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15/15

$$Z = \sum_{k=0}^{N} e^{-\beta k \varepsilon} d(k)$$

polimer posiedł prosto N-k razy, a jak skręcił to miał za każdym razem 2 możliwości, więc degeneracja wynosi $d(k) = \binom{N}{N-k} 2^k$

$$Z = \sum_{k=0}^{N} e^{-\beta k \varepsilon} \frac{N!}{k!(N-k)!} 2^k = \sum_{k=0}^{N} \binom{N}{k} \left(e^{-\beta\varepsilon} \cdot 2\right)^k = \left(1+2e^{-\beta\varepsilon}\right)^N \checkmark$$

$$(1+x)^N = \sum_{k=0}^{N} \binom{N}{k} x^k$$

$$\langle E \rangle = \frac{1}{Z} \sum_{k=0}^{N} k\varepsilon e^{-\beta k \varepsilon} d(k) = \frac{-1}{Z} \frac{\partial}{\partial \beta} \sum_{k=0}^{N} e^{-\beta k \varepsilon} d(k) =$$

$$= -\frac{1}{Z} \frac{\partial}{\partial \beta} Z = -\frac{1}{Z} N \left(1+2e^{-\beta\varepsilon}\right)^{N-1} \cdot 2 \cdot (-\varepsilon) e^{-\beta\varepsilon} =$$

$$= \frac{2N\varepsilon e^{-\beta\varepsilon}}{1+2e^{-\beta\varepsilon}} = \frac{2N\varepsilon}{e^{\beta\varepsilon}+2} \checkmark$$

$$C_V = \frac{\partial \langle E \rangle}{\partial T} = \frac{\partial \langle E \rangle}{\partial \beta} \frac{\partial \beta}{\partial T} = \frac{1}{k_B T^2} 2N\varepsilon^2 \cdot \frac{1}{\left(e^{\beta\varepsilon}+2\right)^2} = \frac{2 k_B N \beta^2 \varepsilon^2}{\left(e^{\beta\varepsilon}+2\right)^2} \checkmark$$