Problems 2

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1. Let $x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}$. $x_{\mu} \equiv \eta_{\mu\nu} x^{\nu}$

$(\eta^{-1})^{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$

Show that :

(a) $x'_{\mu} = x_{\rho} (\Lambda^{-1})^{\rho}{}_{\mu}$, where $\Lambda^{\mu}{}_{\nu} (\Lambda^{-1})^{\nu}{}_{\rho} = \delta^{\mu}_{\rho}$

(b) $\partial'_{\mu} = (\Lambda^{-1})^{\rho}{}_{\mu}\, \partial_{\rho}$ where $\partial_{\rho} = \dfrac{\partial}{\partial x^{\rho}}$

2. Show that the Lorentz group $O(3,1)$ has 4 disconnected components characterized by the following 4 group elements of $O(3,1)$

$\{ \mathbb{1} = \mathrm{diag}(1,1,1,1),\ \mathbb{P} = \mathrm{diag}(1,-1,-1,-1),$

$T = \mathrm{diag}(-1,1,1,1),\ \mathbb{P}\cdot T = \mathrm{diag}(-1,-1,-1,-1) \}$

3. Calculate $e^{i\omega M^{01}}$ and relate it to a Lorentz transformation.

4. $\underline{SL(2,\mathbb{C})}$ is spanned by $\{ \frac{\sigma^a}{2}, i\frac{\sigma^b}{2} \}$ $a,b = 1,2,3$

$\underline{SO(3,1)}$ is spanned by $\{ M^{\mu\nu} \}$

Show that

$\underline{SL(2,\mathbb{C})}$ and $\underline{SO(3,1)}$ are isomorphic i.e. both Lie algebras have the same commutation relations under identification

$$M^{ij} \leftrightarrow \epsilon^{ijk} \frac{\sigma^k}{2}, \quad M^{0k} \leftrightarrow i \frac{\sigma^k}{2}$$

Check that fact for a few values of the indices i, j, k.

5 Show that $U^\dagger \sigma^\mu U = \Lambda^\mu{}_\nu \sigma^\nu$.

maps $SL(2,\mathbb{C}) \ni U \rightarrow \Lambda \in SO^+(3,1)$