# Simple models in Penrose's Conformal Cyclic Cosmology 

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## What is

- CCC or Conformal Cyclic Cosmology is a proposal for a Cosmology Hypothesis which answers the question 'What was before the Big Bang?'.
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## Penrose's onformal yclic Cosmology

- The Universe consists of eons, each being a time oriented spacetime, whose conformal compactifications have spacelike $\mathscr{I}$. The Weyl tensor of the metric on each $\mathscr{I}$ is zero.
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- cer says nothing about this what is the physics in a given eon when the physical age of it is normal; normal meaning that eon is neither too young nor too old. CCC tells what is going on when an eon is either about to die, or had just been born.
- In particular, CCC does not require that the eons have the same history! It is Conformal Cyclic Cosmology, and not Conformal Periodic Cosmology!
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## Penrose's onformal yclic Cosmology

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- Eons are ordered, and the conformal compactifications of consecutive eons, say the past one and the present one, are glued together along $\mathscr{I}^{+}$of the past eon, and $\mathscr{I}^{-}$of the present eon.
- The vicinity of the matching surface (the wound) of the past and the present eons - this region Penrose calls bandaged region for the two eons - is equipped with the following three metrics, which are conformally flat at the wound:
- a Lorentzian metric $g$ which is regular everywhere,
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## Penrose's onformal yclic Cosmology

- In a bandage region, the three metrics $g$, $g$ gand $\hat{g}$, are conformally related on their overlaping domains.
- How to make this relation specific is debatable, but Penrose proposes that $\check{g}=\Omega^{2} g$, and $\hat{g}=\frac{1}{\Omega^{2}} g$, with $\Omega \rightarrow 0$ on the wound.
- The metric $g$ g in the present eon is a physical metric there. Likewise, the metric $\hat{g}$ in the past eon is a physical metric there.
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Penrose's onformal yclic Cosmology


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## Modelling Penrose's CCC scenario

- Question: How to make a model of Penrose's bandaged region of two eons?
- One needs a function $\Omega$, vanishing on some spacelike hypersurface, and a regular Lorentzian 4-metric $g$, such that if $g ̆=\Omega^{2} g$ satisfies Einstein equations with some physically reasonable energy momentum tensor, then $\hat{g}=\frac{1}{\Omega^{2}} g$ also satisfies Einstein equations with possibly different, but still physically reasonable energy momentum tensor.
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## Modelling Penrose's CCC scenario

- Similar question to the question posed and solved by $\mathbf{H}$. Brinkman. In 1925 he asked a question 'when in a conformal class of metrics there could be two nonisometric Einstein metrics?'. Brinkman found all such metrics in dimension four. In every signature.
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## Polytrope perfect fluids in FLRW models

- Let us for a while restrict to the FLRW metrics with $\kappa=1$, $g_{\text {test }}=-\mathrm{d} t^{2}+\Omega^{2}(t) r_{0}^{2}\left(\mathrm{~d} \chi^{2}+\sin ^{2} \chi\left(\mathrm{~d} \theta^{2}+\sin ^{2} \theta \mathrm{~d} \phi^{2}\right)\right)$.
- It is convenient to introduce a conformal time $n=\int \frac{\mathrm{dt}}{\mathrm{dt}}$ so that the FLRW metric looks
i.e. $g_{\text {test }}=\Omega^{2}(\eta) g_{\text {Einst }}$.
- This parametrization is very convenient since taking $u=-\Omega(\eta) \mathrm{d} \eta$, the most general FLRW metric $g$ satisfying Einstein's equations

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\text { Ric }-\frac{1}{2} \text { Rgtest }=(\mu+p) u=u+p g_{t e s t}
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with polytropic equation of state $p=w \mu, w=$ const, is given by

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- Now we go back to the Penrose-Tod's bandage triple ( $\check{g}, g, \hat{g}$ ).
- Take $g$ as $g$ Einst, $g=g_{\text {Einst }}$
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If $\Omega=\Omega(\eta)$ is such that $g \check{g}=\Omega^{2} g_{\text {Einst }}$ satisfies Einstein's equations, with $\Lambda=0$, and with the energy momentum tensor $\bar{T}$ of a perfect fluid, whose presure $\check{p}$ is proportional to the energy density $\check{\mu}$, via $\check{p}=\check{w} \check{\mu}, \check{w}=$ const, then
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## Transformation



Suspiscious points: $\check{W}=-1,1 / 3$ (cosmological constant radiation), since the scalar curvature $R=0$, when $\check{w}=1 / 3$; and $\check{w}=-1 / 3$ (gas of strings), when $\Omega \neq 0$ on $\mathscr{I}$.


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- We come back to the FLRW metric $\check{g}=-\mathrm{d} t^{2}+\Omega^{2}(t) r_{0}^{2} g_{s^{3}}$.
- We write it as $\check{g}=\Omega^{2}(t)\left(-\frac{\mathrm{d}^{2}}{\Omega^{2}(t)}+r_{0}^{2} g_{s^{3}}\right)$, so that it is clear that $\breve{g}=\Omega^{2}(t) g_{\text {Einst }}$.
- Then the condition that g satisfies perfect fluid Eisntein's equations with $\check{u}=-\mathrm{d} t, \check{p}=\check{w} \check{\mu}$, and the cosmological constant $\Lambda$, is equivalent to the following ODE for $\Omega$ :
$2 t^{2} \Omega \Omega^{\prime \prime}-\left(1+3 W_{1}\right)\left(1+t^{-2} \Omega^{\prime 2}\right)+\left(1+\psi_{1} x+2 \Omega^{2}\right.$.
- We want that $\check{W}=$ const and that $\hat{g}=\frac{1}{\Omega^{2}} g_{\text {Einst }}$ satisfies perfect fluid Eisntein's equations with $\hat{u}=-\frac{\mathrm{dt}}{\Omega^{2}}, \hat{p}=\hat{w} \hat{\mu}$, the cosmological constant $\hat{\wedge}$, and $\hat{W}=$ const.
- From the Einstein's equations for $\hat{g}$ we easilly calculate $\hat{w}$, and forcing it to be constant, because of the above ODE satisfied by $\Omega$, we find that it is possible provided that:

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\dot{M}(1+w)(1-3 w)=0 .
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- Thus, a neccessary condition for both $\Omega$ and $\Omega^{-1}$ to describe the polytropes, is that either one of the $\wedge$ s is zero, or $\check{w}$ is of the 'radiation- $\wedge$ ' type.
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2 r_{0}^{2} \Omega \Omega^{\prime \prime}=-(1+3 \check{W})\left(1+r_{0}^{2} \Omega^{\prime 2}\right)+(1+\check{w}) \check{\Lambda} r_{0}^{2} \Omega^{2} .
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- Considering the case $\check{w}=1 / 3$, one shows that remarkably $\hat{w}=1 / 3$ (generalization of the result of Paul Tod). More explicitly this case can be integrated to the very end.
- Theorem. The function $\Omega=\Omega(t)$ given by:

has the property that both $\check{g}=\Omega^{2} g_{\text {Einst }}$ and $\hat{g}=\Omega^{-2} g_{\text {Einst }}$ satisfy Einstein's equations with polytropic perfect fluid equation of state, for which $\hat{w}=w=1 / 3$ (radiation), and with the corresponding cosmological constants $\Lambda$ and $\hat{\wedge}$. Here $g_{\text {Einst }}=-\Omega^{-2} \mathrm{~d} t^{2}+r_{0}^{2} g_{\mathbb{S}^{3}}$.
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\Omega^{2}=\frac{3-3 \cosh \left(2 \sqrt{\frac{\lambda}{3}} t\right)-2 r_{0}^{2} \sqrt{\lambda \hat{\Lambda}} \sinh \left(2 \sqrt{\frac{\lambda}{3}} t\right)}{\hat{\Lambda} r_{0}^{2}}
$$

has the property that both $\check{g}=\Omega^{2} g_{\text {Einst }}$ and $\hat{g}=\Omega^{-2} g_{\text {Einst }}$ satisfy Einstein's equations with polytropic perfect fluid equation of state, for which $\widehat{w}=\check{w}=1 / 3$ (radiation), and with the corresponding cosmological constants $\Lambda$ and $\hat{\Lambda}$.

Colloquially speaking incoherent radiation passes happily through the wound. However, cosmological constants can change from any positive value to any other one. Ha.

- Considering the case $\check{w}=1 / 3$, one shows that remarkably $\hat{w}=1 / 3$ (generalization of the result of Paul Tod). More explicitly this case can be integrated to the very end.
- Theorem. The function $\Omega=\Omega(t)$ given by:

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## Motivation for the next model (picture by R. Penrose)



## Motivation for the next model (picture by R. Penrose)



Motivation for the next model (picture by P.N.)


Motivation for the next model (picture by P.N.)


## Possible generalizations

- I consider two consecutives eons $\hat{M}$ and $M$ from Penrose's Conformal Cyclic Cosmology and study how the matter content of the past eon ( $\bar{M})$ determines the matter content of the present eon $(M)$ by means of the reciprocity hypothesis.
- I assume that the only matter content in the final stages of the past eon is a spherical wave described by Einstein's equations with the pure radiation energy momentum tensor

$$
\hat{T}^{i j}-\hat{\omega} K^{i} K^{i} \quad \quad \hat{\sigma} K^{i} K^{i}-0,
$$

and with cosmological constant $\hat{\wedge}$. I solve these Einstein's equations associating to $\hat{M}$ the metric $\hat{g}=t^{-2}\left(-\mathrm{d} t^{2}+h_{t}\right)$, which is a Lorentzian analog of the Poincaré-Einstein metric known from the theory of conformal invariants. The solution is obtained under the assumption that the 3-dimensional conformal structure [ $h$ ] on the $\mathscr{I}^{+}$of $\hat{M}$ is flat, that the metric $\hat{g}$ admits a power series expansian in the time variable $t$, and that $h_{0}=h_{t=0} \in[h]$. It follows that such a solution depends on precisely one real arbitrary function of the radial variable $r$.

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\hat{T}^{i j}=\hat{\Phi} K^{i} K^{j}, \quad \hat{g}_{i j} K^{i} K^{j}=0,
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## Possible generalizations

- Applying the reciprocal hypothesis, $\hat{g} \rightarrow \check{g}=t^{4} \hat{g}$, I show that the new eon $(M, g)$ created from the one containing a single spherical wave, is filled at its initial state with three types of radiation: (i) the damped spherical wave which continues its life from the previous eon, (ii) the ingoing spherical wave obtained as a result of a colision of the wave from the past eon with the Bang hypersulface and (iii) randomily scattered waves that could be interpreted as perfect fluid with the energy density $\check{p}$ and the isotropic pressure $\check{p}$ such that $\check{p}=\frac{1}{3} \check{\rho}$. The metric $\check{g}$ solves the Einstein's equations without cosmological constant and with the energy-momentum tensor
$\check{T}^{i j}=\check{\phi} K^{i} K^{j}+\breve{\psi} L^{i} L^{j}+(\check{p}+\check{p}) \breve{u}^{i} \check{u}^{j}+\check{p}^{\prime \prime}{ }^{i j}$,
in which $\check{u}^{i} \check{u}^{j} \check{g}_{i j}=-1, \check{g}_{i j} L^{i} L^{j}=0$ and $L^{i} K^{i} \check{g}_{i j}=-2$.
- Applying the reciprocal hypothesis, $\hat{g} \rightarrow \check{g}=t^{4} \hat{g}$, I show that the new eon ( $\check{M}, g ̆ g)$ created from the one containing a single spherical wave, is filled at its initial state with three types of radiation: (i) the damped spherical wave which continues its life from the previous eon, (ii) the ingoing spherical wave obtained as a result of a colision of the wave from the past eon with the Bang hypersurface and (iii) randomly scattered waves that could be interpreted as perfect fluid with the energy density and the isotropic pressure $\check{p}$ such that $\check{p}=\frac{1}{3} \check{p}$. The metric $\check{g}$ solves the Einstein's equations without cosmological constant and with the energy-momentum tensor
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# solves the Einstein's equations without cosmological constant <br> and with the energy-momentum tensor 

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$$
\check{T}^{i j}=\check{\phi} K^{i} K^{j}+\check{\psi} L^{i} L^{j}+(\check{\rho}+\check{p}) \check{u}^{i} \breve{u}^{j}+\check{p} \breve{g}^{i j},
$$

in which $\check{u} \check{i}^{\prime} \check{u} \check{g}_{i j}=-1, \check{g}_{i j} L^{i} L^{j}=0$ and $L^{i} K^{j} \check{g}_{i j}=-2$.

## Possible generalizations

- I start with a conformal class $\left[h_{0}\right]$ represented by the flat 3-dimensional metric

$$
h_{0}=\frac{2 r^{2} \mathrm{~d} z \mathrm{~d} \bar{z}}{\left(1+\frac{z \bar{z}}{2}\right)^{2}}+\mathrm{d} r^{2}
$$

- Then I make Poincar'e anstaz by considering a 1-paramater family of 3-d metrics $h_{t}$. This will be a spherically symmetric family

$$
h_{t}=\frac{2 r^{2}(1+\nu(t, r)) \mathrm{d} z \mathrm{~d} \bar{z}}{\left(1+\frac{z z}{2}\right)^{2}}+(1+\mu(t, r)) \mathrm{d} r^{2}
$$

where the unknown function $\nu=\nu(t, r)$ and $\mu=\mu(t, r)$ are both real analytic in the variable $t$ and such that: $\nu(0, r)=0$ and $\mu(0, r)=0$.

- This satisfies $h_{t=0}=h_{0}$ and because of the analyticity assumption we have $\nu(t, r)=\sum_{i=1}^{\infty} a_{i}(r) t^{i}$ and $\mu(t, r)=\sum_{i=1}^{\infty} b_{i}(r) t^{\prime}$. with a set of differentiable functions $a_{i}=a_{i}(r)$ and $b_{i}=b_{i}(r)$ depending on the $r$ variable only.
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h_{0}=\frac{2 r^{2} \mathrm{dzd} \overline{\bar{z}}}{\left(1+\frac{\frac{z}{2}}{2}\right)^{2}}+\mathrm{d} r^{2} .
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\nu(0, r)=0 \quad \text { and } \quad \mu(0, r)=0 .
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## Possible generalizations

- This leads to the following ansatz for the Poincaré-type metric $\hat{g}$ for the past eon $\hat{M}$ :
$\hat{g}=t^{-2}\left(-\mathrm{d} t^{2}+\frac{2 r^{2}\left(1+\sum_{i=1}^{\infty} a_{i}(r) t^{\prime}\right) \mathrm{d} z \mathrm{~d} \bar{z}}{\left(1+\frac{z \overline{2}}{2}\right)^{2}}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right) \mathrm{d} r^{2}\right)$.
Our (pre)past eon manifold $\hat{M}$ is parameterized by $t>0, r>0$ and $z \in \mathbb{C} \cup\{\infty\}$.
- I now consider the following null vector field $K$ on $\hat{M}$ :

$$
K=\partial_{t}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right)^{-\frac{1}{2}} \partial_{r}
$$

- It is tangent to a congruence of null geodesics without shear and twist, which represents light rays emanating from the source at the surface $r=0$.
- I require that the Poincaré-type metric $\hat{g}$ satisfies the Einstein equations $\hat{R}^{i j}=\hat{\Lambda} \hat{g}^{i j}+\hat{\Phi} K^{i} K^{j}$ with this null vector field $K$ and some functions $\hat{\Phi}$ and $\hat{\Lambda}$. We have the following theorem/conjecture.
- This leads to the following ansatz for the Poincaré-type metric $\hat{g}$ for the past eon $\hat{M}$ :

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- This leads to the following ansatz for the Poincaré-type metric $\hat{g}$ for the past eon $\hat{M}$ :
$\hat{g}=t^{-2}\left(-\mathrm{d} t^{2}+\frac{2 r^{2}\left(1+\sum_{i=1}^{\infty} a_{i}(r)^{\prime}\right) \mathrm{d} z \mathrm{~d} \overline{\mathrm{z}}}{\left(1+\frac{\bar{z}}{2}\right)^{2}}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right) \mathrm{d} r^{2}\right)$.
Our (pre)past eon manifold $\hat{M}$ is parameterized by $t$
and
- I now consider the following null vector field $K$ on $M$ :
- It is tangent to a congruence of null geodesics without shear and twist, which represents light rays emanating from the source at the surface $r=0$.
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$$
K=\partial_{t}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right)^{-\frac{1}{2}} \partial_{r}
$$

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$\hat{g}=t^{-2}\left(-\mathrm{d} t^{2}+\frac{2 r^{2}\left(1+\sum_{i=1}^{\infty} a_{i}(r) t^{t^{\prime}}\right) \mathrm{d} z \mathrm{~d} \overline{\mathrm{z}}}{\left(1+\frac{\bar{z}}{2}\right)^{2}}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right) \mathrm{d} r^{2}\right)$.
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## Possible generalizations

## Theorem 1. <br> If the metric



## satisfies Einstein's equations

$$
\hat{E}_{i j}: \hat{n}_{j j}-\hat{\hat{\Lambda}}_{j i j}-\hat{\Phi} K_{i} \hat{K}_{j}=0
$$

with

then we have:

## Possible generalizations

## Theorem 1.

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## Possible generalizations

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If the metric
$\hat{g}=t^{-2}\left(-\mathrm{d} t^{2}+h_{t}\right)=$

$$
t^{-2}\left(-\mathrm{d} t^{2}+\frac{2 r^{2}\left(1+\sum_{i=1}^{\infty} a_{i}(r) t^{i}\right) \mathrm{d} z \mathrm{~d} \bar{z}}{\left(1+\frac{z \bar{z}}{2}\right)^{2}}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right) \mathrm{d} r^{2}\right)
$$

## satisfies Einstein's equations

with

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## Possible generalizations

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$$

satisfies Einstein's equations

$$
\hat{E}_{i j}:=\hat{R}_{i j}-\hat{\Lambda} \hat{g}_{i j}-\hat{\Phi} \hat{K}_{i} \hat{K}_{j}=0
$$

with

$$
K=K^{i} \partial_{i}=\partial_{t}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right)^{-\frac{1}{2}} \partial_{r}, \quad \hat{K}_{i}=\hat{g}_{i j} K^{j},
$$

then we have:

## Possible generalizations

- The coefficients $a_{1}(r), a_{2}(r) b_{1}(r)$ and $b_{2}(r)$ identically vanish, $a_{1}(r)=a_{2}(r)=b_{1}(r)=b_{2}(r)=0$, and the power series expansion of histarts at the $t^{3}$ terms, $h_{r}=r^{3} \lambda(r)+O\left(t^{\wedge}\right)$.
- The metric $\hat{g}$, or what is the same, the power series expansions $\nu(t, r)=\sum_{i=1}^{\infty} a_{i}(r) t^{i}$ and $\mu(t, r)=\sum_{i=1}^{\infty} b_{i}(r) t^{i}$, are totally determined up to infinite order by an arbitrary differentiable function $f=f(r)$.
- More precisely, the Einstein equations $\hat{E}_{i j}=\mathcal{O}\left(t^{k+1}\right)$ solved up to an order $k$, together with an arbitrary differentiable function $i=!(r)$, uniquely determine $\mu(t, r)$ and $\mu(t, r)$ up to an order $(k+2)$.
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## Possible generalizations

- In the lowest order the solution reads:


The energy function $\hat{\phi}$ and the cosmological constant $\hat{\wedge}$ are:

$$
\hat{\phi}=3 \frac{f^{\prime}}{r^{3}} t^{6}+O\left(t^{7}\right) \text { and } \hat{\Lambda}=3+O\left(t^{k+3}\right)
$$

the Weyl tensor of the solution is

$$
w_{n i} i_{k \mid}=o^{\prime}(t) \text {. }
$$

In particular, the Weyl tensor $W^{i}{ }_{j k l}$ vanishes at $t=0$ and $\hat{\Lambda}=3>0$ there.

## Possible generalizations

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\nu=\frac{f}{r^{3}} t^{3}+\mathcal{O}\left(t^{4}\right) \quad \text { and } \quad \mu=-\frac{2 f}{r^{3}} t^{3}+\mathcal{O}\left(t^{4}\right) ;
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\hat{\Phi}=3 \frac{f^{\prime}}{r^{3}} t^{6}+\mathcal{O}\left(t^{7}\right) \quad \text { and } \quad \hat{\Lambda}=3+\mathcal{O}\left(t^{k+3}\right)
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the Weyl tensor of the solution is

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W^{i}{ }_{j k l}=\mathcal{O}(t) .
$$

In particular, the Weyl tensor $W^{i}{ }_{j k l}$ vanishes at $t=0$ and $\hat{\Lambda}=3>0$ there.

## Possible generalizations

- With the use of computers we calculated this solution up to the order $k=10$, finding explicitly $\nu=\sum_{k=3}^{10} a_{k} t^{k}$ and $\mu=\sum_{k=3}^{10} b_{k} t^{k}$. The formulas are compact enough up to $k=8$ and up to the order $k=8$ they read:


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$$
\begin{aligned}
\nu(t, r)= & f \frac{t^{3}}{r^{3}}-\frac{3}{4} f^{\prime} \frac{t^{4}}{r^{4}}+\frac{1}{10}\left(-2 r f^{\prime}+3 r^{2} f^{\prime \prime}\right) \frac{t^{5}}{r^{5}}+ \\
& \frac{1}{24}\left(3 f^{2}-3 r f^{\prime}+3 r^{2} f^{\prime \prime}-2 r^{3} f^{(3)}\right) \frac{t^{6} 6}{r^{6}}+ \\
& \frac{r}{280}\left(-24 f^{\prime}-105 f^{\prime}+24 f^{\prime \prime}-12 r^{2} f^{(3)}+5 r^{3} f^{(4)}\right) \frac{t^{7}}{r^{7}}- \\
& \frac{r}{960}\left(60 f^{\prime}+288 f f^{\prime}-150 r f^{\prime 2}-60 r f^{\prime \prime}-216 r f f^{\prime \prime}+30 r^{2} f^{(3)}-10 r^{3} f^{(4)}+3 r^{4} f^{(5)}\right) \frac{t^{8}}{r^{8}}+ \\
& \mathcal{O}\left(\left(\frac{t}{r}\right)^{9}\right)
\end{aligned}
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& \frac{1}{24}\left(3 f^{2}-3 r f^{\prime}+3 r^{2} f^{\prime \prime}-2 r^{3} f^{(3)}\right) \frac{t^{6}}{r^{6}}+ \\
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& \mathcal{O}\left(\left(\frac{t}{r}\right)^{9}\right) \\
\mu(t, r)= & -2 f \frac{t^{3}}{r^{3}}+\frac{3}{4} f^{\prime} \frac{4^{4}}{r^{4}}-\frac{1}{5} f^{\prime \prime} \frac{t^{5}}{r^{5}}+\frac{1}{24}\left(39 f^{2}+r^{3} f^{(3)}\right) \frac{t^{6}}{r^{6}}-\frac{r}{280}\left(390 f f^{\prime}+2 r^{3} f^{(4)}\right) \frac{t^{7}}{r^{7}}+ \\
& \frac{r}{960}\left(-18 f f^{\prime}+300 r f^{\prime 2}+378 r f^{\prime \prime}+r^{4} f^{(5)}\right) \frac{t^{8}}{r^{8}}+\mathcal{O}\left(\left(\frac{t}{r}\right)^{9}\right) .
\end{aligned}
$$

## Possible generalizations

- For a solution up to this order we find that:

$\frac{r^{3}}{40}\left(120 f^{\prime}+522 f f^{\prime}-177 r f^{\prime 2}-120 r f^{\prime \prime}-378 r f^{\prime \prime}+93 r^{2} f^{\prime} f^{\prime \prime}+60 r^{2} f^{(3)}+90 r^{2} f f^{(3)}-20 r^{3} f^{(4)}+5 r^{4} f^{(5)}\right.$ mul t$)^{12}$.
- I have no patience to type the Weyl tensor components up to high order. It is enough to say that that up to the 4th order in $t$, modulo a nonzero constant tensor $C^{\prime} j k$, it is equal to:

$$
W^{i}{ }_{j k l}=\left(\frac{f}{r^{2}} \frac{t}{r}-\frac{f^{\prime}}{r} \frac{t^{2}}{r^{2}}+\frac{f^{\prime \prime}}{2} \frac{t^{3}}{r^{3}}\right) C^{i}{ }_{j k l}+\mathcal{O}\left(\left(\frac{t}{r}\right)^{4}\right) .
$$

- Of course, for the positivity of the energy density $\hat{\phi}$ close to the surface $\mathscr{I}^{+}$of $\hat{M}$ we need


## Possible generalizations

- For a solution up to this order we find that:

$$
\begin{aligned}
\hat{\phi}= & 3 r^{3} f^{\prime} \frac{t^{6}}{r^{6}}+3 r^{3}\left(f^{\prime}-r f^{\prime \prime}\right) \frac{t^{7}}{r^{7}}+\frac{3 r^{3}}{2}\left(2 f^{\prime}-2 r f^{\prime \prime}+r^{2} f^{(3)}\right) \frac{t^{8}}{r^{8}}+ \\
& \frac{r^{3}}{2}\left(6 f^{\prime}+6 f f^{\prime}-6 r f^{\prime \prime}+3 r^{2} f^{(3)}-r^{3} f^{(4)}\right) \frac{t^{9}}{r^{9}}+ \\
& \frac{r^{3}}{8}\left(24 f^{\prime}+66 f f^{\prime}-12 r f^{\prime 2}-24 r f^{\prime \prime}-30 r f f^{\prime \prime}+12 r^{2} f^{(3)}-4 r^{3} f^{(4)}+r^{4} f^{(5)}\right) \frac{t^{10}}{r^{10}}+ \\
& \frac{r^{3}}{40}\left(120 f^{\prime}+522 f f^{\prime}-177 r f^{\prime 2}-120 r f^{\prime \prime}-378 r f f^{\prime \prime}+93 r^{2} f^{\prime} f^{\prime \prime}+60 r^{2} f^{(3)}+90 r^{2} f f^{(3)}-20 r^{3} f^{(4)}+5 r^{4} f^{(5)}\right. \\
& \mathcal{O}\left(\left(\frac{t}{r}\right)^{12}\right),
\end{aligned}
$$

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- Of course, for the positivity of the energy density $\hat{\phi}$ close to the surface $\mathscr{I}^{+}$of $\hat{M}$ we need


## Possible generalizations

- For a solution up to this order we find that:

$$
\begin{aligned}
& \hat{\Phi}=3 r^{3} f^{\prime} \frac{t^{6}}{r^{6}}+3 r^{3}\left(f^{\prime}-r f^{\prime \prime}\right) \frac{t^{7}}{r^{7}}+\frac{3 r^{3}}{2}\left(2 f^{\prime}-2 r f^{\prime \prime}+r^{2} f^{(3)}\right) \frac{t^{8}}{r^{8}}+ \\
& \frac{r^{3}}{2}\left(6 f^{\prime}+6 f f^{\prime}-6 r f^{\prime \prime}+3 r^{2} f^{(3)}-r^{3} f^{(4)}\right) \frac{t^{9}}{r^{9}}+ \\
& \frac{r^{3}}{8}\left(24 f^{\prime}+66 f f^{\prime}-12 r f^{\prime 2}-24 r f^{\prime \prime}-30 r f f^{\prime \prime}+12 r^{2} f^{(3)}-4 r^{3} f^{(4)}+r^{4} f^{(5)}\right) \frac{t^{10}}{r^{10}}+ \\
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& \mathcal{O}\left(\left(\frac{t}{r}\right)^{12}\right), \\
& \hat{\Lambda}=3+\mathcal{O}\left(t^{9}\right) .
\end{aligned}
$$

- I have no patience to type the Weyl tensor components up to high order. It is enough to say that that up to the 4th order in $t$, modulo a nonzero constant tensor $C^{i} j k /$, it is equal to:

- Of course, for the positivity of the energy density $\hat{\Phi}$ close to the surface $\mathscr{I}^{+}$of $\hat{M}$ we need
- For a solution up to this order we find that:

$$
\begin{aligned}
\hat{\Phi}= & 3 r^{3} f^{\prime} \frac{t^{6}}{r^{6}}+3 r^{3}\left(f^{\prime}-r f^{\prime \prime}\right) \frac{t^{7}}{r^{7}}+\frac{3 r^{3}}{2}\left(2 f^{\prime}-2 r f^{\prime \prime}+r^{2} f^{(3)}\right) \frac{t^{8}}{r^{8}}+ \\
& \frac{r^{3}}{2}\left(6 f^{\prime}+6 f f^{\prime}-6 r f^{\prime \prime}+3 r^{2} f^{(3)}-r^{3} f^{(4)}\right) \frac{t^{9}}{r^{9}}+ \\
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& \mathcal{O}\left(\left(\frac{t}{r}\right)^{12}\right)
\end{aligned}
$$

$$
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## Possible generalizations

- For a solution up to this order we find that:

$$
\begin{aligned}
\hat{\Phi}= & 3 r^{3} f^{\prime} \frac{t^{6}}{r^{6}}+3 r^{3}\left(f^{\prime}-r f^{\prime \prime}\right) \frac{t^{7}}{r^{7}}+\frac{3 r^{3}}{2}\left(2 f^{\prime}-2 r f^{\prime \prime}+r^{2} f^{(3)}\right) \frac{t^{8}}{r^{8}}+ \\
& \frac{r^{3}}{2}\left(6 f^{\prime}+6 f f^{\prime}-6 r f^{\prime \prime}+3 r^{2} f^{(3)}-r^{3} f^{(4)}\right) \frac{t^{9}}{r^{9}}+ \\
& \frac{r^{3}}{8}\left(24 f^{\prime}+66 f f^{\prime}-12 r f^{\prime 2}-24 r f^{\prime \prime}-30 r f f^{\prime \prime}+12 r^{2} f^{(3)}-4 r^{3} f^{(4)}+r^{4} f^{(5)}\right) \frac{t^{10}}{r^{10}}+ \\
& \frac{r^{3}}{40}\left(120 f^{\prime}+522 f f^{\prime}-177 r f^{\prime 2}-120 r f^{\prime \prime}-378 r f f^{\prime \prime}+93 r^{2} f^{\prime} f^{\prime \prime}+60 r^{2} f^{(3)}+90 r^{2} f f^{(3)}-20 r^{3} f^{(4)}+5 r^{4} f^{(5)}\right. \\
& \mathcal{O}\left(\left(\frac{t}{r}\right)^{12}\right)
\end{aligned}
$$

$$
\hat{\wedge}=3+\mathcal{O}\left(t^{9}\right) .
$$

- I have no patience to type the Weyl tensor components up to high order. It is enough to say that that up to the 4th order in $t$, modulo a nonzero constant tensor $C^{i}{ }_{j k l}$, it is equal to:

$$
W^{i}{ }_{j k l}=\left(\frac{f}{r^{2}} \frac{t}{r}-\frac{f^{\prime}}{r} \frac{t^{2}}{r^{2}}+\frac{f^{\prime \prime}}{2} \frac{t^{3}}{r^{3}}\right) C^{i}{ }_{j k l}+\mathcal{O}\left(\left(\frac{t}{r}\right)^{4}\right) .
$$

- Of course, for the positivity of the energy density $\hat{\phi}$ close to the surface of $\hat{M}$ we need


## Possible generalizations

- For a solution up to this order we find that:

$$
\begin{aligned}
& \hat{\Phi}=3 r^{3} f^{\prime} \frac{t^{6}}{r^{6}}+3 r^{3}\left(f^{\prime}-r^{\prime \prime}\right) \frac{7^{7}}{r^{7}}+\frac{33^{3}}{2}\left(2 f^{\prime}-2 r^{\prime \prime}+r^{2} f^{(3)}\right) \frac{t^{8}}{8^{8}}+ \\
& \frac{r^{3}}{2}\left(6 f^{\prime}+6 f^{\prime}-6 r^{\prime \prime}+3 r^{2} f^{(3)}-r^{3} f^{(4)}\right) \frac{t^{9}}{r^{9}}+ \\
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$$
f^{\prime}>0
$$

## Possible generalizations

The Poincaré-type metric $\hat{g}$ can be interpreted as the ending stage of the evolution of the past eon in Penrose's CCC. The eon has a positive cosmological constant $\hat{\Lambda} \simeq 3$, which is filled with a spherically symmetric pure radiation moving along the null congruence generated by the vector field $K$.

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## Possible generalizations

Theorem 2.
Assume that the metric $\hat{g}$ as before satisfies the Einstein equations $\hat{E}_{i j}=0$. Then, the reciprocal metric

satisfies the Einstein equations

Here $\breve{K}_{i}$ and $K_{i}$ are the null 1-forms corresponding to the pair of outgoing-ingoing null vector fields
via $\check{K}_{i}=\breve{g}_{i j} K^{j}$ and $\check{L}=\breve{g}_{i j} L^{j}$, and the 1 -form vector field $\breve{u}_{i}$ corresponds to the future oriented - Note that now $t<0$ (!) timelike unit vector field

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& \check{g}=t^{2}\left(-\mathrm{d} t^{2}+\frac{2 r^{2}(1+\nu(t, r)) \mathrm{d} z \mathrm{~d} \overline{\mathrm{z}}}{\left(1+\frac{z \bar{z}}{2}\right)^{2}}+(1+\mu(t, r)) \mathrm{d} r^{2}\right)= \\
& \quad t^{2}\left(-\mathrm{d} t^{2}+\frac{2 r^{2}\left(1+\sum_{i=1}^{\infty} a_{i}(r) t^{i}\right) \mathrm{d} z \mathrm{~d} \bar{z}}{\left(1+\frac{z \bar{z}}{2}\right)^{2}}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right) \mathrm{d} r^{2}\right)
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\check{E}_{i j}=\check{R}_{i j}-\Phi \check{K}_{i} \check{K}_{j}-\check{\psi} L_{i} L_{j}-(\check{\rho}+\check{\rho}) \check{u}_{i} \check{u}_{j}-\frac{1}{2}(\check{\rho}-\check{p}) \check{g}_{i j}=0 .
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\end{aligned}
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\check{E}_{i j}=\check{R}_{i j}-\check{\Phi} \check{K}_{i} \check{K}_{j}-\check{\psi} \check{L}_{i} L_{j}-(\check{\rho}+\check{p}) \check{u}_{i} \check{u}_{j}-\frac{1}{2}(\check{\rho}-\check{\rho}) \check{g}_{i j}=0 .
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\end{aligned}
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$$

Here $\check{K}_{i}$ and $\check{L}_{i}$ are the null 1 -forms corresponding to the pair of outgoing-ingoing null vector fields

$$
K=K^{i} \partial_{i}=\partial_{t}+\left(1+\sum_{i=1}^{\infty} b_{i}(r) t^{i}\right)^{-\frac{1}{2}} \partial_{r} \text { and } L=L^{i} \partial_{i}=\partial_{t}-\left(1+\sum_{i=1}^{\infty} b_{i}(r)^{i}\right)^{-\frac{1}{2}} \partial_{r},
$$

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$$
\check{u}=\check{u}^{i} \partial_{i}=-t^{-1} \partial_{t}
$$

$\operatorname{via} \check{u}_{i}=\check{g}_{i j} \check{u}^{j}$.

## Possible generalizations

For the solutions $\nu(t, r), \mu(t, r)$ of the past eon's Einstein's equations, which were given in terms of the power series expansions as
$\nu(t, r)=\sum_{i=3}^{k+2} a_{i}(r) t^{i}+\mathcal{O}\left(t^{k+3}\right)$ and $\mu(t, r)=\sum_{i=3}^{k+2} b_{i}(r) t^{i}+\mathcal{O}\left(t^{k+3}\right)$
in Theorem 1, the formulae for the power series expansions of the energy densities $\varnothing \breve{\psi}, \check{\rho}$ and the pressure $\check{p}$ are as follows:


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$$
\begin{aligned}
\check{\Phi}= & -\frac{9 f}{r^{3}} t^{-3}+\frac{9 f^{\prime}}{r^{3}} t^{-2}+\frac{1}{2 r^{4}}\left(8 f^{\prime}-11 r f^{\prime \prime}\right) t+\frac{3}{4 r^{5}}\left(5 f^{\prime}-5 r f^{\prime \prime}+3 r^{2} f^{(3)}\right)+ \\
& \frac{9}{40 r^{6}}\left(16 f^{\prime}+5 f^{\prime}-16 r f^{\prime \prime}+8 r^{2} f^{(3)}-3 r^{3} f^{(4)}\right) t+ \\
& \frac{1}{120 r^{7}}\left(420 f^{\prime}+1068 f f^{\prime}-30 r f^{\prime 2}-420 r f^{\prime \prime}-384 r f f^{\prime \prime}+210 r^{2} f^{(3)}-70 r^{3} f^{(4)}+19 r^{4} f^{(5)}\right) t^{2}+ \\
& \cdots+\mathcal{O}\left(t^{k-3}\right),
\end{aligned}
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\check{\Psi}= & -\frac{9 f}{r^{3}} t^{-3}+\frac{6 f^{\prime}}{r^{3}} t^{-2}+\frac{1}{2 r^{4}}\left(2 f^{\prime}-5 r f^{\prime \prime}\right) t^{-1}+\frac{3}{4 r^{5}}\left(f^{\prime}-r f^{\prime \prime}+r^{2} f^{(3)}\right)+ \\
& \frac{1}{40 r^{6}}\left(24 f^{\prime}-75 f f^{\prime}-24 r f^{\prime \prime}+12 r^{2} f^{(3)}-7 r^{3} f^{(4)}\right) t+ \\
& \frac{1}{60 r^{7}}\left(30 f^{\prime}+39 f f^{\prime}+75 r f^{\prime 2}-30 r f^{\prime \prime}+33 r f f^{\prime \prime}+15 r^{2} f^{(3)}-5 r^{3} f^{(4)}+2 r^{4} f^{(5)}\right) t^{2}+ \\
& \cdots+\mathcal{O}\left(t^{k-3}\right)
\end{aligned}
$$

## Possible generalizations



In these formulas all the doted terms are explicitly determined in terms of $f$ and its derivatives (I was lazy, and typed only the terms adapted to the choice $k=6$ in Theorem 1).

## Possible generalizations

$$
\begin{aligned}
\check{\rho}= & 3 t^{-4}+\frac{18 f}{r^{3}} t^{-1}-\frac{18 f^{\prime}}{r^{3}}+\frac{-6 f^{\prime}+9 r f^{\prime \prime}}{r^{4}} t-\frac{3}{4 r^{6}}\left(9 f^{2}+3 r f^{\prime}-3 r^{2} f^{\prime \prime}+2 r^{3} f^{(3)}\right) t^{2}+ \\
& \frac{3}{20 r^{6}}\left(-24 f^{\prime}+105 f f^{\prime}+24 r f^{\prime \prime}-12 r^{2} f^{(3)}+5 r^{3} f^{(4)}\right) t^{3}- \\
& \frac{1}{20 r^{7}}\left(60 f^{\prime}+96 f f^{\prime}+120 r f^{\prime 2}-60 r f^{\prime \prime}+72 r f f^{\prime \prime}+30 r^{2} f^{(3)}-10 r^{3} f^{(4)}+3 r^{4} f^{(5)}\right) t^{4}+ \\
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& \cdots+\mathcal{O}\left(t^{k-1}\right), \\
& \check{p}= t^{-4}+\frac{6 f}{r^{3}} t^{-1}+\frac{1}{r^{4}}\left(2 f^{\prime}-r f^{\prime \prime}\right) t+\frac{1}{2 r^{6}}\left(18 f^{2}+3 r f^{\prime}-3 r^{2} f^{\prime \prime}+r^{3} f^{(3)}\right) t^{2}- \\
& \quad \frac{3}{20 r^{6}}\left(-8 f^{\prime}+45 f f^{\prime}+8 r f^{\prime \prime}-4 r^{2} f^{(3)}+r^{3} f^{(4)}\right) t^{3}+ \\
& \frac{1}{30 r^{7}}\left(30 f^{\prime}+57 f f^{\prime}+45 r f^{\prime 2}-30 r f^{\prime \prime}+39 r f f^{\prime \prime}+15 r^{2} f^{(3)}-5 r^{3} f^{(4)}+r^{4} f^{(5)}\right) t^{4}+ \\
& \cdots+\mathcal{O}\left(t^{k-1}\right) .
\end{aligned}
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## Possible generalizations

## Remarks.

- Note that since in $\check{M}$ the time $t<0$, the requirement that the energy densities are positive near the Big Bang hypersurface i = 0 implies that ' $>0$ in additition to "" $>0$, the requirement we got from the past eon. Note also that $f>0$ and $f^{\prime}>0$ are the only conditions needed for the positivity of energy densities, as the leading term in $\check{\rho}$ is $\check{\rho} \simeq 3 t^{-4}$, and is positive regardless of the sign of i.
- Remarkably the leading terms in $\check{\rho}$ and $\check{p}$, i.e. the terms with negative powers in $t$, are proportional to each other with the numerical factor three. We have

$$
\check{p}=\frac{1}{3} \check{\rho}+\mathcal{O}\left(t^{0}\right) .
$$

- This means that immediately after the Bang, apart from the matter content of the two spherical ingoing and outgoing waves in the new eon, there is also a scattered radiation there, described by the perfect fluid with $\check{p}=\frac{1}{3} \check{\rho}$.


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## Possible generalizations

- This solution to the three metrics in Penrose-Tod's bandage region has the following apealing physical property: Immediately after the Bang, the spherical wave from the previous eon not only produces two spherical waves - one is obvious: it is the still expanding but damped wave that survived the Bang; the other is less obvious, becuase although it is still sphereical it focuses - but there is also there a third ingredient: it is a randomly scattered radiation described by the perfect fluid with $p=\frac{1}{3} p$.
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## Literature

- H. W. Brinkman (1925), 'Einstein spaces which are mapped conformally on each other', Math. Ann. 94, 119-145
- P. Tod (2015), 'The equations of Conformal Cyclic Cosmology', Gen. Rel. Grav. 47,https://doi.org/10.1007/s10714-015-1859-7
- P. Tod (2010), 'Conformal methods in General Delativity with application to Conformal Cyclic Cosmology: A minicourse at IX International Meeting on Lorentz Geometry held in Warsaw' (ask Paul Tod for a copy)
- K. Meissner, P. Nurowski (2017), 'Conformal transformations and the beginning of the Universe', Phys. Rev. D 95, Issue 8, 84016, 1-5.
- P. Nurowski (2021), 'Radiative Poincaré type eon and its follower', https://arxiv.org/abs/2101.12670.
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