

# Discussion of “Analytical Solution for Dissolution-Timescale Reactive Transport in Fluid-Saturated Porous Rocks” by Chongbin Zhao, B. E. Hobbs, and A. Ord

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The discussers wish to point out that some of the key results in this paper have been published previously (Lichtner 1988; Szymczak and Ladd 2014; Ladd and Szymczak 2017). In those papers, the governing Eqs. (16), (20), (21) were derived by asymptotic analysis in the limit that  $\gamma_a$  Eq. (2) is small. In particular, these papers show that there can be a diffuse interface between dissolved and undissolved materials, with a nonzero concentration downstream of the reaction front (Sherwood 1987; Hinch and Bhatt 1990). On the other hand, the authors and colleagues have energetically proposed an alternative scenario (Zhao et al. 2008, 2010, 2013, 2014, 2015, 2018; Zhao 2014) where the downstream concentration vanishes and the interface is sharp. In the paper under discussion, the same equations, which were heavily (but incorrectly) criticized recently (Zhao et al. 2018), are reintroduced without comment or citations to closely related work.

The connection between Eq. (21) and prior work, for example Eq. (30) in Ladd and Szymczak (2017), can be made explicit by a different scaling of the dimensionless quantities (Ladd and Szymczak 2017)

$$\tilde{u} = \frac{u}{u_0}, \quad \tilde{x} = \frac{xu_0}{D\phi}$$

where  $u_0$  is the fluid velocity in the undissolved matrix. These dimensionless variables can be connected to the ones in the paper under discussion  $\bar{u}$  and  $\bar{x}$  by the parameter  $Zh$  [Eq. (28)]

$$\bar{u} = Zh \tilde{u}, \quad \bar{x} = Zh^{-1} \tilde{x}$$

Substituting for  $\bar{u}$  and  $\bar{\nabla} = \partial/\partial\bar{x}$  in Eq. (21) we recover Eq. (30) of Ladd and Szymczak (2017), noting that the parameter  $H = D\phi ks_0/u_0^2 = Zh^{-2}$ . The scalings are equivalent for finite  $Zh$ , but the fast-reaction (or thin-front) limit of Eq. (21) cannot be taken directly because the reaction rate appears in the length scale. By contrast, in the scaling from Szymczak and Ladd (2014) and Ladd and Szymczak (2017) the dimensionless velocity is always of order unity and the thin-front limit can be found by taking  $H \rightarrow \infty$  (Ladd and Szymczak 2017).

Finally, we note that the porosity and concentration profiles for a steadily propagating front were derived in Szymczak and Ladd (2013, 2014) under the same conditions of small changes in porosity.

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