Problems 1

wtorek, 2 marca 2021 19:51

① Show that

$SU(2) = \{ g \in \mathrm{Mat}(2 \times 2, \mathbb{C}) ; \quad g = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, a, b \in \mathbb{C}$

and $|a|^2 + |b|^2 = 1 \}$

Check group axioms.

② Find $\underline{SU(2)}$ from the presentation of $su(2)$ as given in ①.

Anti-hint: (do not use e^{iT})

③ Orthogonal groups

Define $O(n) = \{ g \in \mathrm{Mat}(n \times n, \mathbb{R}) ; \quad g^T g = \mathbb{1}_n \}$

ⓐ Show that $O(n)$ has two connected components

Define $SO(n) = \{ g \in O(n) ; \ \det(g) = 1 \}$

ⓑ Find $\underline{SO(n)}$

Hint: use $\det(e^A) = e^{\mathrm{Tr}\, A}$

④ Show that (σ^a - Pauli matrices)

a) $\sigma^a \sigma^b = \delta^{ab} + i \epsilon^{abc} \sigma^c$, $a,b,c = 1,2,3$

b) using a) show that

$$e^{i \omega_a \frac{\sigma^a}{2}} = \cos \frac{|\omega|}{2} + i \sigma^a \frac{\omega_a}{|\omega|} \sin \frac{|\omega|}{2} \in SU(2)$$

where $|\omega|^2 = \sum_a \omega_a^2$

c) Find a and b as defined in problem ①

⑤ Show that $\underline{SO(3)} = \underline{SU(2)}$ i.e. one can choose basis of both algebras such that their commutiation relations are the same.

⑥ SU(2) vs. SO(3)

Let $U \in SU(2)$.

Show that $U^+ \sigma^i U = O^{ij} \sigma^j$ where $O \in SO(3)$

CW.