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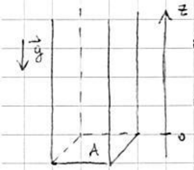
Zadanie 1




$$\Omega(E,A,N) = \int_{H<E} d\Gamma_N = \int_{H<E} \frac{d^{3N}p\, d^{3N}q}{h^{3N}N!} \qquad H = \sum_{i=1}^{N}\left(\frac{|\vec{p}_i|^2}{2m} + mgz_i\right) = \sum_{i=1}^{3N}\frac{p_i^2}{2m} + \sum_{j=1}^{N} mgz_j$$

$$\Omega(E,A,N) = \frac{1}{h^{3N}N!}\underbrace{\int_A d^{2N}x\, d^Ny}_{=A^N}\int_{H<E} d^{3N}p\, d^Nz = \frac{A^N}{h^{3N}N!}\int_{\sum_{i=1}^N\left(\frac{|\vec p_i|^2}{2m}+mgz_i\right)<E} d^{3N}p\, d^Nz$$

Warunek $\sum_{i=1}^{N}\left(\frac{|\vec{p}_i|^2}{2m}+mgz_i\right)<E$ implikuje $\sum_{i=1}^{N} mgz_i < E$. Zamieniamy na całkę iterowaną:

$$\Omega(E,A,N)=\frac{A^N}{h^{3N}N!}\int_{\sum_{i=1}^N z_i<\frac{E}{mg}} d^Nz \int_{\sum_{i=1}^{3N} p_i^2<\left(E-\sum_{i=1}^N mgz_i\right)\cdot 2m} d^{3N}p$$

Skalujemy:

$$\Omega(E,A,N) = \frac{A^N}{h^{3N}N!}\int_{\sum_{i=1}^N z_i<\frac{E}{mg}} d^Nz\left[2m\left(E-\sum_{i=1}^N mgz_i\right)\right]^{\frac{3N}{2}}\int_{\sum_{i=1}^{3N}p_i^2<1} d^{3N}p = \frac{A^N}{h^{3N}N!}\frac{\pi^{\frac{3N}{2}}}{\Gamma(\frac{3N}{2}-1)}\int_{\sum_{i=1}^N z_i<\frac{E}{mg}} d^Nz\left[2m\left(E-\sum_{i=1}^N mgz_i\right)\right]^{\frac{3N}{2}}$$

$$= \frac{A^N}{h^{3N}N!}\frac{\pi^{\frac{3N}{2}}}{\Gamma(\frac{3N}{2}+1)}\left(\frac{E}{mg}\right)^N(2mE)^{\frac{3N}{2}}\int_{\sum_{i=1}^N z_i<1} d^Nz\left(1-\sum_{i=1}^N z_i\right)^{\frac{3N}{2}}$$

Obliczamy całkę iterowaną: (wiemy, że $\forall_i\ z_i \geq 0$)

$$\int_{\sum_{i=1}^N z_i<1} d^Nz\left(1-\sum_{i=1}^N z_i\right)^{\frac{3N}{2}} = \int_0^1 dz_1\int_0^{1-z_1}dz_2\dots\int_0^{1-\sum_{i=1}^{N-1}z_i} dz_N\left(1-\sum_{i=1}^N z_i\right)^{\frac{3N}{2}} = \int_0^1 dz_1\int_0^{1-z_1}dz_2\dots \frac{1}{\frac{3N}{2}+1}\left[-\left(1-\sum_{i=1}^N z_i\right)^{\frac{3N}{2}+1}\right]\Bigg|_{0=z_N}^{1-\sum_{i=1}^{N-1}z_i=z_N} =$$

$$=\int_0^1dz_1\int_0^{1-z_1}dz_2\dots\int_0^{1-\sum_{i=1}^{N-2}z_i}dz_{N-1}\left(1-\sum_{i=1}^{N-1}z_i\right)^{\frac{3N}{2}+1}\frac{1}{\frac{3N}{2}+1} = \dots = \frac{1}{(\frac{3N}{2}+1)(\frac{3N}{2}+2)\dots(\frac{3N}{2}+N)}$$

$$\Omega(E,A,N) = \frac{A^N}{h^{3N}N!}\frac{\pi^{\frac{3N}{2}}}{\Gamma(\frac{3N}{2}+1)}\left(\frac{E}{mg}\right)^N(2mE)^{\frac{3N}{2}}\frac{1}{(\frac{3N}{2}+1)(\frac{3N}{2}+2)\dots(\frac{5N}{2})} = \frac{1}{h^{3N}N!}\left(\frac{AE}{mg}\right)^N(\cancel{\pi}\,2\pi mE)^{\frac{3N}{2}}\frac{1}{\Gamma(\frac{5N}{2}+1)}$$

Przyjmijmy, że N parzyste: $\Omega(E,A,N) = \frac{1}{N!(\frac{5N}{2})!}\left(\frac{A}{h^3mg}\right)^N(2\pi m)^{\frac{3N}{2}}E^{\frac{5N}{2}}$

$$w(E,A,N) = \frac{\partial\Omega(E,A,N)}{\partial E} = \frac{1}{N!(\frac{5N}{2}-1)!}\left(\frac{A}{h^3mg}\right)^N(2\pi m)^{\frac{3N}{2}}E^{\frac{5N}{2}-1}$$

$$S = k\ln w(E,A,N) = Nk\lim_{\infty}\frac{\ln w}{N} = Nk\lim_{\infty}\frac{1}{N}\left[N\ln\left(\frac{A}{h^3mg}\right)+\frac{3N}{2}\ln(2\pi m)+\frac{5N}{2}\ln E-\ln E-\ln N!-\ln\left(\frac{5N}{2}-1\right)!\right]\approx$$

$$\approx Nk\lim_{\infty}\frac{1}{N}\left[N\ln\left(\frac{A}{h^3mg}\right)-N\ln N+N+\frac{3N}{2}\ln(2\pi m)+\left(\frac{5N}{2}-1\right)\ln E-\left(\frac{5N}{2}-1\right)\ln\left(\frac{5N}{2}-1\right)+\frac{5N}{2}-1\right]=$$

$$= kN\ln\left(\frac{A}{h^3Nmg}\right)+k\frac{7N}{2}+k\frac{3N}{2}\ln(2\pi m)+k\frac{5N}{2}\ln\left(\frac{E}{\frac{5N}{2}-1}\right)$$

$$T = \left(\frac{\partial S}{\partial E}\right)^{-1}_{N,A} = \left(\frac{5}{2}Nk\frac{1}{E}\right)^{-1} = \frac{E}{\frac{5}{2}Nk}.$$