X_1, X_2, \dots, X_k)$.
- We say that a vector field $X \in \mathcal{D}$ iff $X_p \in D_p$ for all $p \in M$.
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How to associate G_2 to de Sitter space?

- Take the **Beltrami – de Sitter model** associated with **de Sitter space** in 2D. Take its associated **correspondence space** (T, g) and its **dancing metric** g.
- Go to the **circle twistor bundle** $\mathbb{S}^1 \to \mathbb{T} \to T$ of (T, g), and consider the **twistor distribution** $\mathcal{D}$ on it.
- It is almost everywhere $(2, 3, 5)$ on $\mathbb{T}$ as the dancing metric g on T has **nowhere vanishing** selfdual Weyl tensor.
- Since the twistor distribution is cannonically defined by the **conformal structure** of g, the **conformal Killing symmetries** lift to **symmetries of the twistor distribution**.
- But the dancing metric, **by construction**, have at least $8 = \dim \mathfrak{sl}(\mathbf{3}, \mathbb{R})$ conformal Killing symmetries.
- So, by Cartan's result – $\dim \mathfrak{g} \geq 8 \implies \mathfrak{g}_2$ symmetry – the **twistor distribution of the dancing metric is** locally **split** G_2 symmetric.

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Outline

Short message

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- The interesting story is **how to associate the split G_2 from the de Sitter space when** $d > 2$.
- **I know** how to naturally associate split G_2 to the picture **when** $d = 3$ **and** (with a little sweat) **when** $d = 4$.
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Correspondence space and the dancing metric

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THANK YOU!

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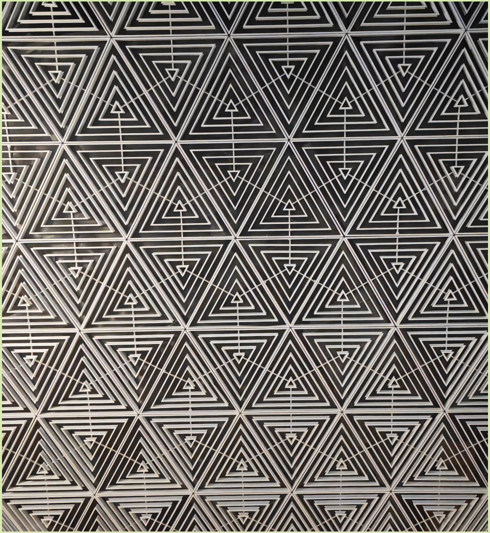

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