INRODUCTION TO FUNDAM. INTERACTIONS

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0.1 Intro

Principles

1. Lorentz invariance: determines (uniquely) group structure of the theory
2. gauge symmetry: necessary ingredient of the construction of Lagrangians – removes negative norm states from non-trivial Lorentz oscilators
3. free particles corresponds to oscilator (Heisenberg) algebra = Lagrangians quadratic in fields
4. part. and anti-part. are necesssary for (Wick rot.) ...???
5. interaction: perturbation of the free propagation
6. the interesting physics follows from the detailed analysis of the above principles.

$\tilde{dk} \equiv đ^3k/2\omega_k$, $\omega_k = \sqrt{\vec{k}^2 + m^2}$, $kx \equiv k_0 x^0 - \vec{k}\vec{x}$,

Chapter 1

Classical Fields

1.1 General properties

1.1.1 Lorentz tensors

https://en.wikipedia.org/wiki/Lorentz_transformation

$$\Lambda_R = \exp\{i/2\,\omega_{\mu\nu}T_R^{\mu\nu}\} \tag{1.1.1}$$

1. scalar: $\phi' = \phi$.
2. vector: $v'^\mu = \Lambda^\mu{}_\nu v^\nu,\ v'_\mu = v_\nu(\Lambda^{-1})^\nu{}_\mu$
3. second rank tensor: $t'^{\mu\nu} = \Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma t^{\rho\sigma}$
4. scalars: $v_\nu v^\mu,\ t^{\rho\sigma}t_{\rho\sigma}$

WARNING. Position x^μ is not a Lorentz tensor. It transforms under full Poincare group as follows: $x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$. According to (1.1.1) the metric η does not transform under Lorentz group i.e. it is a scalar.

With the help of the metric η we can define quantities with the lower indices e.g. co-vectors: $v_\mu \equiv \eta_{\mu\nu}v^\nu$. Transformation properties of these quantities are according to the above definition.
So: $v'_\mu = \eta_{\mu\nu}v'^\nu = \eta_{\mu\nu}\Lambda^\nu{}_\rho\eta^{\rho\sigma}v_\sigma = v_\rho(\Lambda^{-1})^\rho{}_\mu$.

1.1.2 Fields

Scalar field has the same value in two coordinate systems related by $x' = \Lambda x$

$$\phi'(x') = \phi(x), \to \ \phi'(x) = \phi(\Lambda^{-1}x) \qquad [= \exp\{i/2\,\omega_{\mu\nu}T^{\mu\nu}_{\infty}\}\phi(x)]$$

Vector, rep. R fields

$$(V')^{\mu}(x') = \Lambda^{\mu}{}_{\nu}V^{\nu}(x) \tag{1.1.2}$$

$$(V')_{\mu}(x') = (\eta\Lambda)_{\mu\nu}V^{\nu}(x) = (\eta\Lambda\eta^{-1})_{\mu}{}^{\nu}V_{\nu} \tag{1.1.3}$$

$$\Psi'_{R}(x') = \exp\{\,\frac{i}{2}\,\omega_{\mu\nu}T^{\mu\nu}_{R}\}\Psi_{R}(x) \tag{1.1.4}$$

1.1.3 Variational principle

We mimic class. mechanics

1.1.3.1 ACTION AND VARIATIONAL PRINCIPLE are maps from spacetime $M = V \times [t_i, t_f]$ to a space of fields i.e. $\phi: \ x \to \ \phi(x)$.

Action is a functional of fields represented as integral of Lagrangian density $\mathcal{L}$.

$$S[\phi] = \int_{t_i}^{t_f} L[\phi] = \int_{M_V} d^4x \ \mathcal{L}(\phi, \partial_{\mu}\phi, x) \tag{1.1.5}$$

The volume $M = V \times [t_i, t_f]$ where V is the space-like manifold.

Variation of S.

$$\delta S[\phi] \equiv S[\phi + \delta\phi] - S[\phi] = O((\delta\phi)^2) \quad \text{for} \quad \phi = \phi_{cl} \tag{1.1.6}$$

The variations of fields are such that $\delta\phi(t_i) = \delta\phi(t_f) = 0$.

Under this circumstances (1.1.6) is equivalent to classical **equations of motion** (Euler-Lagrange equations).[1]

$$\partial_{\mu}\,\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\phi^a)} - \frac{\partial\mathcal{L}}{\partial\phi^a} = 0 \tag{1.1.7}$$

with some boundary conditions and initial-final conditions $\phi(t_0, \vec{x}\,) = \phi_0(\vec{x}\,),\ \phi(t_f, \vec{x}\,) = \phi_f(\vec{x}\,)$.

[1] The index a shows different species of fields ϕ. We shall usually suppress this index.

1.1.3.2 S MUST BE ... In particle physics applications S must respect certain requirements:

1. $S = \int_{\mathbb{M}^4} d^4x \; \mathcal{L}(\phi(x), \partial_\mu \phi(x))$
2. real $S^* = S$
3. S is Poincare inv.
   (a) no external sources i.e. $\partial_\mu \mathcal{L} = 0$
   (b) relativistic theories we require L (as a function of ϕ, $\partial_\mu\phi$) to be Lorentz invariant: $\mathcal{L}' \equiv \mathcal{L}(\phi'(x), ..) = \mathcal{L}(\phi(y), ...)$, where $y = \Lambda^{-1}x$.
   (c) No boundaries
4. By quantum theory (renormalizability, unutarity):
   (a) contains canonical kinetic terms e.g. $(\partial_\mu\phi)^2$
   (b) only spin $0,\ 1/2,\ 1$ fields
   (c) polynomials in fields
   (d) up dim. 4 "operators".

S can be invariant under actions of some groups farther on called symmetry groups of the model given by $\mathcal{L}$.

$$\mathcal{L} = \frac{1}{2}\,(\partial_\mu\phi^T\partial_\mu\phi - m^2\phi^T\phi), \qquad \phi \in \mathbb{R}^n,\ G = O(n) \tag{1.1.8}$$

$$\mathcal{L} = (\partial_\mu\phi^+\partial_\mu\phi - m^2\phi^+\phi), \qquad \phi \in \mathbb{C}^n,\ G = U(n) \ \longrightarrow\ G_e = O(2n) \tag{1.1.9}$$

1.2 Free fields

1.2.1 Scalars

Free real scalar: $\mathcal{L} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] \to \ (\partial_\mu\partial^\mu + m^2)\phi = 0$

$\phi(x) = \int đ^4k\ \phi_k e^{-ikx}$ is solution iff $(\kappa^2 - m^2)\phi_k = 0 \to\ \phi_k = 2\pi\delta(k^2 - m^2)\tilde{\phi}_k$.

$\underline{k^2 = m^2}$ – is the dispersion relation as for particle of the mass m, $\to k_0 = \pm\omega_k,\ \omega_k = \sqrt{\vec{k}^2 + m^2}$.

$$\begin{aligned}\phi(x) &= \int đ^4k\, 2\pi\delta(k^2 - m^2)\tilde{\phi}_k e^{-ikx} = \int \tilde{d}k\ [\tilde{\phi}_k e^{-ikx}|_{k_0=\omega_k} + \tilde{\phi}_k e^{-ikx}|_{k_0=-\omega_k}] \\ &= \int \tilde{d}k\ [a(\vec{k})e^{-ikx} + a^*(\vec{k})e^{ikx}]|_{k_0=\omega_k} \end{aligned} \tag{1.2.1}$$

Free complex scalar:

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi \to\ (\partial_\mu\partial^\mu + m^2)\phi = 0 \to\ \phi(x) = \int \tilde{d}k\ [a(\vec{k})e^{-ikx} + b^*(\vec{k})e^{ikx}]|_{k_0=\omega_k} \tag{1.2.2}$$

Both terms depends on $kx \equiv k_0x^0 - \vec{k}\vec{x}$ i.e. represent waves going along $\vec{k}$ i.e. have momentum $\vec{k}$.

1.2.2 Teoria Maxwella – Vector fields

1.2.2.1 Maxwell equations

$$\nabla\vec{B} = 0, \quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \tag{1.2.3}$$

$$\nabla\vec{E} = \rho, \quad \nabla\times\vec{B} = \vec{J} + \frac{\partial\vec{E}}{\partial t} \tag{1.2.4}$$

1.2.2.2 Potentials, gauge invariance, field-strenth tensor Eq.(1.2.3) implies existence of the vector potential $\vec{A}$ such that $\vec{B} = \nabla\times\vec{A}$ so $\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ is $\nabla\times(\vec{E} + \frac{\partial\vec{A}}{\partial t}) = 0$, what implies existence of the scalar potential A_0 (sometimes denoted by φ) such that $\vec{E} + \frac{\partial\vec{A}}{\partial t} = -\nabla A_0$. We introduce the 4-potential $A_\mu = (A_0, -\vec{A})$. The potentials A_μ are called gauge fields.

1.2.2.3 Gauge invariance Under substitution:

$$A_\mu \to\ A_\mu + iU\partial_\mu U^{-1}, \quad U = e^{i\chi}. \tag{1.2.5}$$

the field-strengths $\vec{E}, \vec{B}$ do not change. If χ is well defined (see monopole) then: $A_\mu \to\ A_\mu + \partial_\mu\chi$ i.e. $\vec{A} \to\ \vec{A} - \nabla\chi,\ A_0 \to\ A_0 + \partial_t\chi$.

1.2.2.4 TENSOR F AND ITS DUAL

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1.2.6}$$

so $F_{0i} = E^i$, $F_{ij} = F^{ij} = -\epsilon^{ijk}B^k$. Notice that $\epsilon^{\mu\nu\rho\sigma}\partial_\mu F_{\nu\rho} = 0$.

F is the fild-strength tensor, such that $F_{0i} = E^i,\ F_{ij} = -\epsilon^{ijk}B^k$.

$$\text{Lorentz:} \quad (F')^{\mu\nu}(x') = \Lambda^\mu{}_\rho \Lambda^\mu{}_\sigma\, F^{\rho\sigma}(x) \tag{1.2.7}$$

e.g. motionless charge produces $(E \neq 0, B = 0)$ while in rationing frame we here both $(E \neq 0, B \neq 0)$.

$$\mathcal{L} = -\frac{1}{4}F^2 - A_\mu J^\mu \tag{1.2.8}$$

The Euler-Lagrange eqs. for this $\mathcal{L}$ leads to **Maxwell eqs.** (1.2.9) with $J^\nu = (\rho, \vec{J})$.

$$\partial_\mu F^{\mu\nu} = J^\nu, \tag{1.2.9}$$

e.g. $\nu = 0$, $\partial_i F^{i0} = \rho, \to\ \partial_i E^i = \rho$

1.2.2.5 GENERAL FREE FIELD

In the Lorentz gauge $\partial^\mu A_\mu = 0$ Maxwell eq. are $\partial^2 A^\mu = j^\mu$. General solution for free eq. is:

$$A_\mu(x) = \int \tilde{dk}\ \sum_{s=1}^{3} [\epsilon^s_\mu(k) a_s(k) e^{-ikx} + h.c.] \tag{1.2.10}$$

where $\{\epsilon^s_\mu(k)\}$ are fixed three orthonormal ($\epsilon^s(k)\cdot\epsilon^r(k) = \delta_{rs}$) 4-vectors in $\mathbb{M}^4$ such that $k^\mu \epsilon^s_\mu(k) = 0$.

Residual gauge freedom $\delta A_\mu = \partial_\mu \chi$ with $\partial^2\chi = 0$ says that we have freedom to remove one more polarization along k_μ: $\quad \delta \sum \epsilon^s_\mu(k) a_s(k) = k_\mu \chi(k)$.

$$A_\mu(x) = \int \tilde{dk}\ \sum_{s=1}^{2} [\epsilon^s_\mu(k) a_s(k) e^{-ikx} + h.c.] \tag{1.2.11}$$

1.2.3 Massive electrodynamics : Static potential

$$\mathcal{L} = -1/4\, F^2 - jA + \tfrac{1}{2}\, m^2 A^2$$

e.o.m.

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = j^\nu$$

imply $\partial_\mu A^\mu = 0$ thus eom is $(\partial^2 + m^2)A^\nu = j^\nu$. Free sol. are (1.2.10).

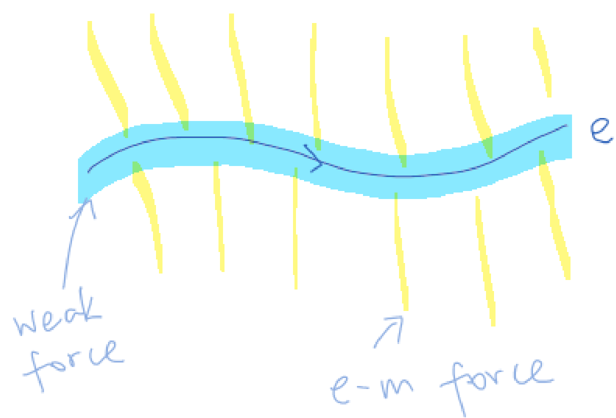
Potential of static point-like charge $j^0 = q\,\delta^{(3)}(\vec{x} - \vec{y})$:

$$(-\Delta + m^2)A^0(\vec{x} - \vec{y}) = q\,\delta^{(3)}(\vec{x} - \vec{y}) \Rightarrow (\vec{k}^2 + m^2)A^0(k) = q \;\Rightarrow\quad A^0(\vec{x} - \vec{y}) = q \int \bar{d}^3k\,\frac{e^{-i\vec{k}\vec{r}}}{\vec{k}^2 + m^2}$$

$\vec{r} = \vec{x} - \vec{y},\ k = |\vec{k}|,\ d^3k = k^2\ dk d\Omega_2^2,\quad \int d\Omega_2^2 = \int_0^{2\pi}\delta\varphi\int_0^{\pi}\sin(\theta)d\theta = \int_0^{2\pi}\delta\varphi\int_{-1}^{1}d(\xi = cos(\theta))$, $\vec{k}\vec{r} = kr\ \cos(\theta),\ \int_{-1}^{1}d\xi e^{-ikr\ \xi} = \frac{1}{-ikr}(e^{-ikr} - e^{ikr})$

$$A^0(\vec{x}-\vec{y}) = q\,(2\pi)^{-3}\cdot 2\pi\cdot\frac{1}{r}\,i\,(\frac{1}{2}\int_{-\infty}^{\infty}dk)\frac{k(e^{-ikr} - e^{ikr})}{k^2 + m^2} = \frac{q}{2(2\pi)^2 r}(-\frac{d}{dr}\int\frac{e^{ikr} + e^{-ikr}}{k^2 + m^2}) = \boxed{\frac{q}{4\pi}\,\frac{e^{-mr}}{r}}$$

[2] Potential of static particle: see Sec.1.2.3.



[2] $\int_{-\infty}^{\infty}dk\ \underbrace{\frac{k}{k^2 + m^2}}_{\text{poles at k=}\pm\text{im}}\ (\underbrace{e^{-ikr}}_{\Im(k)<0} - \underbrace{e^{ikr}}_{\Im(k)>0})$ can be really done due to Jordan's lemma.

1.2.4 Spinors for fermions

Electron has spin. Stern-Gerlach. su(2) vs. so(3)

1.2.4.1 Lorentz vs. $SL(2,\mathbb{C})$ we define

$$SL(2,\mathbb{C}) = \{U \in Mat(2\times 2,\mathbb{C}), \det(U) = 1\} \tag{1.2.12}$$

$SL(2,\mathbb{C})$ is double cover of $SO_+^{\uparrow}(1,3)$: $U = \pm 1 \to \ \Lambda = 1$.

One can show that (see below) ($\sigma^\mu = (1,\vec{\sigma})$)

$$U^+\, \sigma^\mu\, U = \Lambda^\mu{}_\nu \sigma^\nu, \quad \Lambda \in SO_+^{\uparrow}(1,3) \tag{1.2.13}$$

Taking $U = e^\alpha,\ \alpha = \alpha_i\sigma^i,\ \alpha_i \in \mathbb{C}$ we get relation between Lie algebras

$$\alpha^+\sigma^\mu + \sigma^\mu\alpha = iT^\mu{}_\nu\sigma^\nu \tag{1.2.14}$$

1.2.4.2 Two embedding of $\mathbb{M}^4 \to$ hermitian Mat$(2\times 2,\mathbb{C})$

$$x^\mu \to\ X = x^\mu\sigma_\mu, \qquad \sigma_\mu = (1,-\vec{\sigma}) = \overline{\sigma}^{\,\mu} \tag{1.2.15}$$

$$x^\mu \to\ \overline{X} = x^\mu\overline{\sigma}_{\,\mu}, \qquad \overline{\sigma}_{\,\mu} = (1,\vec{\sigma}) = \sigma^\mu, \tag{1.2.16}$$

$$\overline{\sigma}_{\,\mu} = \sigma^2\sigma^*_\mu\sigma^2 \tag{1.2.17}$$

space-time distance — hermiticity

$$\det(X^2) = \det(\overline{X}^{\,2}) = (x^\mu)^2, \qquad X^+ = X,\ \overline{X}^{\,+} = \overline{X} \tag{1.2.18}$$

Transformations preserving space-time distance and hermiticity. Let $U_L,\ U_R \in SL(2,\mathbb{C})$.

$$\begin{aligned} X &= U_R^+\, X'\, U_R \ \Rightarrow \det(X^2) = \det(X'^2) \Rightarrow |\det U_R| = 1 \\ \overline{X} &= U_L^+\overline{X}\,'U_L \end{aligned} \tag{1.2.19}$$

Relation to Lorentz: $\quad X = U_R^+\, X'\, U_R \ \leftrightarrow\ \sigma_\mu x^\mu = (U_R^+\,\sigma_\mu\, U_R)\,\Lambda^\mu{}_\nu x^\nu$

$$\Rightarrow \quad (U_R^+\,\sigma^\mu\, U_R) = \Lambda^\mu{}_\nu\sigma^\nu, \quad (U_L^+\,\overline{\sigma}^{\,\mu}\, U_L) = \Lambda^\mu{}_\nu\overline{\sigma}^{\,\nu}, \tag{1.2.20}$$

$$\Rightarrow \quad U_L = \sigma^2 U_R^*\sigma^2, \quad U_L\, U_R^+ = 1 \tag{1.2.21}$$

$U_R,\ U_L$ are two inequivalent (but conjugate) reps of $SL(2,\mathbb{C})$.

1.2.4.3 WEYL SPINORS

- Space of left-spinors $\mathbf{S_L} = \{\psi'_L(x') = U_L \psi_L\}$, right-spinors $\mathbf{S_R} = \{\psi'_R(x') = U_R \psi_R\}$,
- Conjugate spinor : $(\psi_L)_C \equiv i\sigma^2 \psi_L^* \in S_R$, $(\psi_R)_C \equiv -i\sigma^2 \psi_R^* \in S_L$
- the relative sign sets$\rightarrow$ see charge conjugation

1.2.4.4 DIRAC EQ. ($\overline{\partial} \equiv \overline{\sigma}^{\,\mu}\partial_\mu$): $\overline{\partial}\,\psi_L \in \mathbf{S_R},\ \partial\psi_R \in \mathbf{S_L}$

$$\boxed{\begin{array}{c} i\overline{\partial}\,\psi_L - m\chi_R = 0 \\ i\partial\chi_R - m\psi_L = 0 \end{array}}$$

Eq. for 2 independent spinors - is has U(1) invariance: $\psi_L \rightarrow\ e^{i\alpha}\psi_L,\ \chi_R \rightarrow\ e^{-i\alpha}\chi_R$

1.2.4.5 DIRAC-MAJORANA EQ.

$$\begin{aligned} &i\overline{\partial}\,\psi_L - m\chi_R - m_1(\psi_L)_C = 0 \\ &i\partial\chi_R - m\psi_L - m_2(\psi_R)_C = 0 \end{aligned} \tag{1.2.22}$$

1.2.4.6 γ-AMMAS Minkowski metric $(1, -1, -1, -1)$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \quad (\gamma^i)^\dagger = -\gamma^i, \quad (\gamma^0)^\dagger = \gamma^0 \tag{1.2.23}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^{\,\mu} & 0 \end{pmatrix} = (i\sigma^2 \otimes 1, \sigma^1 \otimes \sigma^i), \qquad \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma^{\mu\nu} = (\sigma^{\mu\nu})_\alpha{}^\beta = \frac{i}{4}(\sigma^\mu\overline{\sigma}^{\,\nu} - \sigma^\nu\overline{\sigma}^{\,\mu}), \qquad \overline{\sigma}^{\,\mu\nu} = (\overline{\sigma}^{\,\mu\nu})^{\dot\alpha}{}_{\dot\beta} = \frac{i}{4}(\overline{\sigma}^{\,\mu}\sigma^\nu - \overline{\sigma}^{\,\nu}\sigma^\mu)$$

Lorentz

$$M_S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] = \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \overline{\sigma}^{\,\mu\nu} \end{pmatrix}, \qquad [M_S^{\mu\nu}, \gamma^\rho] = -(M_V^{\mu\nu})^\rho{}_\sigma\gamma^\sigma$$

$$(\gamma^0 U^+ \gamma^0)\gamma^\mu U = \Lambda^\mu{}_\nu\gamma^\nu, \qquad U = e^{i/2\,\omega_{\mu\nu}M_S^{\mu\nu}} = \begin{pmatrix} U_L & 0 \\ 0 & U_R \end{pmatrix}, \qquad \Lambda = e^{i/2\,\omega_{\mu\nu}M_V^{\mu\nu}}$$

1.2.4.7 DIRAC FIELD

$$\psi = \begin{pmatrix} \psi_L \\ \chi_R \end{pmatrix}, \quad \psi_L = \tfrac{1}{2}\,(1 - \gamma_5)\psi \tag{1.2.24}$$

$$\psi'(x') = U\psi(x), \partial'_\mu\gamma^\mu\psi' = U\partial_\mu\gamma^\mu\psi, \quad \overline{\psi} \equiv \psi^\dagger\gamma^0, \quad \overline{\psi}\,'(x') = \overline{\psi}\,U^{-1}$$

$$(i\partial\!\!\!/ - m)\psi = 0, \quad \text{Dirac eq.,} \quad \psi(x) = \int \tilde{dk}\, [b_k^s u_s(k)\, e^{-ikx} + (d_k^s)^\dagger v_s(k) e^{ikx}]|_{k_0=\omega_k} \tag{1.2.25}$$

$$(p\!\!\!/ - m)u_s(p) = 0,\ (p\!\!\!/ + m)v_s(p) = 0,\ s = 1, 2 \tag{1.2.26}$$

$$\bar{u}_s(p)u_r(p) = 2m\delta_{sr}, \qquad \bar{v}_s(p)v_r(p) = -2m\delta_{sr}, \qquad \bar{u}_s(p)v_r(p) = 0,$$
$$\textstyle\sum_s u_s\bar{u}_s = p\!\!\!/ + m \equiv 2m\Lambda_+, \qquad \sum_s v_s\bar{v}_s = p\!\!\!/ - m \equiv -2m\Lambda_-$$

I/Z 2.2.1. [] [3] General solution [4]

1.2.4.8 Lorentz tensors $\overline{\psi}\,\gamma^{\mu\nu\cdots}\psi$.

1.2.4.9 Right– and Left– Handed Fermions

It is convenient to define the *helicity projectors*:

$$\boxed{P_L \equiv \frac{1}{2}(1-\gamma_5)} \quad , \quad \boxed{P_R \equiv \frac{1}{2}(1+\gamma_5)}\,, \tag{1.2.27}$$

which satisfy the usual properties of projection operators,

$$P_L + P_R = 1\ , \quad , P_R\, P_L = P_L\, P_R = 0\ , \quad , P_L^2 = P_L\ , \quad , P_R^2 = P_R\ .$$

For the conjugate spinors we have,

$$\begin{aligned} \bar{\psi}_L &= (P_L\psi)^\dagger\gamma_0 = \psi^\dagger P_L^\dagger\gamma_0 = \psi^\dagger P_L\gamma_0 = \psi^\dagger\gamma_0 P_R = \bar{\psi}P_R \\ \bar{\psi}_R &= \bar{\psi}P_L\ . \end{aligned}$$

Let us make some general remarks. First of all, we should notice that fermion mass term mixes right– and left–handed fermion components,

$$\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\ . \tag{1.2.28}$$

On the other hand, the electromagnetic (vector) current, does not mix those components, *i.e.*

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_R\gamma^\mu\psi_R + \bar{\psi}_L\gamma^\mu\psi_L\ . \tag{1.2.29}$$

Finally, the $(V - A)$ fermionic weak current can be written in terms of the helicity states as,

$$\bar{\psi}_L\gamma^\mu\psi_L = \bar{\psi}P_R\gamma^\mu P_L\psi = \bar{\psi}\gamma^\mu P_L^2\psi = \bar{\psi}\gamma^\mu P_L\psi = \frac{1}{2}\,\bar{\psi}\gamma^\mu(1-\gamma_5)\psi\ , \tag{1.2.30}$$

what shows that only left–handed fermions play a rôle in weak interactions.

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[3] $(p\!\!\!/-m)\Lambda_+ = 0 \to\ \Lambda_+ = a_0(p\!\!\!/+m) + a_5(p\!\!\!/+m)\gamma_5,\ \Lambda_+(p\!\!\!/-m) = 0 \to\ \Lambda_+ = a_0(p\!\!\!/+m),\ \Lambda_+^2 = \Lambda_+ \to\ a_0 = 1/(2m).$

[4] With the above normalization of spinors we get CCR for b, d operators as in Itzyskon/Zuber 3.3.

Chapter 2

Quanta

2.1 Free Quantum Fields

2.1.1 Basics of quanta

Formalizm kanoniczny mech.klasycznej

$$p = \frac{\partial L(q,\dot{q})}{\partial \dot{q}},\ \{q,p\}_{PB} = 1,\ H(q,p) = p\dot{q} - \mathcal{L},\ eq.m.\ \dot{f}(q,p) = \{f,H\} \tag{2.1.1}$$

de Broigle $\lambda = \frac{h}{p} \rightarrow$,free particle is $e^{-iEt+\vec{p}\vec{x}}$, $E = E(k)$- zwiazek dyspersyjny.

Formalizm kanoniczny mech.kwantowej

$$p = \frac{\partial L(q,\dot{q})}{\partial \dot{q}}, \quad [q,p] = i, \qquad\qquad i\dot{\mathcal{O}} = [\mathcal{O},H] \ \rightarrow \ \mathcal{O}(t) = e^{iHt}\mathcal{O}(0)e^{-iHt}$$

$q, p, H, ...$ are operators on a Hilbert space $\mathcal{H}$.

$$\langle\psi|\mathcal{O}(t)|\psi'\rangle = \langle\psi(t)|\mathcal{O}(0)|\psi'(t)\rangle \tag{2.1.2}$$

where Schrodinger eq. $i\partial_t|\psi(t)\rangle = H(p,q)|\psi(t)\rangle$ i.e. $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$.

2.1.2 Free scalar field – can.quant.

Heisenberg,Dirac $\rightarrow$ quantized field is someting completely different then classical field (relation A.2.1.3) $\Phi = \Phi_{cl} + \phi_q$ particle interpretation comes form the dispertion relation (we shall operate with completely unlocalized waves)

2.1.2.1 REAL FIELD

- Plane waves. Czastki i antyczastki (r.Kleina-Gordona)

$$\mathcal{L}_0 = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] \qquad \text{E.o.m.} \quad (\partial^2 + m^2)\phi = 0. \tag{2.1.3}$$

  Eq.m. solves (see (1.2.1))

$$\phi(x) = \int \tilde{dk}\, [a(\vec{k})e^{-ikx} + a^*(\vec{k})e^{ikx}]|_{k_0=\omega_k}, \qquad \tilde{dk} \equiv \bar{d}^3k/2\omega_k \tag{2.1.4}$$

- Canonical momentum:

$$\pi(x) \stackrel{\text{df}}{=} \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)} = \partial_t\phi \tag{2.1.5}$$

- Canonical commutation relation:

$$[\phi(\vec{x}\,', t), \pi(\vec{x}, t)] = i\delta^{(3)}(\vec{x}\,' - \vec{x}) \tag{2.1.6}$$

- Rep. $\phi(\vec{x}\,, t) = \frac{\delta}{\delta\phi(\vec{x}\,,t)}$.$\rightarrow$ Schrodinger eq. (see A.2.1.2) $\rightarrow$ not GOOD.

- Creation, annihilation operators

$$\phi(x) = \int \tilde{dk}\, [a(\vec{k})e^{-ikx} + a^\dagger(\vec{k})e^{ikx}]|_{k_0=\omega_k}, \qquad a(\vec{k}) = e^{i\omega_k t}\int d^3x\, (i\pi(x) + \omega_k\phi(x))e^{-i\vec{k}\vec{x}}$$
$$[a(\vec{k}), a^\dagger(\vec{k}\,')] = \tilde{\delta}(k - k'),\ [a, a] = 0, \qquad \tilde{\delta}(k - k') \equiv 2\,\omega_k\,(2\pi)^3\delta^{(3)}(\vec{k} - \vec{k}\,') \tag{2.1.7}$$

- Hamiltonian: classical

$$T_{\mu\nu} = \int d^3x\, (\partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L})\,, \quad \partial_\mu T^{\mu\nu} = 0 \tag{2.1.8}$$
$$\text{conserved charges} \quad P_\mu = \int d^3x\, (\pi\partial_\mu\phi - \eta_{0\mu}\mathcal{L})$$
$$H = \int d^3x\, (\pi\partial_0\phi - \mathcal{L}) = \int d^3x \frac{1}{2}\,[\pi^2 + m^2\phi^2 + (\nabla\phi)^2] \tag{2.1.9}$$

  $\boxed{\star}$ $\partial_t H_0 = 0$ []. Quantum

$$H_0 = \frac{1}{2}\int \tilde{dk}\,\omega_k[a^\dagger(k)a(k) + a(k)a^\dagger(k)] \tag{2.1.10}$$

  Momentum operator: $\vec{P} = \int \tilde{dk}\,\vec{k}\,a^\dagger(k)a(k)$ and the particle number operator: $N = \int \tilde{dk}\,a^\dagger(k)a(k)$ so that $a^\dagger(k)a(k)$ is the density of states.

- Interpertation.

$$[P^\mu, a(k)] = -k^\mu\, a(k), \quad [P^\mu, a^\dagger(k)] = k^\mu\, a^\dagger(k), \qquad k^0 = \omega_k \tag{2.1.11}$$

thus if $H_0|\psi\rangle = E|\psi\rangle$ then $a(k)|\psi\rangle$ has energy $(E-\omega_k)$, and $a^\dagger(k)|\psi\rangle$ has energy $(E+\omega_k)$. We say that the state $a(k)|\psi\rangle$ has one particle of momentum k^μ less than $|\psi\rangle$ and the state $a^\dagger(k)|\psi\rangle$ has one particle of momentum k^μ more than $|\psi\rangle$.

The requirement that H_0 is bounded from below implies that there exists a (unique) state of the least energy (ground state, vacuum state: $|0\rangle$) which does not contain any particle so it is annihilated by $a(k)$: $a(k)|0\rangle$. We assign to it null energy: $H_0|0\rangle = 0$. In order be in agreement with (2.1.10) we have to redefine H_0: we normal order it. The procedure applied to any operator amounts to flip positions of a, $a^\dagger$ in such a way that $a^\dagger$ will stand in the leftmost position. Normal ordering is denoted by double dots e.g. $:A:$. Thus the normal ordered Hamiltonian is

$$:H_0: = \int \tilde{dk}\ \omega_k\, a^\dagger(k) a(k) \tag{2.1.12}$$

In fact $:H_0:$ differs from H_0 by an infinite constant which originates from (2.1.6).

2.1.3 Time indep. states - Fock space.

The eigenstates of $:H_0:$ are:

- vacuum state: $|0\rangle$,
- one particle states of momentum $\vec{k}$:$|k> = a^\dagger(k)|0\rangle$. These are normalized as follows: $\langle k'|k\rangle = \tilde{\delta}(\vec{k}\,' - \vec{k})$ i.e. the normalization is different than in non-relativistic quantum mechanics (!!!) - the r.h.s. of the last formula is Lorentz invariant.
- two particle states: $|k_1, k_2> = \frac{1}{2}\, a^\dagger(k_2) a^\dagger(k_1)|0\rangle$, ($\frac{1}{2}$ due to two identical particles)
- etc.

2.1.3.1 Complex field .

$$\begin{aligned} \mathcal{L}_0 &= |\partial_\mu \phi|^2 - m^2 |\phi|^2 && (2.1.13) \\ \phi(x) &= \int \tilde{dk}\ [a(\vec{k})\, e^{-ikx} + b^\dagger\, (\vec{k}) e^{ikx}]|_{k_0=\omega_k} && (2.1.14) \\ P^\mu &= \int \tilde{dk}\ k^\mu [a^\dagger a + b^\dagger b] && (2.1.15) \end{aligned}$$

Notice that the 4-momentum of the state $b^\dagger$ is k and not $-k$ what naively could be inferred from the wave-function standing by $b^\dagger$ in (2.1.14).

2.1.3.2 CHARGE.??? The Hamiltonian for charged particles $H_0 = a_i^\dagger a_i + b_i^\dagger b_i$ has an extra sl_2 symmetry algebra: $J^0 = a_i^\dagger a_i - b_i^\dagger b_i$, $J^+ = a_i^\dagger b_i$, $J^- = b_i^\dagger a_i$. J^0 is the charge ! For free theory the charge, and exchange of particle for anti-particle does not change the results! If this would be exact symmetry of the theory and we could work with finite reps. of the algebra we would have quantized charge (by reps of sl_2). For interaction theory we must work with infinite rep. of this algebra so there is no bound on possible charges.

2.1.4 Fermions

$[A, BC]_{\#BC} = [A, B]_{\#B}C + (-1)^{\#B}B[A, C]_{\#C}$

2.1.4.1 DIRAC

$$\psi(x) = \sum_{s=1}^{2}\int \tilde{dk}\ \left[u^s(k)b^s(k)e^{-ikx} + v^s(k)d^{s+}(k)e^{ikx} \right] \tag{2.1.16}$$

$$\{b^s(k), b^{r+}(k')\} = \delta^{rs}\tilde{\delta}(k-k'), ...etc. \tag{2.1.17}$$

$$\{b^s(k)^+, \psi(x)\} = u^s(k)e^{-ikx}, \quad \{b^s(k), \psi(x)\} = 0 \tag{2.1.18}$$

$$\{d^s(k)^+, \psi(x)\} = 0, \quad \{d^s(k), \psi(x)\} = v^s(k)e^{ikx} \tag{2.1.19}$$

propagator
$$\langle 0|T\psi(x)\overline{\psi}(y)|0\rangle = \int \frac{i(\not{k} + m)}{k^2 - m^2 + i\epsilon} e^{-ik(x-y)} \tag{2.1.20}$$

2.1.4.2 WEYL, MASSLESS LEFT NEUTRINO As above but $m \to 0$, spin $s = 1$ only. Propagator has an extra $P_L = (1 - \gamma_5)/2$.

2.1.5 Gauge fields

Quantum field

$$A_\mu(x) = \sum_s \int \tilde{dk}\ [\epsilon_\mu^s\ a^s(k)e^{-ikx} + \epsilon_\mu^{i*}\ a^{s+}(k)e^{ikx}], \tag{2.1.21}$$

$$
\begin{aligned}
[a_A^{s+}(k), A_\mu(x)] &= \epsilon^s_{A,\mu}(k) e^{-ikx}\,, && \text{photon, } \ Z \\
[a_W^{s+}(k), W^+_\mu(x)] &= \epsilon^s_{W,\mu}(k) e^{-ikx}, \quad [a^s_W(k), W^+_\mu(x)] = 0 && \\
[b^s_W(k)^+, W^-_\mu(x)] &= \epsilon^{s*}_{W,\mu}(k) e^{ikx}\,, \quad [b^{s+}_W(k), W^+_\mu(x)] = 0 && \\
\langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle &= \int d\!\!{}^-{}^4 k \, \frac{-i(\eta_{\mu\nu} - k_\mu k_\nu / k^2)}{k^2} e^{-ik(x-y)} && \text{massless } \ \partial_\mu A_\mu = 0 \\
\langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle &= \int d\!\!{}^-{}^4 k \, \frac{-i(\eta_{\mu\nu} - k_\mu k_\nu / M^2)}{k^2 - M^2} e^{-ik(x-y)} && \text{massive, unitary gauge = no GB}
\end{aligned}
\tag{2.1.22}
$$

2.2 Interacting QF – Feynman diagams

2.2.1 Evolution operator

We are interested in the evolution of the physical states in the interacting theory:

$$H = H_0 + H_I$$

Hamiltonian is time-independent for all isolated systems of intererst here.

Schroedinger eq. $i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$ implies that the evoluton operator $U(t,t')$

$$state \qquad |\psi(t)\rangle = U(t,t')|\psi(t')\rangle, \quad U(t,t) = 1$$

$$operator \qquad \mathcal{O}(t) = U(t,t')\mathcal{O}(t')U(t,t')^+$$

$$\boxed{i\partial_t U(t,t') = H\, U(t,t') \qquad U(t,t') = e^{-iH(t-t')} = U(t)U^+(t')} \tag{2.2.1}$$

H is time independent so e.g. for the scalar theory is given by (2.1.9) with $\phi = \phi(0,\vec{x}\,)$.

2.2.1.1 SCATTERING MATRIX We scatter some particles in full, interacting theory. We assume that at $t' \to -\infty$ (the incomeing state) evolves as free state of interest $U_0(t')|\psi_i\rangle$ (see Sec.2.1.3):

$$|\psi(t')\rangle \quad \overset{t'\to -\infty}{\longrightarrow} \quad |\psi_i(t')\rangle = U_0(t')|\psi_i\rangle. \tag{2.2.2}$$

Fot $t > t'$ if evolves as: $|\psi(t)\rangle_i = U(t,t')|\psi(t')\rangle$. Now we send $t \to \infty$ and ask about the transition amplitude $|\psi_i\rangle \to |\psi_f\rangle$ (also called scattering amplitude):

$$\lim_{t\to\infty} < \psi_f(t)|\psi(t)\rangle_i = \lim_{-t',t\to\infty} \langle\psi_f|U_0^+(t)U(t,t')U_0(t')|\psi_i\rangle \tag{2.2.3}$$

The quantity $U_0^+(t)U(t,t')U_0(t') \equiv U_I(t,t')$ is called evolution operator in the interacting picture (index I). It respects

$$i\partial_t U_I(t,t') = H_I^{(I)}(t)\, U_I(t,t') \tag{2.2.4}$$

where $H_I^{(I)}(t) \overset{\text{df}}{=} U_0^+(t)\, H_I\, U_0(t)$. Notice that for $H_I = 0,\ U_I = 1,\ U_I(t,t) = 1$.

IMPORTANT: Although H is time independent $H_I^{(I)}(t)$ *depends on time* throught all fields which are subject to FREE evolution by $U_0(t)$ i.e. they are free fields !!!

$$U_0^+(t)\,\phi(x,0)\,U_0(t) = \phi(x,t).$$

$\boxed{\star}$ *Do $U_0^+(t)\,\phi(x,0)\,U_0(t) = \phi(x,t)$. Be careful: $U_0(t) = e^{-itH_0}$ not : e^{-itH_0} :. The latter does not give the correct evolution of states.* []

♣ *CCR for free fields are invariant under any evolution $\phi(t) = U_{any}\,\phi(0)U_{any}^{-1}$: the a, a^+ are time independent. S is not normal ordered: i.e. $\langle 0|S|0\rangle$ is non trivial.* ♣

2.2.1.2 Perturbative solution

Formally we can integrate (2.2.4)

$$U_I(t,t') = 1 - ig\int_{t'}^{t} dt\; H_I^{(I)}(t_1)U(t_1,t') \tag{2.2.5}$$

Perturbation series

$$\begin{aligned} U_I^{(0)}(t,t') &= 1 \\ U_I^{(1)}(t,t') &= 1 - i\int_{t'}^{t} dt_1\; H_I^{(I)}(t_1) \\ U_I^{(2)}(t,t') &= 1 - i\int_{t'}^{t} dt H_I^{(I)}(t_1)[1 - i\underbrace{\int_{t'}^{t_1} dt_2\; H_I^{(I)}(t_2)}_{\int_{t'}^{t} dt_2\; H_I^{(I)}(t_2)\theta(t_1-t_2)}\;] \end{aligned} \tag{2.2.6}$$

If we denote $U(t,t') = 1 + \sum_{n=1} u^{(n)}$ then

$$u^{(n)} = (-i)^n \int_{t'}^{t} dt_1 \int_{t'}^{t} dt_2 \ldots \int_{t'}^{t} dt_n\; \theta(t_1-t_2)\theta(t_2-t_3)\ldots\theta(t_{n-1}-t_n)H_I^{(I)}(t_1)\ldots H_I^{(I)}(t_n) \tag{2.2.7}$$

$$= \frac{(-i)^n}{n!}\left(\prod_{k=1}^{n}\int_{t'}^{t} dt_k\right)T(H_I^{(I)}(t_1)\ldots H_I^{(I)}(t_n)) \tag{2.2.8}$$

e.g. $u^{(2)} = \frac{(-ig)^2}{2}\int dt_1 dt_2[\theta(t_1-t_2)H_I^{(I)}(t_1)H_I^{(I)}(t_2) + \theta(t_2-t_1)H_I^{(I)}(t_2)H_I^{(I)}(t_1)]$ changing $1 \leftrightarrow 2$ the second term eqauls the first one thus we get (2.2.7).

Because **we calculate momenta from the free theory** then $H_I = -L_I$.

$$u^{(n)} = \frac{i^n}{n!}\left(\prod_{k=1}^{n}\int d^4x_k\right)T(\mathcal{L}_I(x_1)\ldots\mathcal{L}_I(x_n)) \tag{2.2.9}$$

We are interested in evolution from $t' = -\infty$ to $t = \infty$ then

$$U(t,t') \rightarrow\; S = Te^{iS_I[\phi]} \tag{2.2.10}$$

♣ *No* **normal ordering** *of H_I* ♣

$$\boxed{S_{i\rightarrow f} = \langle f|\,Te^{iS_I[\phi]}\,|i\rangle} \tag{2.2.11}$$

2.2.1.3 PROBLEMS OF (2.2.11) (a) **Haag's theorem.** Expression (2.2.11) assumes that non-interacting multi-particle states at asymptotic times can be unitarily related to interacting states at finite times. This is problematic by Haag's theorem, which states that, under reasonable assumptions, the Hilbert spaces for interacting and non-interacting theory belong to unitarily inequivalent representations of the canonical (anti-)commutation relations. Thus a unitary S-matrix operator that transforms non-interacting states into interacting states does not exist (Duncan 2012, pp. 359–370).

$\rightarrow$ This can be repaired pragmatically by introducing renormalization constants for incomeing (i) and outgoing states (f), e.g. instead of (2.2.2)

$$|\psi(t')\rangle \quad \overset{t' \to -\infty}{\longrightarrow} \quad Z_i|\psi_i(t')\rangle. \tag{2.2.12}$$

(b) **UV (ultra-violet) Problem.** For many of the types of interacting QFTs of interest, the terms in the power series (1) diverge. **UV vs. IR divergences.** $\rightarrow$ Renormalization needed.
(c) **Convergence Problem**. For the types of interacting QFTs of interest, there is a consensus that the power series (1) does not converge. $\rightarrow$ Non-perturbative contributions $\rightarrow$ $\boxed{\star}$ $\int e^{-x^2-\lambda x^4}$ [].

2.2.2 Feynman rules

2.2.2.1 SCALAR THEORY We would like to calculate scattering $1+1\to\ 1+1$

$$\langle k_1,k_2|Te^{iS_I}|p_1,p_2\rangle = \langle 0|a(k_1)a(k_2)(1+iS_I+\frac{i^2}{2}T(S_I)^2+\ldots)a^\dagger(p_1)a^\dagger(p_2)|0\rangle \tag{2.2.13}$$

in e.g. $\mathcal{L}=(\partial_\mu\phi\partial^\mu\phi^*-m^2\phi\phi^*)-\frac{\lambda}{4}(\phi\phi^*)^2\to\ \mathcal{L}_I=-\frac{\lambda}{4}(\phi\phi^*)^2$.
We need to commute $a(k)$, ... to the other side of the expression. We write $\langle 0|\,a(k_2)\,\mathcal{E}|0\rangle=\langle 0|[a(k_2),\mathcal{E}]\,|0\rangle$. Then we use the fact that the commutator defines differentation

$$[A,BCD...]=[A,B]CD...+B[A,C]D...+$$

(and similarly for $[BCD...,A]=[B,A]CD...+$) thus a_k act on each term of the $\mathcal{E}$ as diff. according to (2.2.14).

Free the field $\phi(x)=\int\tilde{dk}[a(k)e^{-ikx}+b^\dagger(k)e^{ikx}]$ we need

$$[a_k,\phi^\dagger(x)]=e^{ikx}=[b_k,\phi(x)],\quad [a_k,\phi(x)]=0=[b^+,\phi(x)], \tag{2.2.14}$$

$[a_k,a_p^\dagger]=\tilde{\delta}_{kp}$ and the same for $b,\ b^\dagger$. $\tilde{\delta}_{kp}$ means that $k=q$ i.e. this particle does not interact: we dismiss these contributions.

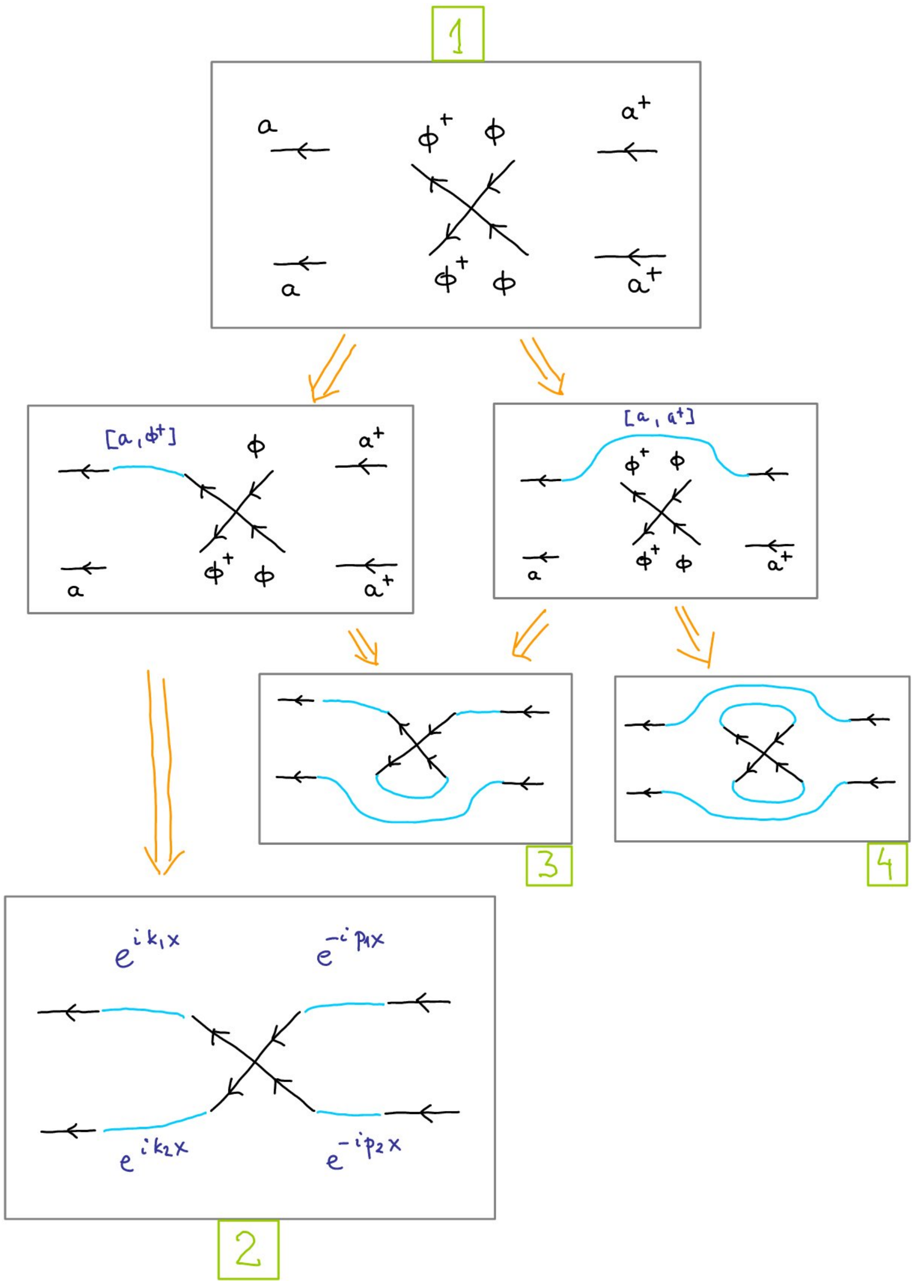
$\leftarrow$(?)they factorize into $\mathbb{I}\otimes$ ...

The firs non-trivial contribution comes from

$$\langle 0|a(k_1)a(k_2)\left(i\frac{\lambda}{4}\int d^4x\phi^2(\phi^\dagger)^2(x)\right)a^\dagger(k_1)a^\dagger(k_2)|0\rangle \tag{2.2.15}$$

The only diagram which takes into account interaction between particles is $\boxed{2}$. It gives

$$i\lambda\int d^4xe^{ix(k_3+k_4-k_1-k_2)}=(2\pi)^4\delta^{(4)}(k_1+k_2-k_3-k_4)\,i\lambda \tag{2.2.16}$$



Example: ϕ^3 **theory.**

$$\mathcal{L}_I = \frac{\lambda}{2}[(\phi_1^*)^2\phi_2 + (\phi_1)^2\phi_2^*]$$

$1+1 \to\ 1+1$ is easy because there is only two diagrams with propagator.

$$\langle k_1, k_2|Te^{iS_I}|p_1, p_2\rangle = \langle 0|a(k_1)a(k_2)(1 + iS_I + \frac{i^2}{2}T(S_I)^2 + \ldots)a^\dagger(p_1)a^\dagger(p_2)|0\rangle \qquad (2.2.17)$$

2.2.2.2 FEYNMAN PROPAGATOR

$$
\begin{aligned}
\langle 0|T\phi(x)\phi^\dagger(y)|0\rangle =&\int \tilde{dk}\ \tilde{dq}\langle 0|\, a_k a_q^\dagger\, e^{-ikx+iqy}\theta(x^0-y^0) + b_q b_k^\dagger\, e^{-iqy+ikx}\theta(y^0-x^0)|0\rangle \\
=&\int \tilde{dk}\ (e^{-ik(x-y)}\theta(x^0-y^0) + e^{ik(x-y)}\theta(y^0-x^0)) = \int đ^4k\ \frac{i}{k^2-m^2+i\epsilon}\, e^{-ik(x-y)}
\end{aligned}
\tag{2.2.18}
$$

$\boxed{\star}$ *Show that*

$$
(\partial_x^2+m^2)G(x-y)=\delta^{(4)}(x-y), \quad \textit{where } G(x-y)\equiv i\langle 0|T\phi(x)\phi^\dagger(y)|0\rangle \tag{2.2.19}
$$

by simple differentiation !!! (see also **??***)* [] *Time-dependent Green's functions: [2].*

Propagators: Description of different Greens functions: Greiner, Field quantization,4.4-6, 9.2. Bogolubov, Shirkov, sec.16.

Chapter 3

Standard model and beyond

3.1 A Chronology of the Weak Interactions

We will present in this section the main steps given towards a unified description of the electromagnetic and weak interactions. In order to give a historical flavor to the presentation, we will mention some parallel achievements in Particle Physics in this century, from theoretical developments and predictions to experimental confirmation and surprises. The topics closely related to the evolution and construction of the model will be worked with more details.

The chronology of the developments and discoveries in Particle Physics can be found in the books of Cahn and Goldhaber [**?**] and the annotated bibliography from COMPAS and Particle Data Groups [**?**]. An extensive selection of original papers on Quantum Electrodynamics can be found in the book edited by Schwinger [**?**]. Original papers on gauge theory of weak and electromagnetic interactions appear in Ref. [**?**].

1896 Becquerel: evidence for spontaneous radioactivity effect in uranium decay, using photographic film.

1897 * Thomson: discovery of the electron in cathode rays.

1900 * Planck: start of the quantum era.

1905 Einstein: photoelectric effect – quanta of light – photons

1911 * Millikan: measurement of the electron charge.

1911 Rutherford: evidence for the atomic nucleus.

1913 * Bohr: invention of the quantum theory of atomic spectra.

1914 Chadwick: first observation that the β spectrum is continuous. Indirect evidence on the existence of neutral penetrating particles.

1919 Rutherford: discovery of the proton, constituent of the nucleus.

1923 * Compton: experimental confirmation that the photon is an elementary particle in $\gamma + C \to \gamma + C$.

1923 * de Broglie: corpuscular–wave dualism for electrons.

1925 * Pauli: discovery of the exclusion principle.

1925 * Heisenberg: foundation of quantum mechanics.

1926 * Schrödinger: creation of wave quantum mechanics.

1927 Ellis and Wooster: confirmation that the β spectrum is continuous.

1927 Dirac: foundations of Quantum Electrodynamics (QED).

1928 * Dirac: discovery of the relativistic wave equation for electrons; prediction of the magnetic moment of the electron.

1929 Skobelzyn: observation of cosmic ray showers produced by energetic electrons in a cloud chamber.

1930 Pauli : first proposal, in an open letter, of the existence of a light, neutral and feebly interacting particle emitted in β decay.

1930 Oppenheimer: self–energy of the electron: the first ultraviolet divergence in QED.

1931 Dirac: prediction of the positron and anti–proton.

1932 * Anderson: first evidence for the positron.

1932 * Chadwick: first evidence for the neutron in $\alpha + Be \to \ C + n$.

1932 Heisenberg: suggestion that nuclei are composed of protons and neutrons.

1934 Pauli : explanation of continuous electron spectrum of β decay — proposal for the neutrino.

$$n \to \ p + e^- + \bar{\nu}_e \ .$$

1934 Fermi: field theory for β decay, assuming the existence of the neutrino. In analogy to "the theory of radiation that describes the emission of a quantum of light from an excited atom", $eJ_\mu A^\mu$, Fermi proposed a current–current Lagrangian to describe the β decay:

$$\mathcal{L}_{\rm weak} = \frac{G_F}{\sqrt{2}} \left(\bar{\psi}_p \, \gamma_\mu \, \psi_n\right) \left(\bar{\psi}_e \, \gamma^\mu \, \psi_\nu\right) \ .$$

1936 Gamow and Teller: proposed an extension of the Fermi theory to describe also transitions with $\Delta J^{\rm nuc} \neq 0$. The vector currents proposed by Fermi are generalized to:

$$\mathcal{L}_{\rm weak} = \frac{G_F}{\sqrt{2}} \sum_i C_i \left(\bar{\psi}_p \, \Gamma^i \, \psi_n\right) \left(\bar{\psi}_e \, \Gamma^i \, \psi_\nu\right) \ ,$$

with the scalar, pseudo–scalar, vector, axial and tensor structures:

$$\Gamma^S = 1 \, , \quad \Gamma^P = \gamma_5 \, , \quad \Gamma^V_\mu = \gamma_\mu \, , \quad \Gamma^A_\mu = \gamma_\mu \gamma_5 \, , \quad \Gamma^T_{\mu\nu} = \sigma_{\mu\nu} \ .$$

Nuclear transitions with $\Delta J = 0$ are described by the interactions $S.S$ and/or $V.V$, while $\Delta J = 0, \pm 1 \ (0 \not\to 0)$ transitions can be taken into account by $A.A$ and/or $T.T$ interactions ($\Gamma^P \to 0$ in the non–relativistic limit). However, interference between them are proportional to m_e/E_e and should increase the emission of low energy electrons. Since this behavior was not observed, the weak Lagrangian should contain,

$$S.S \ \underline{\text{or}} \ \ V.V \ \ \underline{\text{and}} \ \ A.A \ \underline{\text{or}} \ \ T.T \ .$$

1937 Neddermeyer and Anderson: first evidence for the muon.

1937 Majorana: Majorana neutrino theory.

1937 Bloch and Nordsieck: treatment of infrared divergences.

1940 Williams and Roberts: first observation of muon decay

$$\mu^- \to \ e^- + (\bar{\nu}_e + \nu_\mu) \ .$$

1943 Heisenberg: invention of the S–matrix formalism.

1943 * Tomonaga: creation of the covariant quantum electrodynamic theory.

1947 Pontecorvo : first idea about the universality of the Fermi weak interactions *i.e.* decay and capture processes have the same origin.

1947 Bethe : first theoretical calculation of the Lamb shift in non–relativistic QED.

1947 * Kusch and Foley: first measurement of $g_e - 2$ for the electron using the Zeeman effect: $g_e = 2(1 + 1.19 \times 10^{-3})$.

1947 * Lattes, Occhialini and Powell: confirmation of the π^- and first evidence for pion decay $\pi^\pm \to \mu^\pm + (\nu_\mu)$.

1947 Rochester and Butler: first evidence for V events (strange particles).

1948 Schwinger: first theoretical calculation of $g_e - 2$ for the electron: $g_e = 2(1+\alpha/2\pi) = 2(1 + 1.16 \times 10^{-3})$. The high–precision measurement of the anomalous magnetic moment of the electron is the most stringent QED test. The present theoretical and experimental value of $a_e = (g_e-2)/2$, are ,

$$
\begin{aligned}
a_e^{\rm thr} &= (115\,965\,215.4 \pm 2.4) \times 10^{-11} \ , \\
a_e^{\rm exp} &= (115\,965\,219.3 \pm 1.0) \times 10^{-11} \ ,
\end{aligned}
$$

where we notice the impressive agreement at the 9 digit level!

1948 * Feynman ; Schwinger ; Tati and Tomonaga : creation of the covariant theory of QED.

1949 Dyson : covariant QED and equivalence of Tomonaga, Schwinger and Feynman methods.

1949 Wheeler and Tiomno; Lee, Rosenbluth and Yang: proposal of the universality of the Fermi weak interactions. Different processes like,

$$
\begin{aligned}
\beta - \text{decay} &: \quad n \to p + e^- + \bar{\nu}_e \ , \\
\mu - \text{decay} &: \quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \ , \\
\mu - \text{capture} &: \quad \mu^- + p \to \nu_\mu + n \ ,
\end{aligned}
$$

must have the same nature and should share the same coupling constant,

$$
G_F = \frac{1.03 \times 10^{-5}}{M_p^2} \ ,
$$

the so–called Fermi constant.

50's A large number of new particles where discovered in the 50's: π^0, $K^\pm$, Λ, K^0, Δ^{++}, Ξ^-, $\Sigma^\pm$, $\bar{\nu}_e$, $\bar{p}$, $K_{L,S}$, $\bar{n}$, Σ^0, $\bar{\Lambda}$, Ξ^0, $\cdots$

1950 Ward: Ward identity in QED.

1953 Stückelberg; Gell–Mann: invention and exploration of renormalization group.

1954 Yang and Mills: introduction of local gauge isotopic invariance in quantum field theory. This was one of the key theoretical developments that lead to the invention of non–abelian gauge theories.

1955 Alvarez and Goldhaber; Birge *et al.* : $\theta - \tau$ puzzle: The "two" particles seem to be a single state since they have the same width ($\Gamma_\theta = \Gamma_\tau$), and the same mass ($M_\theta = M_\tau$). However the observation of different decay modes, into states with opposite parity:

$$\begin{aligned} \theta^+ &\rightarrow \pi^+ + \pi^0 \ , \qquad\qquad J^P = 0^+ \ , \\ \tau^+ &\rightarrow \pi^+ + \pi^+ + \pi^- \ , \quad J^P = 0^- \ , \end{aligned}$$

suggested that parity could be violated in weak transitions.

1955 Lehmann, Symanzik and Zimmermann: beginnings of the axiomatic field theory of the S–matrix.

1955 Nishijima: classification of strange particles and prediction of Σ^0 and Ξ^0.

1956 * Lee and Yang: proposals to test spatial parity conservation in weak interactions.

1957 Wu et.al.: obtained the first evidence for parity nonconservation in weak decays. They measured the angular distribution of the electrons in β decay,

$$^{60}\text{Co (polarized)} \rightarrow \ ^{60}\text{Ni} + e^- + \bar{\nu}_e \ ,$$

and observed that the decay rate depend on the pseudo–scalar quantity: $< \vec{J}_{\text{nuc}} > . \ \vec{p}_e$.

1957 Garwin, Lederman and Weinrich; Friedman and Telegdi : confirmation of parity violation in weak decays. They make the measurement of the electron asymmetry (muon polarization) in the decay chain,

$$\begin{aligned} \pi^+ \quad &\rightarrow \quad \mu^+ + \nu_\mu \\ &\qquad \hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu \ . \end{aligned}$$

1957 Frauenfelder et.al. : further confirmation of parity nonconservation in weak decays. The measurement of the longitudinal polarization of the electron ($\vec{\sigma}_e.\vec{p}_e$) emitted in β decay,

$$^{60}\text{Co} \rightarrow \ e^- \ (\text{long. polar.}) + \bar{\nu}_e + X \ ,$$

showed that the electrons emitted in weak transitions are mostly left–handed.

The confirmation of the parity violation by the weak interaction showed that it is necessary to have a term containing a γ_5 in the weak current:

$$\mathcal{L}_{\text{weak}} \ \rightarrow \ \frac{G_F}{\sqrt{2}} \sum_i C_i \left(\bar{\psi}_p \, \Gamma^i \, \psi_n \right) \left[\bar{\psi}_e \, \Gamma^i \, (1 \pm \gamma_5) \, \psi_\nu \right] \ .$$

Note that CP remains conserved since C is also violated.

1957 Salam ; Lee and Yang ; Landau: two–component theory of neutrino. This requires that the neutrino is either right or left–handed.

Since it was known that electrons (positrons) involved in weak decays are left (right) handed, the leptonic current should be written as:

$$J^i_{\rm lept} \equiv \left[\bar{\psi}_e \, \Gamma^i \, (1 \pm \gamma_5) \, \psi_\nu\right] \to \left[\bar{\psi}_e \, \frac{(1+\gamma_5)}{2} \, \Gamma^i \, (1 \pm \gamma_5) \, \psi_\nu\right] \; .$$

Therefore the measurement of the neutrino helicity is crucial to determine the structure of the weak current. If Γ^i = V or A then $\{\gamma_5, \Gamma^i\} = 0$ and the neutrino should be left–handed, otherwise the current is zero. On the other hand, if Γ^i = S or T, then $[\gamma_5, \Gamma^i] = 0$, and the neutrino should be right–handed.

1957 Schwinger ; Lee and Yang: development of the idea of the intermediate vector boson in weak interaction. The four–fermion Fermi interaction is "point–like" *i.e.* a s–wave interaction. Partial wave unitarity requires that such interaction must give rise to a cross section that is bound by $\sigma < 4\pi/p^2_{\rm cm}$. However, since G_F has dimension of M^{-2}, the cross section for the Fermi weak interaction should go like $\sigma \sim G_F^2 p^2_{\rm cm}$. Therefore the Fermi theory violates unitarity for $p_{\rm cm} \simeq 300$ GeV.

This violation can be delayed by imposing that the interaction is transmitted by a intermediate vector boson (IVB) in analogy, once again, with the quantum electrodynamics. Here, the IVB should have quite different characteristics, due to the properties of the weak interaction. The IVB should be charged since the β decay requires charge–changing currents. They should also be very massive to account for short range of the weak interaction and they should not have a definite parity to allow, for instance, a $V - A$ structure for the weak current.

With the introduction of the IVB, the Fermi Lagrangian for leptons,

$$\mathcal{L}_{\rm weak} = \frac{G_F}{\sqrt{2}} \left[J^\alpha(\ell) J^\dagger_\alpha(\ell') + \text{h.c.}\right] \; ,$$

where $J^\alpha(\ell) = \bar{\psi}_{\nu_\ell} \Gamma^\alpha \psi_\ell$, becomes:

$$\mathcal{L}^W_{\rm weak} = g_W \left(J^\alpha W^+_\alpha + J^{\dagger\,\alpha} W^-_\alpha\right) \; , \tag{3.1.1}$$

with a new coupling constant g_W.

Let us compare the invariant amplitude for μ–decay, in the low–energy limit in both cases. For the Fermi Lagrangian, we have,

$$\mathcal{M}_{\rm weak} = i \frac{G_F}{\sqrt{2}} J^\alpha(\mu) J_\alpha(e) \; . \tag{3.1.2}$$

On the other hand, when we take into account the exchange of the IVB, the invariant amplitude should include the vector boson propagator,

$$\mathcal{M}^{W}_{\rm weak} = [i\, g_W J^{\alpha}(\mu)] \left[\frac{-i}{k^2 - M_W^2}\left(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{M_W^2}\right)\right] \left[i\, g_W J^{\beta}(e)\right] \ .$$

At low energies, *i.e.* for $k^2 \ll M_W^2$,

$$\mathcal{M}^{W}_{\rm weak} \ \longrightarrow \ i\,\frac{g_W^2}{M_W^2} J^{\alpha}(\mu) J_{\alpha}(e) \ , \tag{3.1.3}$$

and, comparing (3.1.3) with (3.1.2) we obtain the relation

$$\boxed{g_W^2 = \frac{M_W^2 G_F}{\sqrt{2}}} \ , \tag{3.1.4}$$

1958 Feynman and Gell–Mann ; Marshak and Sudarshan ; Sakurai : universal $V-A$ weak interactions.

$$J^{+\,\mu}_{\rm lept} = \left[\bar{\psi}_e\, \gamma^{\mu}(1-\gamma_5)\, \psi_\nu\right] \ . \tag{3.1.5}$$

1958 Leite Lopes : hypothesis of neutral vector mesons exchanged in weak interaction. Prediction of its mass of $\sim 60\ m_{\rm proton}$.

1958 Goldhaber, Grodzins and Sunyar : first evidence for the negative ν_e helicity. As mentioned before, this result requires that the structure of the weak interaction is $V - A$.

1959 * Reines and Cowan: confirmation of the detection of the $\bar{\nu}_e$ in $\bar{\nu}_e + p \to \ e^+ + n$.

1961 Goldstone : prediction of unavoidable massless bosons if global symmetry of the Lagrangian is spontaneously broken.

1961 Salam and Ward : invention of the gauge principle as basis to construct quantum field theories of interacting fundamental fields.

1961 * Glashow: first introduction of the neutral intermediate weak boson (Z^0).

1962 * Danby *et al.*: first evidence of ν_μ from $\pi^{\pm} \to \ \mu^{\pm} + (\nu/\bar{\nu})$.

1963 SU(3) flavour Symmetry (Gell-Mann, George Zweig and independently Ne'eman, in 1961). Quarks (u,d,s).

1963 Cabibbo : introduction of the Cabibbo angle and hadronic weak currents.

It was observed experimentally that weak decays with change of strangeness ($\Delta s = 1$) are strongly suppressed in nature. For instance, the width of the neutron is much larger than the Λ's,

$$\Gamma_{\Delta s=0}\left(n_{udd} \to \ p_{uud}\; e\bar{\nu}\right) \gg \Gamma_{\Delta s=1}\left(\Lambda_{uds} \to \ p_{uud}\; e\bar{\nu}\right) \ ,$$

which yield a branching ratio of 100% in the case of neutron and just $\sim 8 \times 10^{-4}$ for the Λ.

The hadronic current, in analogy with leptonic current (3.1.5), can be written in terms of the u, d, and s quarks,

$$J_\mu^H = \bar{d}\gamma_\mu(1-\gamma_5)u + \bar{s}\gamma_\mu(1-\gamma_5)u \ , \tag{3.1.6}$$

where the first term is responsible for the $\Delta s = 0$ transitions while the latter one gives rise to the $\Delta s = 1$ processes. In order to make the hadronic current also universal, with a common coupling constant G_F, Cabibbo introduced a mixing angle to give the right weight to the $\Delta s = 0$ and $\Delta s = 1$ parts of the hadronic current,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} , \tag{3.1.7}$$

where d', s' (d, s) are interaction (mass) eigenstates. Now the transition $\bar{d} \leftrightarrow u$ is proportional to $G_F \cos\theta_C \simeq 0.97\ G_F$ and the $\bar{s} \leftrightarrow u$ goes like $G_F \sin\theta_C \simeq 0.24\ G_F$.

The hadronic current should now be given in terms of the new interaction eigenstates,

$$\begin{aligned} J_\mu^H &= \bar{d}'\gamma_\mu(1-\gamma_5)u \\ &= \cos\theta_C\ \bar{d}\gamma_\mu(1-\gamma_5)u + \sin\theta_C\ \bar{s}\gamma_\mu(1-\gamma_5)u \ . \end{aligned} \tag{3.1.8}$$

1964 Bjorken and Glashow : proposal for the existence of a charmed fundamental fermion (c).

1964 Higgs ; Englert and Brout ; Guralnik, Hagen and Kibble: example of a field theory with spontaneous symmetry breakdown, no massless Goldstone boson, and massive vector boson.

1964 * Christenson, Cronin, Fitch and Turlay : first evidence of CP violation in the decay of K^0 mesons.

1964 * Salam and Ward : Lagrangian for the electroweak synthesis, estimation of the W mass.

1964 * Gell–Mann; Zweig: introduction of quarks as fundamental building blocks for hadrons.

1964 Greenberg; Han and Nambu: introduction of color quantum number and colored quarks and gluons.

1967 Kibble : extension of the Higgs mechanism of mass generation for non–abelian gauge field theories.

1967 * Weinberg : Lagrangian for the electroweak synthesis and estimation of W and Z masses.

1967 Faddeev and Popov: method for construction of Feynman rules for Yang–Mills gauge theories.

1968 * Salam: Lagrangian for the electroweak synthesis.

1969 Bjorken: invention of the Bjorken scaling behavior.

1969 Feynman: birth of the partonic picture of hadron collisions.

1970 Glashow, Iliopoulos and Maiani: introduction of lepton– quark symmetry and the proposal of charmed quark (GIM mechanism).

1971 * 't Hooft: rigorous proof of renormalizability of the massless and massive Yang– Mills quantum field theory with spontaneously broken gauge invariance.

1973 Kobayashi and Maskawa : CP violation is accommodated in the Standard Model with six favours.

1973: first experimental indication of the existence of weak neutral currents.

$$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^- \quad , \quad \nu_\mu + N \to \nu_\mu + X \ .$$

This was a dramatic prediction of the Standard Model and its discovery was a major success for the model. They also measured the ratio of neutral–current to charged–current events giving a estimate for the Weinberg angle $\sin^2\theta_W$ in the range 0.3 to 0.4.

1973 Gross and Wilczek; Politzer: discovery of asymptotic freedom property of interacting Yang–Mills field theories.

1973 Fritzsch, Gell–Mann and Leutwyler: invention of the QCD Lagrangian.

1974: confirmation of the existence of weak neutral currents in the reaction

$$\nu_\mu + N \to \nu_\mu + X \ .$$

1974 * Aubert *et al.* (Brookhaven); Augustin *et al.* (SLAC): evidence for the J/ψ ($c\bar{c}$).

1975 * Perl *et al.* (SLAC): first indication of the τ lepton.

1977 Herb *et al.* (Fermilab): first evidence of Υ ($b\bar{b}$).

1979 Barber *et al.* (MARK J Collab.); Brandelik *et al.* (TASSO Collab.); Berger *et al.* (PLUTO Collab.); W. Bartel (JADE Collab.): evidence for the gluon jet in $e^+e^- \to 3$ jet.

1983 * Arnison *et al.* (UA1 Collab.) ; Banner *et al.* (UA2 Collab.): evidence for the charged intermediate bosons $W^\pm$ in the reactions

$$p + \bar{p} \to W(\to \ell + \nu) + X \ .$$

They were able to estimate the W boson mass ($M_W = 81 \pm 5$ GeV) in good agreement with the predictions of the Standard Model.

_1983_ * Arnison *et al.* (UA1 Collab.) ; Bagnaia *et al.* (UA2 Collab.): evidence for the neutral intermediate boson Z^0 in the reaction

$$p + \bar{p} \to \ Z(\to \ \ell^+ + \ell^-) + X \ .$$

This was another important confirmation of the electroweak theory.

_1986_ * Van Dyck, Schwinberg and Dehmelt : high precision measurement of the electron $g_e - 2$ factor.

_1987_ Albrecht *et al.* (ARGUS Collab.) : first evidence of $B^0 - \bar{B}^0$ mixing.

_1989_ Abrams *et al.* (MARK-II Collab.): first evidence that the number of light neutrinos is 3.

_1992_ : precise determination of the Z^0 parameters.

_1995_ : observation of the top quark.

_2012_ Discovery of the Higgs particle of mass 125 GeV

3.2 Model standardowy

$$\text{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

$(3, 2, \frac{1}{6})_L$	$(3, 1, \frac{2}{3})_R$	$(3, 1, -\frac{1}{3})_R$	$(1, 2, -\frac{1}{2})_L$	$(1, 1, -1)_R$	$(1, 2, \frac{1}{2})_R$
Q_L	u_R	d_R	L	e_R	Φ
	Quarks		Leptons		Higgs

$$\mathcal{L} = \mathcal{L}_{gauge}(3.2.4) + \mathcal{L}_{fermions}(3.2.8) + \mathcal{L}_{Higgs}(3.2.9) + \mathcal{L}_{Yukawa}(3.3.1) + \mathcal{L}_{neutrinos}(\textbf{??}) \quad (3.2.1)$$

3.2.1 Constructing the Model

In fact, there were several attempts to construct a gauge theory for the (electro)weak interaction. In 1957, Schwinger [**?**] suggested a model based on the group $O(3)$ with a triplet gauge fields (V^+, V^-, V^0). The charged gauge bosons were associated to weak bosons and the neutral V^0 was identified with the photon. This model was proposed before the structure $V - A$ of the weak currents have been established [**?**, **?**, **?**].

The first attempt to incorporate the $V - A$ structure in a gauge theory for the weak interactions was made by Bludman [**?**] in 1958. His model, based on the $SU(2)$ weak isospin group, also required three vector bosons. However in this case the neutral gauge boson was associated to a new massive vector boson that was responsible for weak interactions without exchange of charge (neutral currents). The hypothesis of a neutral vector boson exchanged in weak interaction was also suggested independently by Leite Lopes [**?**] in the same year. This kind of process was observed experimentally for the first time in 1973 at the CERN neutrino experiment [**?**].

Glashow [**?**] in 1961 noticed that in order to accommodate both weak and electromagnetic interactions we should go beyond the $SU(2)$ isospin structure. He suggested the gauge group $SU(2) \otimes U(1)$, where the $U(1)$ was associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge through the analogous of the Gell-Mann–Nishijima formula ($Q = T_3 + Y/2$). The theory now requires four gauge bosons: a triplet (A^1, A^2, A^3) associated to the generators of $SU(2)$ and a neutral field (B) related to $U(1)$. The charged weak bosons appear as a linear combination of A^1 and A^2, while the photon and a neutral weak boson Z^0 are both given by a mixture of A^3 and B. A similar model was proposed by Salam and Ward [**?**] in 1964.

3.2.2 Choosing the gauge group

Let us investigate which gauge group would be able to unify the electromagnetic and weak interactions. We start with the charged weak current for leptons. Since electron–type and muon–type lepton numbers are separately conserved, they must form separate representations of the gauge group. Therefore, we refer as ℓ any lepton flavor ($\ell = e,\, \mu,\, \tau$), and the final Lagrangian will be given by a sum over all these flavors.

From Eq. (1.2.30), we see that the weak current (3.1.5), for a generic lepton ℓ, is given by,

$$J_\mu^+ = \bar{\ell}\gamma_\mu(1-\gamma_5)\nu = 2\,\bar{\ell}_L\gamma_\mu\nu_L\ . \tag{3.2.2}$$

We introduce the left–handed isospin doublet,

$$\mathsf{L} \equiv \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L = \begin{pmatrix} L\,\nu \\ L\,\ell \end{pmatrix} = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}\ , \tag{3.2.3}$$

We have just chosen the candidate for the gauge group,

$$\boxed{SU(2)_L \otimes U(1)_Y} \, .$$

The next step is to introduce *gauge fields* corresponding to each generator, that is,

$$\begin{aligned} SU(2)_L \quad &\longrightarrow \quad A^1_\mu\ ,\ \ A^2_\mu\ ,\ \ A^3_\mu\ , \\ U(1)_Y \quad &\longrightarrow \quad B_\mu\ . \end{aligned}$$

Defining the *strength tensors* for the gauge fields according to (**??**) and (**??**),

$$\begin{aligned} W^i_{\mu\nu} \ &\equiv\ \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,\epsilon^{ijk}\,A^j_\mu A^k_\nu\ , \\ B_{\mu\nu} \ &\equiv\ \partial_\mu B_\nu - \partial_\nu B_\mu\ , \end{aligned}$$

we can write the free Lagrangian for the gauge fields following the results (**??**) and (**??**),

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^i_{\mu\nu} W^{i\ \mu\nu} + \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_{QCD}\ . \tag{3.2.4}$$

3.2.2.1 COVARIANT DERIVATIVES ,

$$D_\mu = \partial_\mu - i\,\frac{g}{2}\,\sigma^a\,A^a_\mu - i\,g'\,Y\,B_\mu\ , \tag{3.2.5}$$

$$D_\mu = \partial_\mu - i\,g'\,Y\,B_\mu\ , \tag{3.2.6}$$

where g and g' are the coupling constant associated to the groups $SU(2)_L$ and $U(1)_Y$ respectively. Their values are

$$g \approx 0.65,\ g' \approx 0.35,\ \sin^2(\theta_w) \approx 0.23. \tag{3.2.7}$$

The fermion Lagrangian becomes

$$\mathcal{L}_{fermions} = \bar{L} i\gamma^\mu D_\mu L + \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{u}_R i\gamma^\mu D_\mu u_R \tag{3.2.8}$$

The Higgs doublet Lagrangian is

$$\mathcal{L}_{\text{scalar}} = (D_\mu\Phi)^\dagger\ D_\mu\Phi - \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2\ , \tag{3.2.9}$$

Physical

$$\lambda \approx 0.13,\ v \approx 245\ GeV \tag{3.2.10}$$

3.2.3 Higgs, W and Z masses

After SSB+BEH the scalar Lagrangian can be as

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \left|\left(\partial_\mu - ig\frac{\sigma^i}{2}A^i_\mu - i\frac{g'}{2}YB_\mu\right)\frac{(v+H)}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\right|^2 \\ &\quad -\mu^2\frac{(v+H)^2}{2} - \lambda\frac{(v+H)^4}{4}\ .\end{aligned} \tag{3.2.11}$$

The second term of (3.2.11) gives rise to terms involving exclusively the scalar field H, namely,

$$-\frac{1}{2}(-2\mu^2)H^2 + \frac{1}{4}\mu^2 v^2\left(\frac{4}{v^3}H^3 + \frac{1}{v^4}H^4 - 1\right)\ . \tag{3.2.12}$$

In (3.2.12) we can also identify the Higgs boson mass term with

$$\boxed{M_H^2 = 2\lambda v^2}\ , \tag{3.2.13}$$

and the self–interactions of the H field. In spite of predicting the existence of the Higgs boson, the Standard Model does not give a hint on the value of its mass since μ^2 is *a priori* unknown.

The *charged gauge bosons* are

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp A^2_\mu)\ \ , \tag{3.2.14}$$

Neutral gauge bosons are

$$\begin{pmatrix}A_\mu\\Z_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}B_\mu\\A^3_\mu\end{pmatrix}\ , \tag{3.2.15}$$

where θ_W is called the Weinberg angle.

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}\ . \tag{3.2.16}$$

The quadratic terms in the vector fields, are,

$$\frac{g^2v^2}{4}W^+_\mu W^{-\,\mu} + \frac{g^2v^2}{8\cos^2\theta_W}Z_\mu Z^\mu\ , \tag{3.2.17}$$

When compared with the usual mass terms for a charged and neutral vector bosons, $M_W^2 W^+_\mu W^{-\,\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$, and we can easily identify

$$\boxed{M_W = \frac{gv}{2}} \quad \boxed{M_Z = \frac{gv}{2c_W} = \frac{M_W}{c_W}}\ . \tag{3.2.18}$$

We can see from that no quadratic term in A_μ appears, and therefore, the photon remains massless, as we could expect since the $U(1)_{\text{em}}$ remains as a symmetry of the theory.

Taking into account the low–energy phenomenology via the relation (**??**), we obtain for the vacuum expectation value

$$\boxed{v = \left(\sqrt{2}G_F\right)^{1/2} \simeq 246 \text{ GeV}} \,, \tag{3.2.19}$$

and the Standard Model predictions for the W and Z masses are

$$M_W^2 = \frac{e^2}{4s_W^2}v^2 = \frac{\pi\alpha}{s_W^2}v^2 \simeq \left(\frac{37.2}{s_W} \text{ GeV}\right)^2 \sim (80 \text{ GeV})^2 \,,$$

$$M_Z^2 \simeq \left(\frac{37.2}{s_W c_W} \text{ GeV}\right)^2 \sim (90 \text{ GeV})^2 \,,$$

where we assumed a experimental value for $s_W^2 \equiv \sin^2\theta_W \sim 0.22$.

3.3 Fermion Masses and V_{CKM}

V_{CKM} is called the Cabibbo–Kobayashi–Maskawa matrix.

Note that the explicit mass term for fermions i forbidden by gauge invariance e.g.

$$m_\ell \,\bar{\ell}\,\ell = m_\ell \,(\bar{\ell}_R\,\ell_L + \bar{\ell}_L\,\ell_R) \,,$$

mixes L and R components and breaks gauge invariance. A way to give mass in a gauge invariant way is via the Yukawa coupling of fermions with the Higgs field (**??**),

$$\mathcal{L}_{\text{Yukawa}} = -L^i\Phi\, y_{ij}^\ell \ell_R^j - Q_L^i\Phi\, y_{ij}^d d_R^j - Q_L^i\Phi_c\, y_{ij}^u u_R^j + \ .c.c.\ (i,j=1,2,3) \tag{3.3.1}$$

where $\Phi_c = i\,\sigma_2\,\Phi^* = \begin{pmatrix} \phi^{0^*} \\ -\phi^- \end{pmatrix}$. After SSB+BEH

$$\mathcal{L}_{\text{yuk}} = (-\bar{\ell}_L^i y_{ij}^\ell \ell_R^j - \bar{d}_L^i\, y_{ij}^d d_R^j - \bar{u}_L^i\, y_{ij}^u u_R^j + \ .c.c.)\,\frac{1}{\sqrt{2}}(v+H) \tag{3.3.2}$$

All Yukawa couplings $y_{ij}^\ell,\ y_{ij}^d,\ y_{ij}^u$ can be diagonalized with two rotations yielding

$$= -\frac{1}{\sqrt{2}}(v+H)(y_i^\ell\ \bar{\ell}^i\ell^i + y_i^d\bar{d}^i d^i + y_i^u\bar{u}^i u^i) \,. \tag{3.3.3}$$

Thus, we can identify the fermion masses,

$$\boxed{m_i = \frac{y_i\ v}{\sqrt{2}}}\ . \tag{3.3.4}$$

The only side effect of the diagonalization is the CKM matrix in charges currents couplings of L quarks

$$\frac{g}{\sqrt{2}} \bar{u}_L^i\, \gamma^\mu V_{CKM}^{ij}\, d_L^j W_\mu^+ + c.c. \tag{3.3.5}$$

Using unitarity constraints and assuming only three generations the experimental value for the elements of the matrix V_{CKM}, with 90% of C.L., can be extract from weak quark decays and from deep inelastic neutrino scattering [**?**]. Absolute values of the V_{CKM} are

$$\begin{pmatrix} 0.9742 - 0.9757 & 0.219 - 0226 & 0.002 - 0.005 \\ 0.219 - 0.225 & 0.9734 - 09749 & 0.037 - 0.043 \\ 0.004 - 0.014 & 0.035 - 0.043 & 0.9990 - 0.9993 \end{pmatrix} .$$

The Cabibbo–Kobayashi–Maskawa matrix can be parameterized as

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\ e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\ e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\ e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\ e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\ e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} ,$$

where $s_{ij},\ c_{ij}) \equiv \sin\theta_{ij},\ \cos\theta_{ij}$. Notice that, in the limit of $\theta_{23} = \theta_{13} \to\ 0$, we associate $\theta_{12} \to\ \theta_C$, the Cabibbo angle (3.1.7). We should notice that, for three generations there appears complex phase δ, and therefore the weak interaction can violate CP thus also T.

3.4 Weak decays of hadrons

Appendix A

Appendices

A.1 Classical fields

A.1.1 *** Helicity-NOT READY

I/Z 2.2.1

Angular momentum operator

$$W_\mu = -\tfrac{1}{2}\,\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma \tag{A.1.1}$$

W^2 is Casimir of Poincare algebra. We enumerate states (of momentum k) by stating value of

$$W \cdot n = -\tfrac{1}{2}\,\gamma_5 \not{n}\not{k}, \quad n^2 = -1, \quad k \cdot n = 0 \tag{A.1.2}$$

then: $\tfrac{1}{2}\,\gamma_5 \not{n} u^\alpha = \alpha u^\alpha,\ \tfrac{1}{2}\,\gamma_5 \not{n} v^\alpha = -\alpha v^\alpha$ **←(?)**. One defines projection

$$P(n) = \tfrac{1}{2}\,(1 + \gamma_5 \not{n}) \tag{A.1.3}$$

$[\Lambda_\pm, P(n)] = 0, \quad \Lambda_+(k)P(n) + \Lambda_-(k)P(n) + \Lambda_+(k)P(-n) + \Lambda_-(k)P(-n) + = 1.$

For $n = n_k = (|\vec{k}|, k_0\vec{k}/|\vec{k}|)/m$ polarizaton states are called HELICITY.

$$P(n_k)\Lambda_\pm(k) \longrightarrow \tfrac{1}{2}\,(1 \pm h_k)\Lambda_\pm(k), \quad h_k = \frac{2\vec{J}\vec{k}}{k} \overset{s=1/2}{=} \frac{\vec{\Sigma}\vec{k}}{k} \tag{A.1.4}$$

FOR MASSLESS FERMION HELICITY STATES ARE CHIRAL STATES

$$\gamma_5 \psi = \mathbf{h_k} \psi$$

. $(\gamma^0 k - \gamma \cdot k)\psi = 0 \rightarrow\ (1 - \gamma^0 \vec{\gamma}\frac{\vec{k}}{k})\psi = 0$. Use $\gamma^0\gamma_i = \gamma_5 \Sigma_i$ gives $(1 - \gamma_5 \vec{\Sigma}\frac{\vec{k}}{k})\psi = 0 \rightarrow\ \gamma_5\psi = h_k\psi$.

A.1.2 Charge conjugation

$$\text{bispinor} \quad \phi_c = C\overline{\psi}^T,\ C^{-1}\gamma^\mu C = -(\gamma^\mu)^T \tag{A.1.5}$$

$$\text{for} \quad C = -i\gamma^0\gamma^2 = \sigma^3 \otimes i\sigma^2,\ C\gamma^{0\,T} = i\gamma^2$$

$$(\psi_c)_L = (\psi_R)_c = i\sigma^2\psi_R^*, \quad (\psi_c)_R = (\psi_L)_c = -i\sigma^2\psi_L^* \tag{A.1.6}$$

$\psi_L^+\psi_R \stackrel{C}{\to}\ \psi_R^+\psi_L$

A.1.3 Yang-Mills

A.1.3.1 Lagrangian and eq.m.

$$\begin{aligned} \mathcal{L} &= \frac{1}{4\,g^2}\mathrm{tr}(F^2) + J^{a\,\mu}A^a_\mu \\ \mathcal{D}_\mu F^{a\,\mu\nu} &= -J^{a\,\mu} \end{aligned} \tag{A.1.7}$$

The fastest way of deriving (**??**) is to write $\mathcal{L}_{gauge} = \frac{1}{4\,g^2}\mathrm{tr}(F^2) = \frac{1}{4\,g^2}\mathrm{tr}([D_\mu, D_\nu]^2)$. Variation is $\delta\mathcal{L} = \frac{1}{g^2}\mathrm{tr}([\delta A_\mu, D_\nu][D_\mu, D_\nu]) = \frac{1}{g^2}\mathrm{tr}([D_\nu, [D_\mu, D_\nu]]\delta A_\mu)$. This gives

$$\frac{\delta\mathcal{L}_{gauge}}{\delta A_\nu} = \frac{1}{g^2}[D_\mu, [D_\mu, D_\nu]] = \frac{-i}{g}\mathcal{D}_\mu F^{a\,\mu\nu}T^a$$

Also

- The external current (independent on A) must be covariantly conserved [1] i.e.

$$\mathcal{D}_\mu J^{a\,\mu} = 0. \tag{A.1.8}$$

- Bianchi identities: $\mathcal{D}_\mu \tilde{F}^{a\,\mu\nu} = 0.$ [★] []

- Hamiltonian

$$H \sim \int_V [F_{0i}^2 + \tfrac{1}{2}\,F_{ij}^2] \tag{A.1.9}$$

  Finite energy solutions $F_{0i}, F_{ij} \stackrel{|x|\to\infty}{\longrightarrow} 0$. Pure Y-M system ((**??**) with $J = 0$) does not have **static solutions** except $F = 0$.

---

[1] due to an identity $0 = \mathcal{D}_\nu\mathcal{D}_\mu F^{a\,\mu\nu}\ lhs \equiv [D_\nu, [D_\mu, [D^\mu, D^\nu]]] \stackrel{Jacobi}{=} [D_\mu, [D_\nu, [D^\mu, D^\nu]]] - [[D^\mu, D^\nu], [D_\nu, D_\mu]]$. On the rhs the second term is 0, the first $= -\ lhs$ thus lhs=0, which together with eq.m. leads to the conclusion.

- For topologically trivial spaces:

$$F = 0 \quad \Leftrightarrow \quad A = U^{-1}\partial_\mu U \tag{A.1.10}$$

Let $U \to 1$ for $|\vec{x}| \to \infty$ then the space R^3 is compactified to S^3. Moreover $\pi_3(G) = Z$ $\boxed{\star}$ [] for most of the non-abelian groups. We get Z distinct ground states of the Y-M theory. The topological number corresponds to elements of $\pi_3(G)$ and it is given by

$$n = \frac{1}{24\pi^2}\int_{S^3} \mathrm{tr}((U^{-1}dU)^3) \tag{A.1.11}$$

For $G = SU(2)$ one can explicitly calculate n in terms of the degree of the map $S^3 \to S^3$. $U = \phi^\mu \sigma^\mu$, where $|\phi| = 1$, $\sigma^\mu = (1, i\sigma^i)$, $\sigma^{(\mu}\overline{\sigma}^{\,\nu)} = \overline{\sigma}^{\,(\mu}\sigma^{\nu)} = \delta^{\mu\nu}$. $\boxed{\star}$ []

A.1.4 *** Massive U(1) gauge boson: Stuckelberg

We introduce an extra scalar real field ϕ:

$$\mathcal{L} = -\frac{1}{4}F^2 + \tfrac{1}{2}\left(\partial_\mu \phi - mA_\mu\right)^2 \tag{A.1.12}$$

This has gauge symmetry $\delta A_\mu = \partial_\mu \chi,\ \delta\phi = m\chi$. We can fix it choosing $\phi = 0$ and get

$$\mathcal{L} = -\frac{1}{4}F^2 + \tfrac{1}{2}\, m^2 (A_\mu)^2 \tag{A.1.13}$$

E.o.m. leads to

$$\partial^\mu F_{\mu\nu} = -m^2 A_\nu \ \Rightarrow 0 = \partial^\nu \partial^\mu F_{\mu\nu} = -m^2 \partial^\nu A_\nu \tag{A.1.14}$$

i.e. $\partial^\nu A_\nu = 0$. Free field solutions are as (1.2.10) but $s = 1, 2, 3$ due to lack of the residual gauge symmetry.

A.1.5 BEH (Higgs) mechanism – Massive gauge bosons

We consider a theory possessing (see figure below):

1. global, continuous, inner symmetry G
2. local, inner symmetry $G_{loc} \subseteq G$
3. ground state $\langle\phi\rangle$ invariant under $H \subseteq G$

BEH mechanism. Goldsotne bosons corresponding to $\underline{X} \cap \underline{G}_{loc}$ vanish from the theory and the appropriate gauge fields get masses.

Proof of the above result. The Lagrangain density is:

$$\mathcal{L} = |D_\mu \phi|^2 - V(\phi) \tag{A.1.15}$$

We consider the Goldstone bosons $\pi = \pi^\alpha(x)\ X^\alpha$ and the ground state part of ϕ: $\phi_\pi = e^{i\pi}\langle\phi\rangle$. We have $V(\phi_\pi) = V(\langle\phi\rangle)$ and

$$D_\mu(A)\phi_\pi = D_\mu(A)\, e^{i\pi}\langle\phi\rangle \tag{A.1.16}$$

By $\{X^\alpha\}$ we denoted the basis of $\underline{X}$. It splits into $\{X_l^a\} = \underline{X} \cap \underline{G}_{loc}$ and the rest $\{X_G^a\}$ (see the figure). We can write:

$$e^{i\pi} = e^{i\tilde{\pi}^a X_l^a}\ e^{i\tilde{\pi}^\beta X_G^\beta} \tag{A.1.17}$$

with $\tilde{\pi}$ being some combinations of π, then (A.1.16) is:

$$D_\mu(A)\phi_\pi = D_\mu(A) e^{i\tilde{\pi}^a X_l^a}\ e^{i\tilde{\pi}^\alpha X_G^\alpha}\langle\phi\rangle \tag{A.1.18}$$

If we make the gauge transformation by $e^{-i\tilde{\pi}^a X_l^a}$ on ϕ and A then the dependence on $\tilde{\pi}^a$ (do not mix with $\tilde{\pi}^\alpha$) will vanish everywhere. In other words $e^{-i\tilde{\pi}^a X_l^a}$ is a part of gauge transformation so $\mathcal{L}$ does not depend on it. This ends the proof.

From (A.1.15) after substitution $\phi \to \ \langle\phi\rangle$ one gets **mass matrix of the gauge bosons**.

$$M_{ab} = \frac{1}{2}\langle\phi\rangle^\dagger (t^{\dagger a} t^b + t^{\dagger b} t^a)\langle\phi\rangle \tag{A.1.19}$$

where t^a's are generators of G_l. $\underline{G}_{loc}$ splits into $(\underline{G}_{loc} \cap \underline{H}) \cup (\underline{G}_{loc} \cap \underline{X})$. Elements of $\underline{G}_{loc} \cap \underline{X}$ denoted by X_l^a does not annihilate the ground state $\langle\phi\rangle$. They correspond to non-null eigenvalues of (A.1.19). The other gauge bosons stay massless.

A.2 Quanta

A.2.1 *** Gauge bosons, currents

A.2.1.1 (N)INTERACTION ENERGY Plugging back to the interaction Lagrangain we get energy of interaction then force (Darwin....)

$$S = \int -\tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - j^\mu A_\mu = -\tfrac{1}{2}\int j^\mu A_\mu = -\tfrac{1}{2}\int dxdy\ j^\mu(x) G(x-y) j_\mu(y) \tag{A.2.1}$$

Let $j = j_1 + j_2$

$$S_{int} = -\int dxdy\, j_1^\mu(x) j_2^\mu(y) G(x-y) + (\text{samooddzialywanie} \rightarrow\ 0) \tag{A.2.2}$$

(see jackson for the explicit formulae p. 585, prawo Biota-Savarta p.178-180, Lagrangian Darwina p.570)

Localized current (moving charge,....) from relat. interaction of point particle with a background

$$j^\mu(x) = e\, c \int_C d\tau \delta^{(4)}(x - z(\tau)) \partial_\tau z^\mu \tag{A.2.3}$$

If currents are fixed we can solve (**??**) for A^μ

$$A^\mu = \int G(x-x') dx' j^\mu(x'), \quad G(x) = \int đ^k\, e^{-ikx} \frac{1}{k^2} \tag{A.2.4}$$

G_F rozwiazuje problem modelu samooddzialywanie czastki rozciaglej (Abrahamaa-Lorentza) - see Feynman course II/2 p.142

A.2.1.2 VACUUM STATE IN POSITION REPRESENTATION We express $a(p)$ by ϕ and π. Take $\pi(\vec{x}\,) = -i \frac{\delta}{\delta\phi(\vec{x}\,)}$ [2] , then

$$\forall \vec{p},\ a(p)|0\rangle = 0,\ \rightarrow\ \ |0\rangle = \exp\left\{-\frac{1}{2} \int d^3x \phi(\vec{x}\,) \sqrt{m^2 - \nabla_y^2}\, \phi(\vec{y})\right\} \tag{A.2.5}$$

It is time independent state because $H_0|0\rangle = 0$. If one suppressed x-dependence then it is like ordinary harmonic oscylator.

[2] we can suppress time dependence here because $H_0|0\rangle = 0$.

A.2.1.3 (N)RELATION TO CLASSICAL FIELDS We define coherent states [3]

$$a(k)|koh\rangle_f = f(k)|koh\rangle_f, \rightarrow \; |koh(k)\rangle_f = e^{-f^*(k)a(k)}e^{f(k)a^\dagger(k)}|0\rangle \tag{A.2.6}$$

then

$$|koh\rangle_f \;=\; \prod_k |koh(k)\rangle_f \sim e^{-\int d^3k f^*(k)a(k)}e^{\int d^3k f(k)a^\dagger(k)}|0\rangle \tag{A.2.7}$$

$$_f\langle koh|\widehat{\phi}(x)|koh\rangle_f = f(x) \in C^\infty(M) \tag{A.2.8}$$

$f(x)$ respect classical eq. of motion.

What about charge ?←(?)

A.2.2 *** Born approx.

$$U_I^{(1)}(t,t') \;=\; 1 - ig\int_{t'}^{t} dt_1\; H_I(t_1), \rightarrow \;\; |\psi(t)\rangle = e^{-iH_0(t-t')}|\psi(t')\rangle - ig\int_{t'}^{t} dt_1\; e^{-iH_0(t-t_1)}H_I e^{-iH_0(t_1-t')}|\psi(t')\rangle$$

$$interact. \qquad \rightarrow \;\; ig\int_{t'}^{t} dt_1\; e^{-iH_0(t-t_1)}H_I e^{-iE_k(t_1-t')}|k\rangle$$

In position rep we get for $t \rightarrow \; \infty$ and free outgoing wave (only forward propagation so we insert extra $\theta(t-t_1)$ so $t > t'$)

$$\psi(\vec{x}\,,t) = e^{-iE_k(t-t')}\psi_k(\vec{x}\,) \quad - \quad ig\int dt_1\; d\vec{x}_1\; \underline{\theta(t-t_1)} < x|e^{-iH_0(t-t_1)}H_I(t_1)|x_1 > e^{-iE_k(t_1-t')}\psi_k(\vec{x}_1)$$

$$interact. \quad \rightarrow \quad -ige^{-iE_k(t-t')}\int dt_1\; dx_1\; \theta(t-t_1) < x|e^{-i(H_0-E_k)(t-t_1)}H_I(t_1)|x_1 > \psi_k(\vec{x}_1)$$

Next we assume (locality) $H_I(t_1)|x_1 >= |x_1 > H_I(\vec{x}_1, t_1)$. For time indep. H_I we can ntegrate over time getting (see below)

$$e^{iE_k(t-t')}\psi(\vec{x}\,,t) = \psi_k(\vec{x}\,) + g\int dx_1\; G(x,x_1)H_I(x_1)\psi_k(\vec{x}_1)$$

the formula obtained previously.

We have

$$(i\partial_t - H_0) < x|e^{-iH_0 t}|x_1 >= 0, \quad < x|e^{-iH_0 t}|_{t=0}|x_1 >= \delta(\vec{x} - \vec{x}_1) \tag{A.2.9}$$

[3] Derivation: $a\mathcal{O} = \mathcal{O}a + f\mathcal{O}$, $\mathcal{O}^{-1}a\mathcal{O} = a + f$, $\rightarrow \; \mathcal{O} = e^{fa^\dagger + c}$, c commuts with a i.e. $\sim a$. If $|koh\rangle$ is normalized to 1 then $e^{fa^\dagger+c}$ is unitary so $c = -f^*a$.

thus

$$(i\partial_t - H_0)\left(-i\theta(t-t_1) < x|e^{-iH_0(t-t_1)}|x_1 >\right) = \delta^{(4)}(x-x_1) \tag{A.2.10}$$

For time-independent H_I we have can integrate over time.

$$G(\vec{x}\,,\vec{x}\,_1) \stackrel{\text{df}}{=} -i\int dt_1\; \theta(t-t_1) < x|e^{-i(H_0-E_k)(t-t_1)}|x_1 >$$

Notice that

$$\int_R dt\; e^{iE_k(t-t_1)}(i\partial_t - H_0)\left(-i\theta(t-t_1) < x|e^{-iH_0(t-t_1)}|x_1 >\right) = \delta^{(3)}(x-x_1) \tag{A.2.11}$$

i.e. $(E_k - H_0)G(x,x_1) = \delta^{(3)}(x-x_1)$. [4]

Feynman rules:
propagator $\frac{i}{k^2-m^2}$, $\quad V = \frac{i\lambda}{4}$, + zachowania 4-pedu.

A.2.3 Wick's theorem

$$\langle 0|T\phi_1(x_1)\ldots\phi_n(x_n)|0\rangle = \sum_{\{all\ choices\}}\;\prod_{\{all\ pairs\}}\langle 0|T\phi_i(x_i)\phi_j(x_j)|0\rangle, \quad \text{even } n \tag{A.2.12}$$

e.g. $\langle 0|T\phi(x_1)\phi^\dagger(x_2)\phi(x_3)\phi^\dagger(x_2)|0\rangle = \langle 0|T\phi(x_1)\phi^\dagger(x_2)|0\rangle\langle 0|\phi(x_3)\phi^\dagger(x_2)|0\rangle + (1\leftrightarrow 3)$.

In our case $\langle 0|T(\phi(x)\phi^\dagger(x))^2|0\rangle = 2\langle 0|T\phi(x)\phi^\dagger(x)|0\rangle^2$

Second order: overal $\frac{(i\lambda)^2}{4^2\cdot 2!}$

$$\int dxdy\; e^{i(k_1+k_2)x}e^{-i(kp_1+p_2)y}\langle 0|T\phi\phi^\dagger(x)\phi\phi^\dagger(y)|0\rangle$$

[4] Equivalently
$(E_k - H_0)G(\vec{x}\,) = -i\int dt_1\; \theta(t-t_1)(i\partial_{t_1}) < x|e^{-i(H_0-E_k)(t-t_1)}|x_1 >= \int dt_1\; \delta(t-t_1) < x|e^{-i(H_0-E_k)(t-t_1)}|x_1 >= \delta^{(3)}(\vec{x} - \vec{x}\,')$

Problems:

1. Calculate: $< x|e^{-iH_0t}|x' >$ by (a) QM method, (b) Fourier transform, and check (A.2.10)
2. Calculate $G(x,x')$ by (a) solving spherical symmetric diff. eq. $(E-H_0)G(x,x') = \delta^{(3)}(x-x')$ (b) Fourier transfor as for Yukawa

$$
\begin{aligned}
&= \int dxdy\; e^{i(k_1+k_2)x}e^{-i(kp_1+p_2)y}2\langle 0|T\phi(x)\phi^\dagger(y)|0\rangle\langle 0|T\phi^\dagger(x)\phi(y)|0\rangle \\
&= 2\int \text{đ}^4k\text{đ}^4q\int dxdy\; e^{i(k_1+k_2)x}e^{-i(p_1+p_2)y}e^{-i(k+q)(x-y)}\frac{i}{k^2-m^2}\frac{i}{q^2-m^2} \\
&= 2\int \text{đ}^4k\text{đ}^4q(2\pi)^8\delta^{(4)}(k_1+k_2-k-q)\delta^{(4)}(p_1+p_2+k+q)\frac{i}{k^2-m^2}\frac{i}{q^2-m^2} \\
&= (2\pi)^4\delta^{(4)}(k_1+k_2-p_1-p_2)2\int \text{đ}^4k\frac{(i)^2}{(k^2-m^2)((k-k_1-k_2)^2-m^2)} \qquad \text{(A.2.13)}
\end{aligned}
$$

A.2.4 Scalar QED: $\mathcal{L} = |D_\mu\phi|^2 - \frac{1}{4}F_{mu\nu}^2$

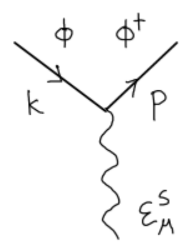
$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu - igA_\mu)\phi(\partial^\mu + igA^\mu)\phi^* \\
&= -\frac{1}{2}(\partial_\mu A_\nu)^2 + \partial_\mu\phi\partial^\mu\phi^* + (igA_\mu(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) + g^2A_\mu^2\phi\phi^*) \qquad \text{(A.2.14)}
\end{aligned}
$$

Quantum photon field

$$
\begin{aligned}
A_\mu(x) &= \int \tilde{dk}\;\sum_i\left[\epsilon_\mu^i\, a^i(k)e^{ikx} + h.c.\right], \qquad \text{(A.2.15)} \\
\langle 0|TA_\mu(x)A_\nu(y)|0\rangle &= \int \text{đ}^4k\;\frac{-i(\eta_{\mu\nu} - k_\mu k_\nu/k^2)}{k^2}e^{-ik(x-y)} \qquad \text{(A.2.16)}
\end{aligned}
$$

$$
L_I \supset (igA_\mu(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) + g^2A_\mu^2\phi\phi^*) \quad \rightarrow\; V_1 = ig(k+p)_\mu,\; V_2 = i\,g^2\eta_{\mu\nu} \qquad \text{(A.2.17)}
$$

We read off the Feynman rules (gauge $\partial_\mu A_\mu = 0$)



A.3 SM

A.3.1 V_{CKM}

A.3.1.1 DIAGONALIZATION OF YUKAWA COUPLINGS The weak eigenstates (q') are linear superposition of the mass eigenstates (q) given by the unitary transformations:

$$V^u_{L,R}\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L,R} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} , \qquad V^d_{L,R}\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L,R} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} ,$$

where $U(D)_{L,R}$ are unitary matrices to preserve the form of the kinetic terms of the quarks (**??**). These matrices diagonalize the mass matrices, *i.e.*,

$$V^{u+}_L y^u V^u_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad V^{d+} y^d V^d_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} .$$

V is the Cabibbo–Kobayashi–Maskawa matrix [**?**, **?**], that can be parametrized as

$$V = R_1(\theta_{23})R_2(\theta_{13}, \delta_{13})R_3(\theta_{12}) ,$$

where $R_i(\theta_{jk})$ are rotation matrices around the axis i, the angle θ_{jk} describes the mixing of the generations j and k and δ_{13} is a phase.

Bibliography

[1] Weinberg

[2] time-dependent Green's functions:

$$\partial_x^2 G(x,x') = \delta^{(4)}(x-x') \tag{A.3.1}$$

$$G(x) = \int đ^4 k \frac{1}{k^2-m^2}\, e^{-ikx} = \int đ^3 k\, e^{i\vec{k}\vec{x}}\, đ^k_0\, \frac{e^{-ik_0x_0}}{k_0^2-(\vec{k}^2+m^2)} \tag{A.3.2}$$

(A.3.3)

Different contour of integration = different boundary conditions (below $\omega \equiv \sqrt{\vec{k}^2+m^2}$)

$$đk_0 \to \begin{cases} opozniony: & \frac{1}{2\omega}(-i)\Theta(x_0)(e^{-i\omega x_0}-e^{i\omega x_0}) \\ przedwczesna: & \frac{1}{2\omega}(+i)\Theta(-x_0)(e^{-i\omega x_0}-e^{i\omega x_0}) \\ Feynmanowski: & \frac{1}{2\omega}((-i)\Theta(x_0)e^{-i\omega x_0}+(+i)\Theta(-x_0)e^{i\omega x_0}) \end{cases} \tag{A.3.4}$$

As above we can interato over θ:

$$G(\vec{x}-\vec{y}) = (2\pi)^{-3}\cdot 2\pi \cdot \frac{1}{r}\, i\, (\frac{1}{2}\int_{-\infty}^{\infty})dk\, k(e^{-ikr}-e^{ikr})\text{“}above''\tag{A.3.5}$$

Massless – we use $\int dke^{-ikx} = 2\pi\delta(x)$ and $\frac{1}{2\omega}(e^{-i\omega x_0}-e^{i\omega x_0}) = \frac{1}{2k}(e^{-ikx_0}-e^{ikx_0})$

$$\begin{aligned} opozniony: \quad G_{ret}(x) &= -\tfrac{1}{4\pi}\Theta(x_0)\tfrac{\delta(x_0-|\vec{x}|)}{r} \\ przedwczesna: \quad G_{adv}(x) &= -\tfrac{1}{4\pi}\Theta(-x_0)\tfrac{\delta(x_0+|\vec{x}|)}{r} \\ Feynmanowski: \quad G_F(x) &= -\tfrac{1}{4\pi}(....) \end{aligned} \tag{A.3.6}$$

(....) is a bit more complicated because in this case (**??**) is not even under $\omega \to -\omega$.

PICTURE: propagation of particles forwad in time and anti-particles backward in time.