Simple models in Penrose's Conformal Cyclic Cosmology

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What is *CCC*?

- *CCC* or *C*onformal *C*yclic *C*osmology is a proposal for a Cosmology Hypothesis which answers the question 'What was before the Big Bang?'.
- The scheme of **Penrose**'s *CCC* is as follows:[1]

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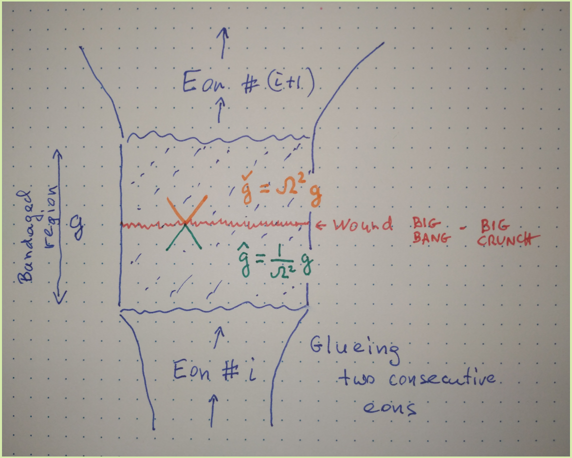
Eon # (i+1)
Bandaged region
g
$\check{g} = \Omega^2 g$
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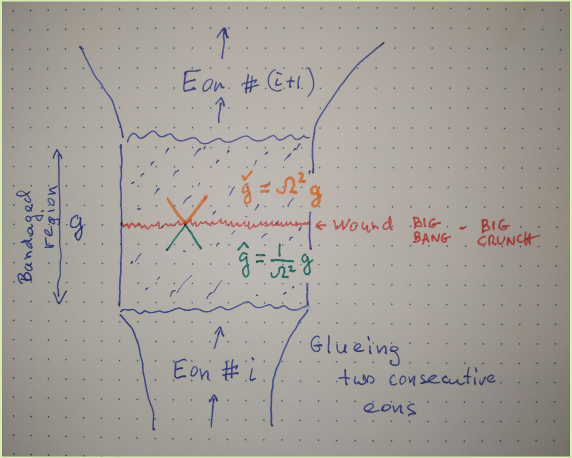
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If $\Omega = \Omega(\eta)$ is such that $\boxed{\check{g} = \Omega^2 g_{Einst}}$ **satisfies Einstein's equations**, with $\Lambda = 0$, and with **the energy momentum tensor** $\check{T}$ **of a perfect fluid**, whose presure $\check{p}$ is proportional to the energy density $\check{\mu}$, via $\check{p} = \check{w}\check{\mu}$, $\check{w} = const$, then $\boxed{\hat{g} = \frac{1}{\Omega^2} g_{Einst}}$ **satisfies Einstein's equations**, with $\Lambda = 0$, and with **the energy momentum tensor** $\hat{T}$ **of a perfect fluid**, whose presure $\hat{p}$ and the energy density $\hat{\mu}$ are related by $\hat{p} = \hat{w}\hat{\mu}$ with

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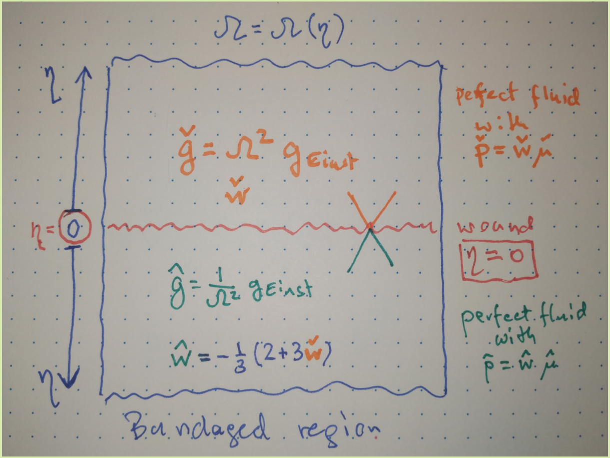
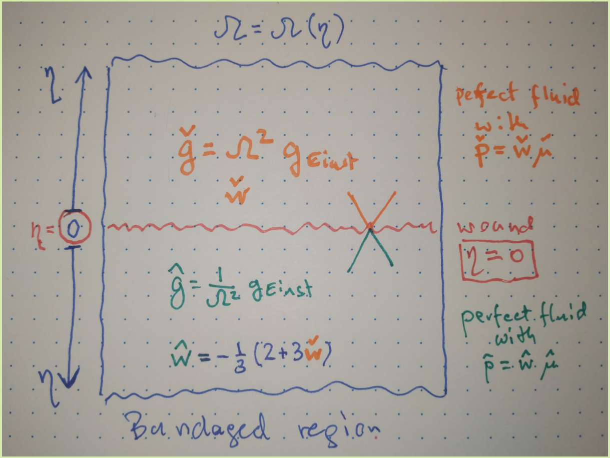
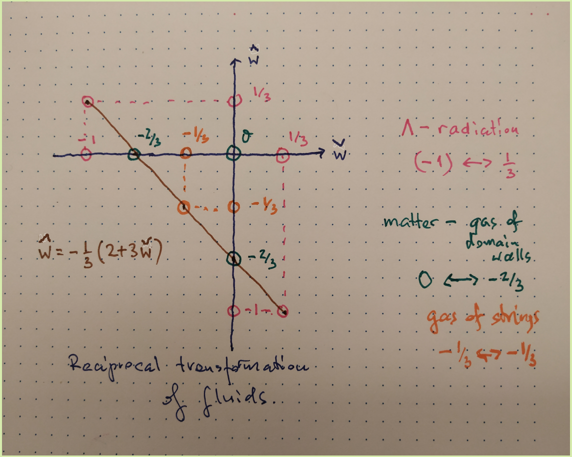
Transformation $\check{g} \to \hat{g} \to \check{g} \to \hat{g} \to \dots$ of fluids




Ω = Ω(η)
perfect fluid
with
p̌ = w̌ μ̌
wound
η = 0
perfect fluid
with
p̂ = ŵ. μ̂
ǧ = Ω² g_Einst
w̌
ĝ = 1/Ω² g_Einst
ŵ = −1/3 (2+3w̌)
η = 0
Bandaged region



Suspiscious points: $\breve{w} = -1, 1/3$ (cosmological constant - radiation), since the scalar curvature $R = 0$, when $\breve{w} = 1/3$; and $\breve{w} = -1/3$ (gas of strings), when $\Omega \neq 0$ on $\mathscr{I}$.

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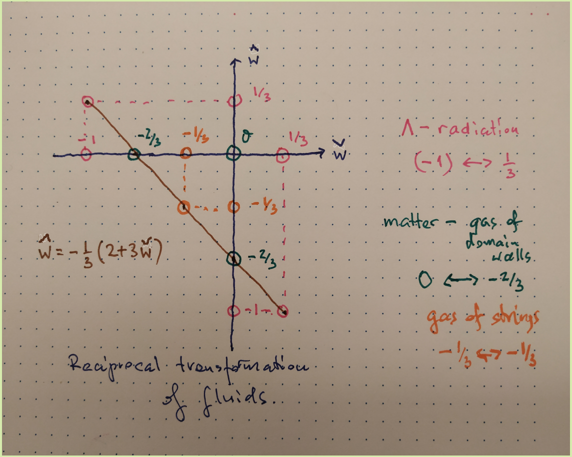

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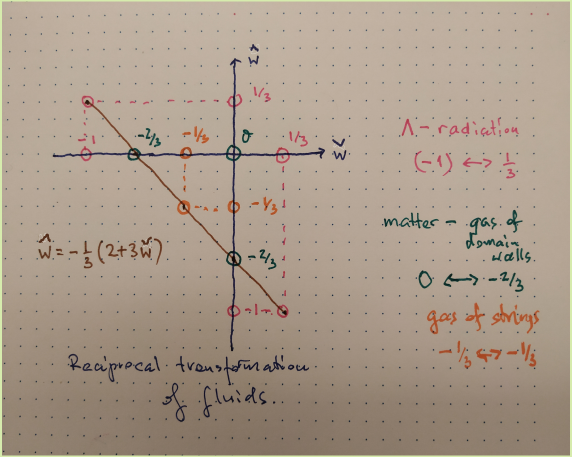

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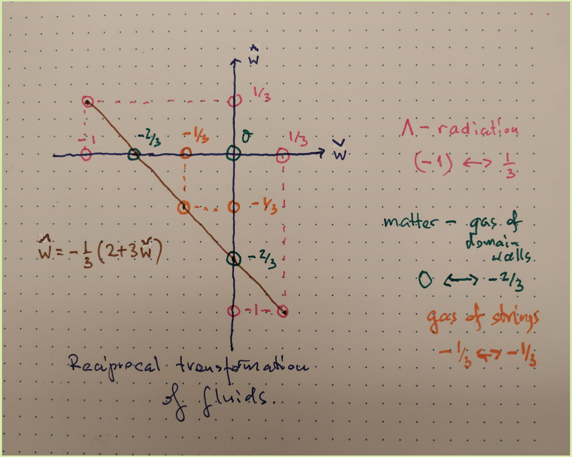

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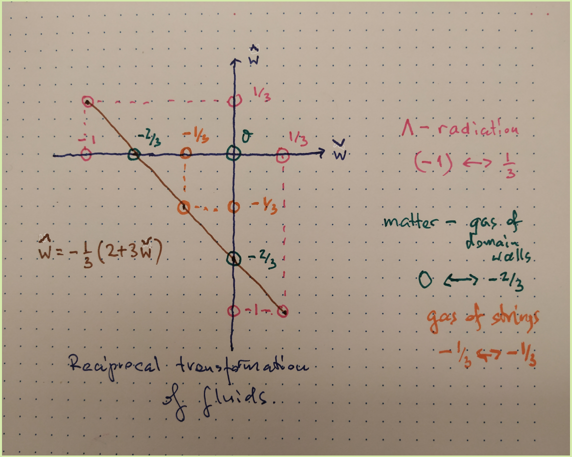

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Transformation $\check{g} \to \hat{g} \to \check{g} \to \hat{g} \to \dots$ of fluids: more careful approach

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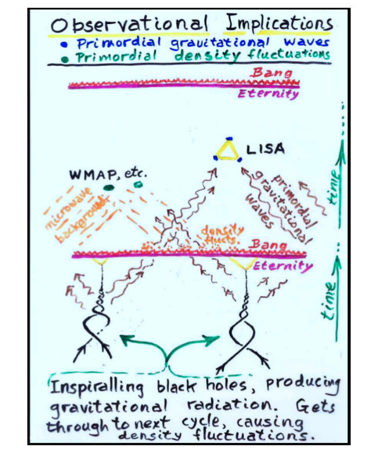

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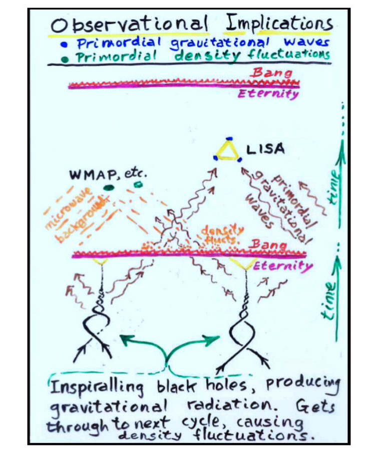
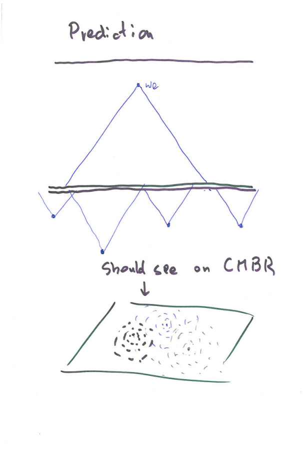
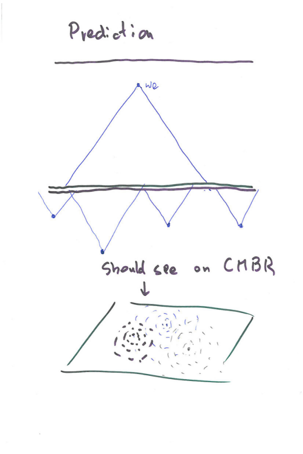


Motivation for the next model (picture by P.N.)

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we
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- Applying the reciprocal hypothesis, $\hat{g} \to \check{g} = t^4 \hat{g}$, I show that the **new eon** $(\check{M}, \check{g})$ created from the one containing a single spherical wave, **is filled** at its initial state **with three types of radiation**: (i) the **damped spherical wave** which continues its life from the previous eon, (ii) the **ingoing spherical wave** obtained as a result of a colision of the wave from the past eon with the Bang hypersurface and (iii) **randomly scattered waves** that could be interpreted as perfect fluid with the energy density $\check{\rho}$ and the isotropic pressure $\check{p}$ such that $\check{p} = \frac{1}{3}\check{\rho}$. The metric $\check{g}$ solves the Einstein's equations **without cosmological constant** and with the energy-momentum tensor

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- I start with a conformal class $[h_0]$ represented by the flat 3-dimensional metric

$$h_0 = \frac{2r^2 \mathrm{d}z\mathrm{d}\bar{z}}{(1+\frac{z\bar{z}}{2})^2} + \mathrm{d}r^2.$$

- Then I make Poincar'e anstaz by considering a 1-paramater family of 3-d metrics h_t. This will be a *spherically symmetric* family

$$h_t = \frac{2r^2\Big(1+\nu(t,r)\Big)\mathrm{d}z\mathrm{d}\bar{z}}{(1+\frac{z\bar{z}}{2})^2} + \Big(1+\mu(t,r)\Big)\mathrm{d}r^2,$$

where the unknown function $\nu = \nu(t,r)$ and $\mu = \mu(t,r)$ are both *real analytic* in the variable t and such that:
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Possible generalizations

- I start with a conformal class $[h_0]$ represented by the flat 3-dimensional metric
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- This leads to the following ansatz for the Poincaré-type metric $\hat{g}$ for the past eon $\hat{M}$:

$$\hat{g} = t^{-2}\Big(-\mathrm{d}t^2 + \frac{2r^2\big(1+\sum_{i=1}^{\infty} a_i(r)t^i\big)\mathrm{d}z\mathrm{d}\bar{z}}{(1+\frac{z\bar{z}}{2})^2} + \big(1+\sum_{i=1}^{\infty} b_i(r)t^i\big)\mathrm{d}r^2\Big).$$

  Our (pre)past eon manifold $\hat{M}$ is parameterized by $t>0$, $r>0$ and $z \in \mathbb{C} \cup \{\infty\}$.

- I now consider the following null vector field K on $\hat{M}$:

$$K = \partial_t + \Big(1+\sum_{i=1}^{\infty} b_i(r)t^i\Big)^{-\frac{1}{2}}\partial_r.$$

- It is tangent to a congruence of null geodesics without shear and twist, which represents light rays emanating from the source at the surface $r=0$.
- I require that the Poincaré-type metric $\hat{g}$ satisfies the Einstein equations $\hat{R}^{ij} = \hat{\Lambda}\hat{g}^{ij} + \hat{\Phi}K^iK^j$ with this null vector field K and some functions $\hat{\Phi}$ and $\hat{\Lambda}$. We have the following theorem/conjecture.

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If the metric

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satisfies Einstein's equations

$$\hat{E}_{ij} := \hat{R}_{ij} - \hat{\Lambda}\hat{g}_{ij} - \hat{\Phi}\hat{K}_i\hat{K}_j = 0$$

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$$\hat{E}_{ij} := \hat{R}_{ij} - \hat{\Lambda}\hat{g}_{ij} - \hat{\Phi}\hat{K}_i\hat{K}_j = 0$$

with

$$K = K^i\partial_i = \partial_t + \left(1+\sum_{i=1}^{\infty} b_i(r)t^i\right)^{-\frac{1}{2}}\partial_r, \qquad \hat{K}_i = \hat{g}_{ij}K^j,$$

then we have:

- The coefficients $a_1(r)$, $a_2(r)$ $b_1(r)$ and $b_2(r)$ identically vanish, $a_1(r) = a_2(r) = b_1(r) = b_2(r) = 0$, and the power series expansion of h_t starts at the t^3 terms, $h_t = t^3\chi(r) + \mathcal{O}(t^4)$.
- **The metric** $\hat{g}$, or what is the same, the power series expansions $\nu(t,r) = \sum_{i=1}^{\infty} a_i(r)t^i$ and $\mu(t,r) = \sum_{i=1}^{\infty} b_i(r)t^i$, are **totally determined up to infinite order** by an arbitrary differentiable function $f = f(r)$.
- More precisely, the Einstein equations $\hat{E}_{ij} = \mathcal{O}(t^{k+1})$ solved up to an order k, together with an arbitrary differentiable function $f = f(r)$, uniquely determine $\nu(t,r)$ and $\mu(t,r)$ up to an order $(k+2)$.

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$$\nu = \frac{f}{r^3}t^3 + \mathcal{O}(t^4) \quad \text{and} \quad \mu = -\frac{2f}{r^3}t^3 + \mathcal{O}(t^4);$$

The energy function $\hat{\Phi}$ and the cosmological constant $\hat{\Lambda}$ are:

$$\hat{\Phi} = 3\frac{f'}{r^3}t^6 + \mathcal{O}(t^7) \quad \text{and} \quad \hat{\Lambda} = 3 + \mathcal{O}(t^{k+3});$$

the Weyl tensor of the solution is

$$W^i{}_{jkl} = \mathcal{O}(t).$$

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- With the use of computers we calculated this solution up to the order $k = 10$, finding explicitly $\nu = \sum_{k=3}^{10} a_k t^k$ and $\mu = \sum_{k=3}^{10} b_k t^k$. The formulas are compact enough up to $k = 8$ and up to the order $k = 8$ they read:

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- Of course, for the positivity of the energy density $\hat{\Phi}$ close to the surface $\mathscr{I}^+$ of $\hat{M}$ we need

$$f' > 0.$$

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Possible generalizations

For the solutions $\nu(t,r)$, $\mu(t,r)$ of the past eon's Einstein's equations, which were given in terms of the power series expansions as $\nu(t,r) = \sum_{i=3}^{k+2} a_i(r)t^i + \mathcal{O}(t^{k+3})$ and $\mu(t,r) = \sum_{i=3}^{k+2} b_i(r)t^i + \mathcal{O}(t^{k+3})$ in Theorem 1, the formulae for the power series expansions of the energy densities $\check{\Phi}$ $\check{\Psi}$, $\check{\rho}$ and the pressure $\check{p}$ are as follows:

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$$
\begin{aligned}
\check{p} =& t^{-4} + \frac{6f}{r^3}t^{-1} + \frac{1}{r^4}\left(2f' - rf''\right)t + \frac{1}{2r^6}\left(18f^2 + 3rf' - 3r^2f'' + r^3f^{(3)}\right)t^2 - \\
& \frac{3}{20r^6}\left(-8f' + 45ff' + 8rf'' - 4r^2f^{(3)} + r^3f^{(4)}\right)t^3 + \\
& \frac{1}{30r^7}\left(30f' + 57ff' + 45rf'^2 - 30rf'' + 39rff'' + 15r^2f^{(3)} - 5r^3f^{(4)} + r^4f^{(5)}\right)t^4 + \\
& \cdots + \mathcal{O}(t^{k-1}).
\end{aligned}
$$

In these formulas all the *doted* terms are explicitly determined in terms of f and its derivatives (I was lazy, and typed only the terms adapted to the choice $k = 6$ in Theorem 1).

Possible generalizations

$$\begin{aligned}\check{\rho} =& 3t^{-4} + \frac{18f}{r^3}t^{-1} - \frac{18f'}{r^3} + \frac{-6f' + 9rf''}{r^4}t - \frac{3}{4r^6}\left(9f^2 + 3rf' - 3r^2f'' + 2r^3f^{(3)}\right)t^2 + \\ & \frac{3}{20r^6}\left(-24f' + 105ff' + 24rf'' - 12r^2f^{(3)} + 5r^3f^{(4)}\right)t^3 - \\ & \frac{1}{20r^7}\left(60f' + 96ff' + 120rf'^2 - 60rf'' + 72rff'' + 30r^2f^{(3)} - 10r^3f^{(4)} + 3r^4f^{(5)}\right)t^4 + \\ & \cdots + \mathcal{O}(t^{k-1}),\end{aligned}$$

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$$\begin{aligned}\check{\rho} =& 3t^{-4} + \frac{18f}{r^3}t^{-1} - \frac{18f'}{r^3} + \frac{-6f' + 9rf''}{r^4}t - \frac{3}{4r^6}\left(9f^2 + 3rf' - 3r^2f'' + 2r^3f^{(3)}\right)t^2 + \\ & \frac{3}{20r^6}\left(-24f' + 105ff' + 24rf'' - 12r^2f^{(3)} + 5r^3f^{(4)}\right)t^3 - \\ & \frac{1}{20r^7}\left(60f' + 96ff' + 120rf'^2 - 60rf'' + 72rff'' + 30r^2f^{(3)} - 10r^3f^{(4)} + 3r^4f^{(5)}\right)t^4 + \\ & \cdots + \mathcal{O}(t^{k-1}),\end{aligned}$$

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Remarks.

- Note that since in $\check{M}$ the time $t < 0$, the requirement that the energy densities are positive near the Big Bang hypersurface $t = 0$ implies that $f > 0$ in addition to $f' > 0$, the requirement we got from the past eon. Note also that $f > 0$ and $f' > 0$ are the only conditions needed for the positivity of energy densities, as the leading term in $\check{\rho}$ is $\check{\rho} \simeq 3t^{-4}$, and is positive regardless of the sign of t.
- Remarkably the leading terms in $\check{\rho}$ and $\check{p}$, i.e. the terms with negative powers in t, are proportional to each other with the numerical factor *three*. We have

$$\check{p} = \tfrac{1}{3}\check{\rho} + \mathcal{O}(t^0).$$

- This means that immediately after the Bang, apart from the matter content of the two spherical ingoing and outgoing waves in the new eon, there is also a scattered *radiation* there, described by the perfect fluid with $\check{p} = \frac{1}{3}\check{\rho}$.

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- This solution to the three metrics in Penrose-Tod's bandage region has the following apealing physical property: Immediately after the Bang, the spherical wave from the previous eon not only produces two spherical waves - one is obvious: it is the still expanding but damped wave that survived the Bang; the other is less obvious, becuase although it is still sphereical it **focuses** - but there is also there a third ingredient: it is a randomly scattered *radiation* described by the perfect fluid with $\check{p} = \frac{1}{3}\check{\rho}$.
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