

# Wire Regelation at Low Temperatures

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Received November 6, 1979; accepted January 23, 1980

The motion of a wire through ice by the regelation process has been measured in the temperature range  $-0.006$  to  $-35^{\circ}\text{C}$ . The motion of the wire was interpreted as an indication of a finite mobility of water in a liquid layer between the wire and the ice over this entire temperature range. The measured values of the mobility would indicate that the viscosity of the water in this layer is equal to the viscosity in the bulk water at least down to  $-15^{\circ}\text{C}$  or to a layer thickness of about 1 nm. A good approximation to the liquid layer thickness over the range of the experimental results is  $h = 3.5 (-T)^{-1/2.4}$  where  $h$  is in nm and  $T$  is in degrees Celsius.

## 1. INTRODUCTION

There are a number of phenomena associated with the behavior of an ice surface that have been the subject of continuing controversy over the past 100 years. These phenomena include the low sliding friction of ice, the sintering of ice particles, frost heave in soils, and the passage of a weighted wire through ice. The controversy originally centered around the explanation of Faraday's (1) experiments on the sintering of ice particles. The explanation put forward by Faraday (1) and Tyndall (2) was that ice melted on some parts of the ice particle, the water flowed along the surface of the ice in a thin "liquid-like" layer, and refroze at the contact between ice particles. Tyndall introduced the term regelation for the process by which ice can change geometry by undergoing simultaneous melting and refreezing. J. Thomson (3) and W. Thomson (4) attacked the suggestion that a "liquid-like" layer existed on the ice surface and suggested that the phenomenon could be explained entirely by pressure melting consideration. The debate over the nature of an ice surface and in particular the region of contact between ice and a substrate has con-

tinued to the present. One of the problems that has been used to test the various theories is the problem of the motion of a wire through ice by the regelation process.

The passage of a weighted wire through ice was first reported by Bottomley (5). Since then there have been numerous studies of this phenomenon. In most of the studies the ice through which the wire passed was maintained at or very near to  $0^{\circ}\text{C}$ . For ice at  $0^{\circ}\text{C}$  the mechanism for the phenomenon which has been widely accepted in the past depends on the pressure melting theory. In this theory the pressure of the wire on the ice produces the melting in front of the wire and refreezing behind it. The heat required for the melting of the ice on the front side of the wire comes from the heat released by refreezing of the water behind the wire. As a result the rate of passage of the wire is largely controlled by the rate that heat can be conducted across the wire. In several of the more recent studies this process has been quantified (6, 7). When quantitative predictions based on this theory are compared with measurements some very significant discrepancies between theoretical and experimental values have been found.

In experiments by Nunn and Rowell (8) measured and predicted velocities agreed for wires of low conductivity materials, however, for wires made of high conductivity materials, such as copper, which move at large regelation velocities, the measured velocity is less than predicted. This discrepancy appeared to be associated with the formation of a visible track of vapor or air bubbles in the wake of the wire. Frank (9) has noted that there is a potential instability of the refreezing front behind the wire that might be responsible for these observations.

A second anomaly was observed by Drake and Shreve (7) for wires subjected to low applied pressures. They observed that these wires could proceed through the ice at regelation velocities a factor of 100 smaller than would be predicted from the pressure melting theory. This depression of the regelation velocity was attributed to the influence of impurities being trapped in the water layer around the wire. They observed a transition from these slow velocities to velocities much closer to those predicted by the pressure melting theory at pressures of about 0.1 MPa. This transition they suggest occurs when the concentration of the impurities behind the wire reaches a point at which rejection of impurities in the wake of the wire can occur. Townsend and Vickery (10) have also reported regelation velocities for spheres which in some cases are as much as a factor of 30 less than expected.

The anomalously low regelation velocities mentioned above have all occurred in ice at or nearly at 0°C. It has been known for some time that regelation may also occur, at very low velocities, in ice at temperatures considerably below 0°C. Telford and Turner (11) have done a study of the regelation velocity as a function of ice temperature and report measurable regelation velocities down to -4°C. They suggest that regelation at these low temperatures proceeds because of the mobility of the water in the "liquid-

like" layer between the wire and the ice as suggested by Faraday (1) and Tyndall (2). A quantitative theory of the "liquid-like" layer was developed by Fletcher (12) from a consideration of the entropy of molecular orientations near a surface. Telford and Turner (11) used the predicted liquid layer thickness to calculate regelation velocities for comparison with their measurements. They found that calculated regelation velocities were substantially lower than their measurements. Also, in their measurements they found that the regelation velocity was proportional to the third power of the force applied to the wire whereas the predictions based on Newtonian flow in the liquid layer would suggest a linear dependence. It should be noted, however, that at the pressures used in these experiments (4.6 MPa) some plastic deformation of the ice may have occurred. Telford and Turner (11) observed a factor of 100 increase in regelation velocity for temperatures warmer than -0.5°C which they attributed to the effect of pressure melting.

A review of the regelation experiments would then suggest that two different modes of regelation exist—a fast mode which occurs for high pressures and in ice near 0°C and a much slower mode (several orders of magnitude slower) that occurs at lower pressures and, or at lower ice temperatures. Using a model which includes both an equilibrium "liquid-like" layer and the effect of pressure melting on the thickness of this layer Gilpin (13) has predicted this two-mode behavior. Regelation velocities for the faster mode are quantitatively predicted by the pressure melting theory—at least for cases in which refreezing behind the wire is complete. In this paper experimental observations on the slower mode will be examined with the objectives of comparing its behavior with predictions and of determining the thickness and water mobility in the liquid layer at an ice-substrate interface.

## 2. EXPERIMENTS

A schematic diagram of the apparatus used is shown in Fig. 1. The wire which moves in the ice was stretched across the open end of a fork with was in turn attached to the core of a linear displacement transducer. The ice sample which was held in a U-shaped holder oriented at  $90^\circ$  to the fork holding the wire was 10 mm wide in the direction of the wire. The wire could be put under tension so that the deflection of the wire when under load was normally less than 1 mm.

A water-tight cylinder containing the entire apparatus was immersed in a constant temperature bath during the tests. The regelation velocity was determined from a chart recording of the displacement transducer output versus time. The bath used was a calibration standard bath which could maintain a set temperature to within  $\pm 0.002^\circ\text{C}$  for a period of a week or more. The temperature uniformity in the test volume of the bath was  $0.002^\circ\text{C}$ . Temperatures were measured with a resistance thermometer with a NBS traceable calibration. The specified accuracy of the temperature measurement system was  $\pm 0.001^\circ\text{C}$ .

Ice specimens for the test were grown by slowly freezing a well-stirred container of triply distilled water from the bottom up. Freezing was stopped when approximately one-third of the water was frozen. By regulating the temperature of the cold surface on which the ice formed a constant rate of growth of about 10 mm per day was maintained. The ice produced had a columnar grains oriented parallel to the *c* axis. The diameter of these grains ranged from 2 to 10 mm. The direction of the regelation relative to the crystal orientation could also be varied by cutting samples in different directions from the ice block that was grown.

Regelation measurements were made in the temperature range  $-0.005$  to  $-35^\circ\text{C}$ . In

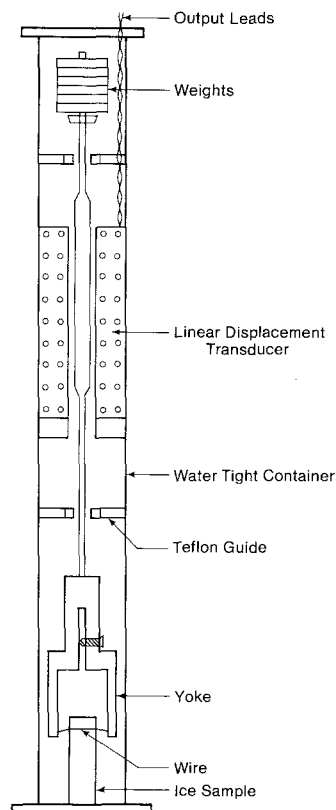


FIG. 1. The wire regelation apparatus.

addition to ice temperature the wire diameter, the wire material, the surface preparation of the wire, and the weight applied to the wire were varied.

## 3. THEORY

In (13) it was found that the effects of pressure melting on the regelation velocity could be neglected if  $P' < 1$ .  $P'$  is the ratio of the pressure on the wire cross-section,  $P$ , to the pressure required for pressure melting,  $P_c$ :

$$P' = \frac{P}{P_c} \quad [1]$$

where

$$P = \frac{F}{2lR} \quad [2]$$

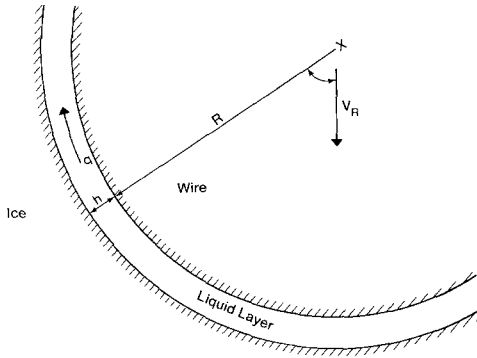


FIG. 2. A schematic representation of a wire moving through ice by regelation.

and

$$P_c = \frac{L(T_f - T)}{\Delta v T_a} + \frac{v_s}{\Delta v} \sigma_{SL} \bar{K}. \quad [3]$$

If the effects of pressure melting can be neglected the thickness of the liquid layer around the wire can be assumed to be constant and equal to its equilibrium value for zero applied pressure. In this case the thickness,  $h$ , is just a function of the characteristic temperature

$$T_c = T - T_f - \frac{T_a}{L} v_s \sigma_{SL} \bar{K}. \quad [4]$$

For wires of the diameter used 12.7 to 381  $\mu\text{m}$  the curvature term in Eq. [4] alters the temperature at most by 0.004°C. Therefore, its effect can be neglected except when the ice temperature is very near to 0°C.

In Fig. 2 a schematic drawing of a wire passing through ice is shown. In this drawing the thickness of the liquid layer,  $h$ , is exaggerated. In actual fact the ratio of  $h/R$  is approximately  $10^{-3}$ . A derivation of the regelation velocity,  $V_R$ , was given in (13); however, it will be convenient to review that derivation here and then consider various possible corrections to it. In the derivation in (13) it was implicitly assumed that the regelation velocity was much smaller than the flow velocity in the liquid layer. This assumption follows directly from the fact that  $h/R$  is assumed small. With this

assumption the flow in the liquid layer is as shown in Fig. 3a—that is channel flow between two stationary walls. In (13) the equation for the flow rate at some angular position,  $\theta$  in such a channel was

$$q/l = -K \frac{v_s}{v_L} \frac{dP}{R d\theta} \quad [5]$$

where  $K = h^3/12\eta$  and  $l$  is the length of the wire in the ice. Equation [5] is the same as one would expect for normal Newtonian flow in a parallel plate channel except for the factor  $v_s/v_L$ . This factor which is neglectable for most practical purposes arises because the variation in pressure across the liquid layer predicted by thermodynamic equilibrium equations was included in the calculation (13). From continuity consideration the flow rate in Eq. [5] is related to the regelation velocity by

$$q/l = \frac{v_L}{v_s} V_R R \sin\theta. \quad [6]$$

Combining [5] and [6] and integrating over  $\theta$  gives the pressure distribution around the wire

$$P = \left(\frac{v_L}{v_s}\right)^2 \frac{V_R R^2 12\eta}{h^3} (\cos\theta + C). \quad [7]$$

As noted in (13) this calculation determines the pressure only to within an arbitrary constant. The value of this constant pressure in the liquid layer may be important for the situations where the wire is near the pressure melting condition, however, the net upward force,  $F_0$ , produced by the pressure distributed around the wire is independent of the constant.

$$F_0 = \left(\frac{v_L}{v_s}\right)^2 V_R l \left(\frac{R}{h}\right)^3 12\pi\eta \quad [8]$$

This is the equation derived in (13). There are, however, some additional components to the upward force on the wire the magnitudes of which should be evaluated and compared to Eq. [8].

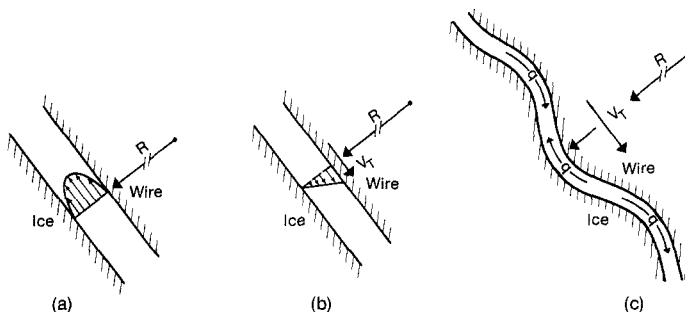


FIG. 3. (a) Basic flow assumed in liquid layer. (b) Additional shear flow caused by relative motion of wire and ice. (c) Effect of relative motion for a rough wire surface.

In Fig. 2 it can be seen that the water flow in the liquid layer produces a shear stress on the wire surface. The magnitude of the shear stress,  $Y$ , is

$$Y = \frac{h}{2} \frac{1}{R} \frac{dP}{d\theta} \quad [9]$$

Integrating this stress over the wire surface gives an upward force of

$$F_1 = \left(\frac{v_L}{v_s}\right)^2 V_R \left(\frac{R}{h}\right)^2 6\pi\eta \quad [10]$$

and ratio

$$\frac{F_1}{F_0} = \frac{1}{2} \frac{h}{R} \quad [11]$$

That is  $F_1$  is of the order of  $h/R$  smaller than  $F_0$  and since the assumption has already been made that  $h/R$  is very small  $F_1$  can be neglected.

In the derivation of Eq. [8] it was assumed, as was indicated in Fig. 3a, that to a first approximation the ice and the wire surface were not moving relative to each other. In actual fact the two surfaces are slipping by each other at each point around the wire. The shear flow that this would generate is shown in Fig. 3b. The shear stress produced by the differential movement of the ice and the wire is

$$Y = V_T \frac{\eta}{h} \quad [12]$$

where  $V_T = V_R \sin\theta$  and the upward force it produces is

$$F_2 = V_R \frac{R}{h} \pi\eta.$$

The ratio of this force to the force  $F_0$  produced by the pressure distribution,

$$\frac{F_2}{F_0} = \frac{1}{12} \left(\frac{v_s}{v_L}\right)^2 \left(\frac{h}{R}\right)^2 \quad [13]$$

is of second order in terms of the small parameter  $h/R$ . The effect of the relative motion of the ice and the wire is, however, much more significant if the surface roughness of the wire is included. Figure 3c shows the situation that occurs when a rough surface slides by the ice. Such a surface moves by regelation with melting occurring on the upslope of the roughness elements and refreezing occurring on the downslope. If the roughness is assumed to be of a small amplitude and sinusoidal, that is

$$R = \Delta R \sin \frac{2\pi R\theta}{\lambda} + \bar{R}. \quad [14]$$

An effective shear stress of

$$Y_{\text{eff}} = 3 \frac{\Delta R^2}{h^3} \eta \left(\frac{v_L}{v_s}\right)^2 V_T \quad [15]$$

is created at the wire surface. The net upward force on the wire produced by surface roughness is then

$$F_3 = \left(\frac{v_L}{v_s}\right)^2 V_R \frac{\Delta R^2 R}{h^3} 3\pi\eta. \quad [16]$$

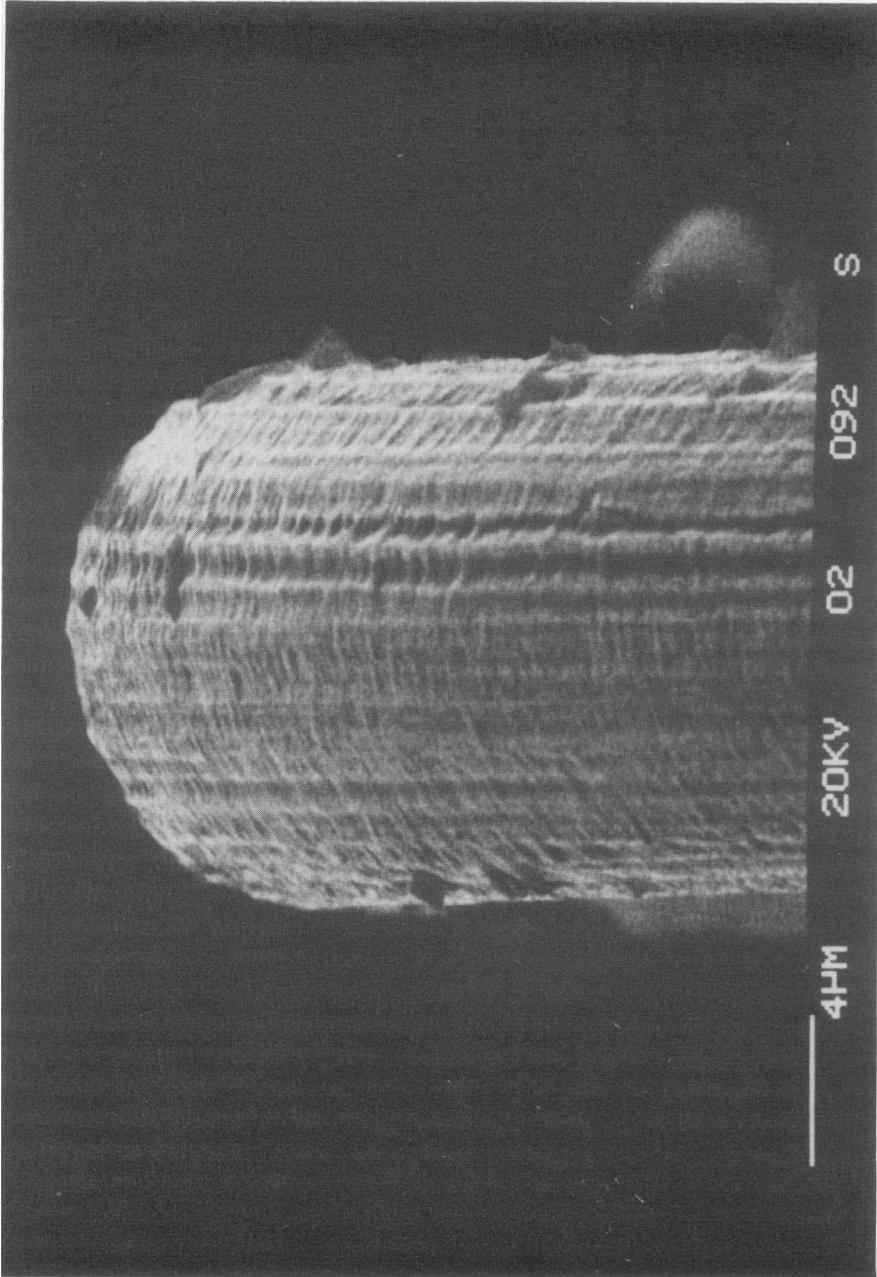


FIG. 4. Scanning electron microscope picture of one of the wires used (12.7  $\mu\text{m}$  diameter tungsten).

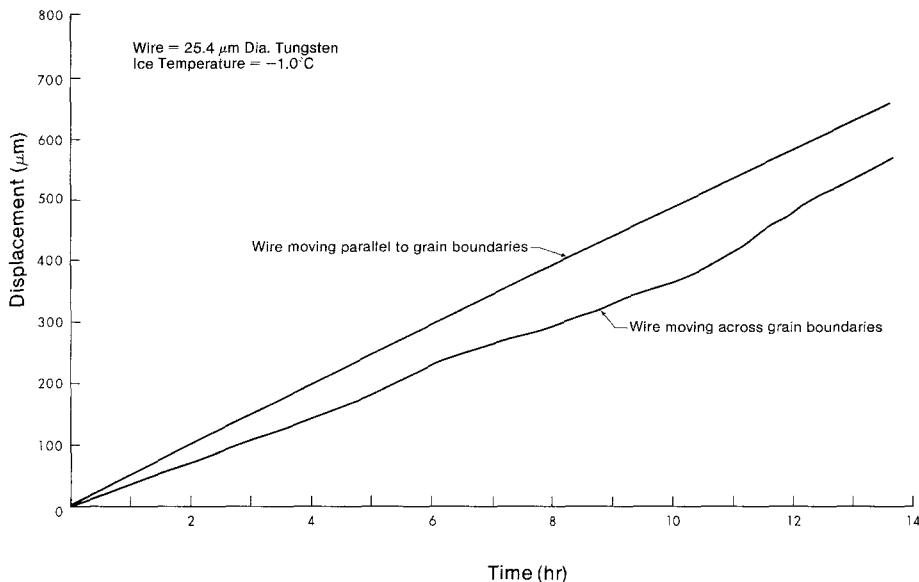


FIG. 5. Displacement versus time curves for regelation across and parallel to grain boundaries.

The ratio of this force to  $F_0$  is then

$$\frac{F_3}{F_0} = \frac{1}{4} \left( \frac{\Delta R}{R} \right)^2. \quad [17]$$

This means that the amplitude of the roughness on the wire can be as much as 10% of the wire radius and it will still have less than a 1% effect on the drag force on the wire. Scanning electron microscope pictures (Fig. 4) of the smallest diameter wire (12.7  $\mu\text{m}$ ) used suggest that the amplitude of the roughness of these wires was of the order of 0.2  $\mu\text{m}$  which is much less than 10% of the radius of this wire.

#### 4. RESULTS

##### 4.1. The Effects of Grain Boundaries

The displacement of the wire through ice as a function of time is shown for two different ice samples in Fig. 5. As mentioned previously the ice used in the present experiment consisted of long columnar crystals oriented roughly parallel to the  $c$  axis. If the wire was moving in a single crystal or parallel to the grain boundaries in an ice

sample with more than one crystal a very uniform rate of progression of the wire was observed. The variation of the slope of the displacement versus time curve in this case was typically less than 1% over a period of several days.

A less uniform rate of motion was obtained if the wire traversed grain boundaries as it often did if the ice was cut with the  $c$  axis parallel to the wire. At the end of a test the ice sample was removed from the apparatus and allowed to melt. During the melting the grain boundary pattern and any other crystal imperfections become visible. Periods of decreased regelation velocity could be related to the passage of the wire through such regions. The rate of regelation in the center of a crystal away from grain boundaries did, however, appear to be independent of the crystal orientation. In the data that will be presented the regelation velocity was obtained by taking the slope of the displacement versus time curve. These data were considered as acceptable only if a uniform rate of motion of the wire was observed for a period of 8 hr or more.

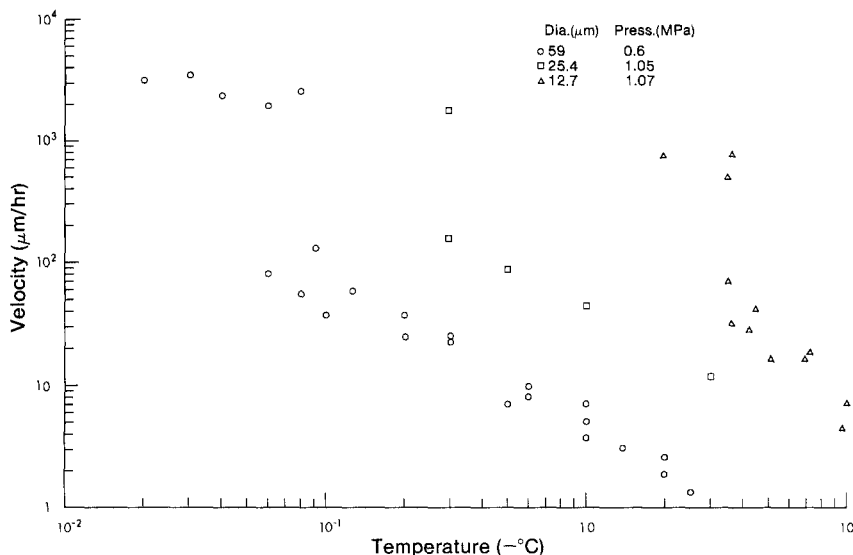


FIG. 6. The effect of ice temperature on the regelation velocities for several different wire diameters. Note fast and slow modes of behavior.

#### 4.2. Effect of Ice Temperature on Regelation Velocity

Figure 6 shows the variation of regelation velocity with ice temperature for several different conditions. As was expected there are clearly two different modes of regelation—a fast and a slow mode. The condition for the transition between modes is quite different than expected and will be discussed in a later section. The main interest of this paper is, however, the slow mode where it is assumed that the liquid layer is at its equilibrium thickness and viscosity in this layer limits the regelation velocity.

The advantage of using a range of wire diameters is clearly seen in Fig. 6 as this results in measurable regelation velocities occurring over a wide range in temperatures.

#### 4.3. The Effect of Pressure on Regelation Velocity

Telford and Turner (11) presented results for only one condition of pressure applied to the wire. They did, however, state that the velocity appeared to vary with the cube of

the applied pressure rather than linearly as would be expected from Eq. [8]. In Fig. 7 the variation of velocity with pressure is shown for two ice temperatures. These results were obtained in the slow regelation mode. It can be seen from these results that a linear variation of velocity with pressure occurred over the pressure range used in the present tests.

#### 4.4. Water Mobility in the Liquid Layer

In Eq. [5] a mobility coefficient,  $K$ , was introduced. If the assumptions about the liquid layer are correct this coefficient should be a function only of temperature and be independent of wire diameter and of the pressure exerted on the wire. The coefficient can be calculated from the measured regelation velocity using Eq. [8],

$$K = \frac{h^3}{12\eta} = \frac{\pi}{2} \left( \frac{v_L}{v_S} \right)^2 \frac{V_R R^2}{P} \quad [18]$$

Figure 8 shows the mobility coefficient as a function of ice temperature. This figure includes wire regelation tests taken on 13 different ice samples using the range of wire

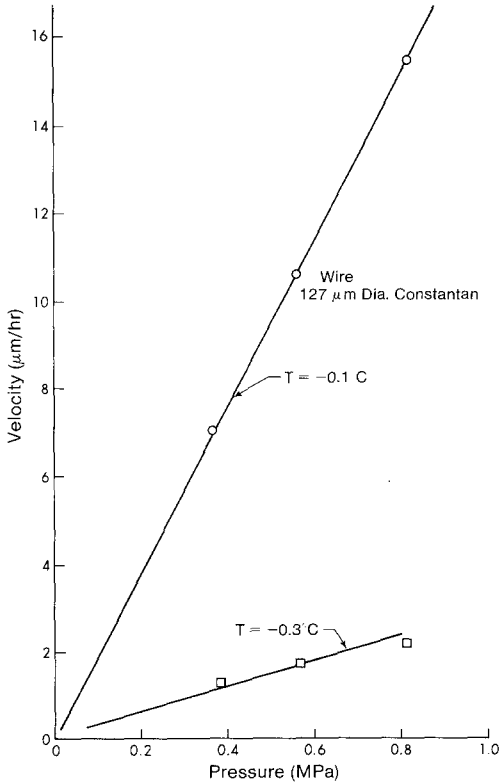


FIG. 7. The effect of pressure applied to the wire on the regelation velocity.

diameters, wire materials, and loading pressures shown on the legend. It can be seen that the mobility, as calculated from the equation, is unaffected by any of these parameters.

For various runs different methods of wire preparation were also tried. These included no cleaning of the wire, washing the wire in dilute sulfuric acid followed by distilled water, and ultrasonic cleaning in acetone. There was no detectable influence of any of these surface preparation methods.

The major factor that controls the water mobility at the ice-wire interface is therefore the ice temperature. In Fig. 8 two different domains of behavior are apparent for the wire regelation tests. From about  $T = -0.02$  to  $-7^\circ\text{C}$  the linear relationship between the logarithms of mobility and temperature suggests a power law relationship. For temperatures colder than about  $T = -7^\circ\text{C}$  the mobility decreases more abruptly.

In the present experiment the stability of the temperature bath limited the maximum ice temperature which could be studied to

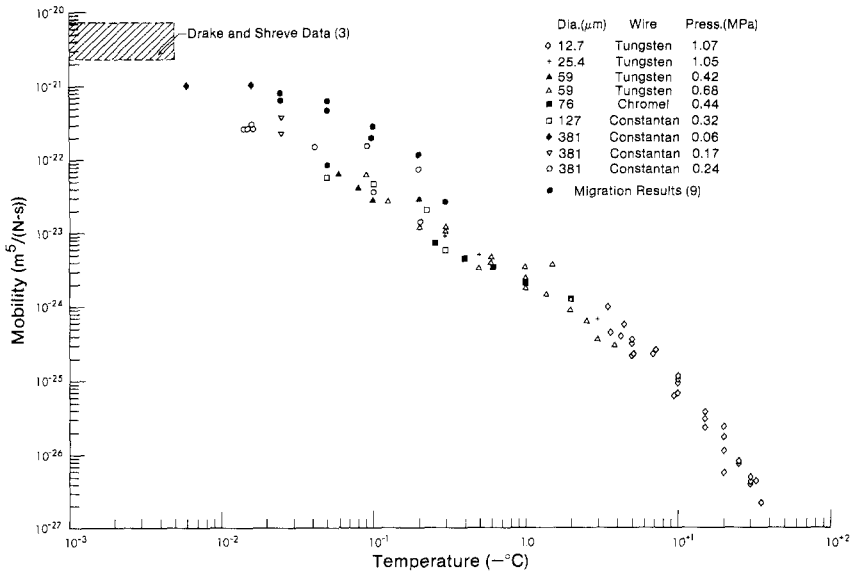


FIG. 8. The mobility of the water in the liquid layer as a function of ice temperature.

TABLE I

Slow Regelation Results of Drake and Shreve (7)

Material	Diameter ( $\mu\text{m}$ )	Pressure (MPa)	Velocity ( $\mu\text{m/hr}$ )	Mobility ( $\text{m}^2/(\text{N} - \text{sec})$ ) ( $\times 10^{-21}$ )	Film thick- ness (nm)
Nylon	120	0.2	54	3.6	42
	250	0.2	25	7.2	53
Chromel	120	0.2	36	2.4	32
	320	0.2	11	5.1	47
Copper	160	0.2	36	4.2	44
	500	0.5	7	3.3	41

about  $-0.005^\circ\text{C}$ . The uncertainty in the temperature is approximately equal to  $\pm 0.003^\circ\text{C}$ . Results for temperatures very close to  $0^\circ\text{C}$  have, however, been obtained by Drake and Shreve (7) by surrounding their ice sample with an ice-water mixture. They observed the two modes of regelation under these conditions. Table I summarizes their results for the lowest velocity mode. Applying Eq. [18] to this data gives the values of mobility shown in Table I and plotted at the left of Fig. 8. The Drake and Shreve [7] results apply ostensibly to ice at  $0^\circ\text{C}$ , however, as was pointed out in (13) surface tension effects could result in effective temperature for these results being  $0.0015$  to  $0.005^\circ\text{C}$  below the equilibrium freezing point. It is not, therefore, certain that these values would represent the asymptotic values as the ice temperature approaches  $0^\circ\text{C}$ .

In (13) a theory relating pressure-induced and temperature gradient-induced regelation was developed. If this theory is correct the mobility can also be calculated from measurements of the rate of migration of particles in ice. Romkins and Miller (14) have made such measurements for glass beads. From Ref. (13) the mobility can be related to the regelation velocity in a temperature gradient,  $G$ , such that

$$K = \frac{h^3}{12\eta} = \frac{2k_i + k_p}{6k_i} \frac{v_L}{v_s} \frac{v_L RT_a}{LG} V_R. \quad [19]$$

The mobility calculated from the data in Ref. (14) is plotted in Fig. 8 for comparison with the wire regelation results. The mobility calculated from the migration data is somewhat higher than that obtained from the wire regelation experiment, however, the general trend of the results is quite similar. Romkins and Miller (14) did note that the migration rates varied considerably from particle to particle. Considering the present observations that grain boundaries appear to affect the regelations the variability observed may be due to the fact that the ice used in (14) had a small grain size. It should also be noted that there may be differences in the liquid layer thicknesses and thus the mobility between the silica surface of the beads employed in the migration tests and the metal surface of the wire used in the present experiments.

#### 4.5. Liquid Layer Thickness

The decrease in mobility with temperature that is observed in Fig. 8 could occur in a number of ways. The first and most obvious is that the thickness of the liquid film,  $h$ , decreases with temperature. The viscosity of the water in the film could also be changed substantially either due to an increase in viscosity with decreasing temperature or by an increase in viscosity caused by the liquid film becoming very thin. Possible changes in solid or liquid phase density are assumed to have a negligible effect compared to the viscosity changes. The variation of bulk viscosity with temperature has been measured for supercooled water down to  $-10^\circ\text{C}$  (15). A second-order polynomial fitted to the viscosity data for water between  $-10$  and  $+10^\circ\text{C}$  gives

$$\eta_b = \eta_0(1 + 0.0361(-T) + 0.00083(-T)^2) \quad [20]$$

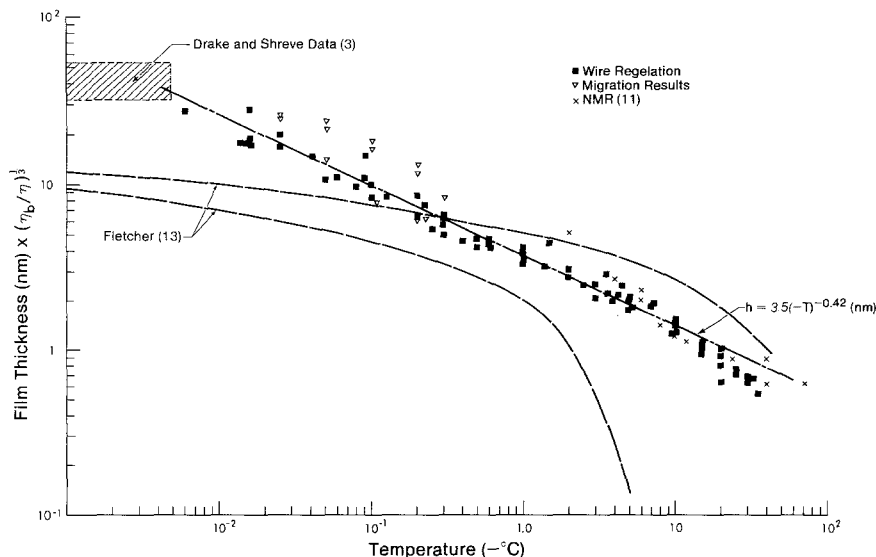


FIG. 9. The thickness of the liquid layer as interpreted from mobility measurements and NMR measurements.

where  $\eta_0$  is the viscosity at  $T = 0^\circ\text{C}$ . Introducing this viscosity relationship into the mobility equation the liquid layer thickness can be written

$$h \left( \frac{\eta_b}{\eta} \right)^{1/3} = (12K\eta_0(1 + 0.0361(-T) + 0.00083(-T)^2)^{1/3}. \quad [21]$$

Then if the viscosity in the liquid layer is equal to the bulk viscosity,  $\eta_b$ , the R.H.S. of Eq. [21] gives the layer thickness. Figure 9 shows the results plotted in this form. The liquid layer thickness implied from wire regelation experiments with the assumption of  $\eta = \eta_b$  range from a maximum of about 40 nm at temperatures approaching  $0^\circ\text{C}$  to about 0.5 nm at  $-35^\circ\text{C}$ .

Nuclear magnetic resonance (NMR) measurements have been made of the amount of unfrozen water in a silica power dispersed in ice. Baurer *et al.* (15) have used these measurements to calculate the thickness of the liquid layer. Their results which provide a totally independent measure of the liquid layer thickness are in

remarkably good agreement with the present results in the overlapping temperature range of  $-2$  to  $-20^\circ\text{C}$  (see Fig. 9).

There is some indication that at temperatures below about  $-15^\circ\text{C}$  that the results obtained from the wire regelation experiment fall below the NMR measurements. This would indicate that the viscosity in the liquid film is increasing more rapidly than would be predicted by Eq. [20]. At  $-35^\circ\text{C}$  the viscosity implied from Eq. [18] using the mobility obtained from the wire regelation experiment and the liquid layer thickness obtained from NMR measurements is 9.5 times the bulk water viscosity at  $0^\circ\text{C}$ . Equation [20] would project that the bulk viscosity of water at  $-35^\circ\text{C}$  is 3.28 times its value at  $0^\circ\text{C}$ . The implication is that the increase in viscosity caused by the thinness of the liquid layer results in the additional increase of about a factor of 3 at the  $-35^\circ\text{C}$  temperature.

These results are rather surprising in that they suggest that water behaves as a normal Newtonian fluid in layers of a thickness down to about 1.0 nm or about 10 times the

O-H band length and only in layers thinner than this is there any suggestion of a viscosity greater than bulk values. This result is quite contrary to results obtained from measurements made of sliding resistance. Generally those studies have suggested that the viscosity in the liquid layers is much larger, 10 to 1000 times larger, than the bulk values (15, 17), and this is in the temperature range 0 to  $-5^{\circ}\text{C}$  where no effect is seen in the present experiments. A possible explanation of this divergence of results may be obtained by comparing Eqs. [12] and [15] for the sliding resistances of a smooth surface,  $Y$ , and that of a rough surface,  $Y_{\text{eff}}$ . The ratio of these shear stresses is

$$\frac{Y_{\text{eff}}}{Y} = 3 \left( \frac{v_L}{v_s} \right)^2 \left( \frac{\Delta R}{h} \right)^2. \quad [22]$$

Therefore for the sliding experiments to be used as a measure of the simple shear stress,  $Y$ , in a liquid layer the value of the surface unevenness,  $\Delta R$ , must be significantly smaller than the layer thickness,  $h$ . That is, the surface over which the ice is sliding must be smooth and flat over the entire area of the ice specimen to better than 1 nm. The large variability of the results obtained from sliding tests would also suggest that the surface unevenness has an important effect on the results.

The results in Fig. 9 were obtained from a number of different sources and apply to several different types of surfaces (silica, metal, and nylon). On the basis of these results there does not appear to be a large effect of surface type on the liquid layer thickness.

The data obtained by Telford and Turner (11) was not shown on either Fig. 8 or 9. Their data would lie above the present data by a factor of 25 on the mobility and a factor of 3 in film thickness. This difference is perhaps the result of the fact that large wires under high loads were used in (11). A few tests with similar conditions that were

tried in the present experiment produced similar results.

#### 4.6. Liquid Layer Models

The results obtained in Fig. 9 can be compared with the liquid layer models developed by Fletcher. He has considered a number of approaches to the problem of trying to predict the thickness of such a layer and concludes that the predicted layer thickness should fit a model (18)

$$h(\text{nm}) = (2 \text{ to } 5) - 2.5 \log_{10} (-T) \quad [21]$$

where  $T$  is in degrees Celcius. These curves are shown in Fig. 9. It can be seen that the exponential form of the relationship does not provide a good fit to the data; however, the order of magnitude of the prediction is correct at least in the temperature range 0.1 to  $1.0^{\circ}\text{C}$ .

A much better fit to the data is obtained by assuming a power law relationship of the form

$$h = a (-T)^{-1/\alpha} \quad [22]$$

in which  $h$  and  $a$  are in nm and  $T$  is the ice temperature in degrees Celcius. A power law curve with  $a = 3.5$  nm and  $\alpha = 2.4$  is shown in Fig. 9. This expression fits the values obtained from wire regelation measurements over the temperature range  $-0.005$  to  $-20.0^{\circ}\text{C}$  within the experimental scatter of the results.

#### 4.7. High-Speed Regelation Mode

The results discussed thus far concern only the low-speed regelation mode. This was the mode that was assumed to be associated with the viscous flow in the equilibrium liquid layer. In Fig. 6 it was observed that under certain conditions a sudden transition to a much faster regelation mode did occur. It was originally thought that this fast mode was associated with pressure melting, however, an analysis of the present data indicates that the transition and the fast

TABLE II  
Transition between Slow to Fast Regelation Modes

Data	Wire diameter ( $\mu\text{m}$ )	Ice temperature ( $^{\circ}\text{C}$ )	Applied pressure (MPa)	Pressure required for melting (MPa)	Regelation velocity ( $\mu\text{m/hr}$ )	
					Slow	Fast
Present experiment	59	-0.06 to -0.08	0.6	0.8 to 1.0	100	2000
Present experiment	25.4	-0.3	1.05	4.0	120	1500
Present experiment	12.7	-3.5	1.07	47.4	70	600
Drake and Shreve (7)	120 to 500	$\sim 0$	0.05 to 0.1	$\sim 0$	70 to 180	35000 to 18000
Telford and Hunter (11)	450	-0.5	4.6	6.8	80	14000

mode cannot be simply related to pressure melting. Table II summarizes some facts about the transition condition as it was observed in this and other experiments. In this table the applied pressure can be compared with the pressure required to produce pressure melting. It can be seen from this comparison that pressure melting may be involved in some cases but not in all. The only relatively constant factor in all cases is the maximum regelation velocity for the slow mode. There would appear to be an upper limit of about  $100 \mu\text{m/hr}$  for regelation in the slow mode. It should be noted, however, that only a very limited amount of data was taken on the fast mode and the transition phenomenon in the present experiment and any conclusion drawn from this data would be highly speculative.

## 5. CONCLUSIONS

Wire regelation experiments, particularly with very small diameter wires, provide a useful tool for exploring the behavior of water in the liquid layer between ice and a substrate. The present experiments indicate that the water in this layer remains mobile down to at least  $-35^{\circ}\text{C}$ . At this temperature the viscosity of the water in the layer which is only about  $0.75 \text{ nm}$  thick is at most a few

times greater than bulk values. These results are quite contradictory to estimates obtained from sliding experiments. It was, however, pointed out in this paper that results from sliding experiments may be controlled by surface roughness whereas in the wire regelation experiments the surface roughness can at most introduce a second order effect.

Working with wires of smaller diameter than were used in the present experiment (that is, less than  $12.7 \mu\text{m}$ ) it may be possible to explore the mobility in the liquid layer at even lower temperatures than were studied in this experiment. Also there are some unexplained phenomena concerning the fast regelation mode and the transition between the modes that remain to be explored.

## APPENDIX 1: NOMENCLATURE

$F_0$	Force on wire required to produce flow around wire
$F_1, F_2, F_3$	Forces produced by second-order effects
$G$	Temperature gradient in the ice
$h$	Thickness of liquid layer
$k_i$	Thermal conductivity of ice
$k_p$	Thermal conductivity of particle

$K$	Mobility of water in liquid layer
$\bar{K}$	Mean curvature of ice surface
$l$	Length of wire in ice
$L$	Latent heat of fusion
$P$	Pressure on wire cross section
$P_c$	Characteristic pressure required for melting
$P'$	$P/P_c$
$q$	Flow rate in liquid layer
$R$	Radius of wire
$\bar{R}$	Mean radius of wire
$\Delta R$	Amplitude of surface roughness
$T$	Ice temperature
$T_a$	Absolute temperature
$T_c$	Characteristic temperature
$v_L$	Specific volume of liquid (water)
$v_s$	Specific volume of solid (ice)
$\Delta v$	$v_s - v_L$
$V_R$	Regelation velocity
$V_T$	Relative velocity of ice and surface of wire
$\eta$	Viscosity of water in layer
$\eta_0$	Viscosity of water at 0°C
$\eta_b$	Bulk viscosity of water
$\sigma_{SL}$	Ice-water interfacial tension
$Y$	Shear stress at wire surface
$Y_{\text{eff}}$	Effective shear stress caused by surface roughness

## ACKNOWLEDGMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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