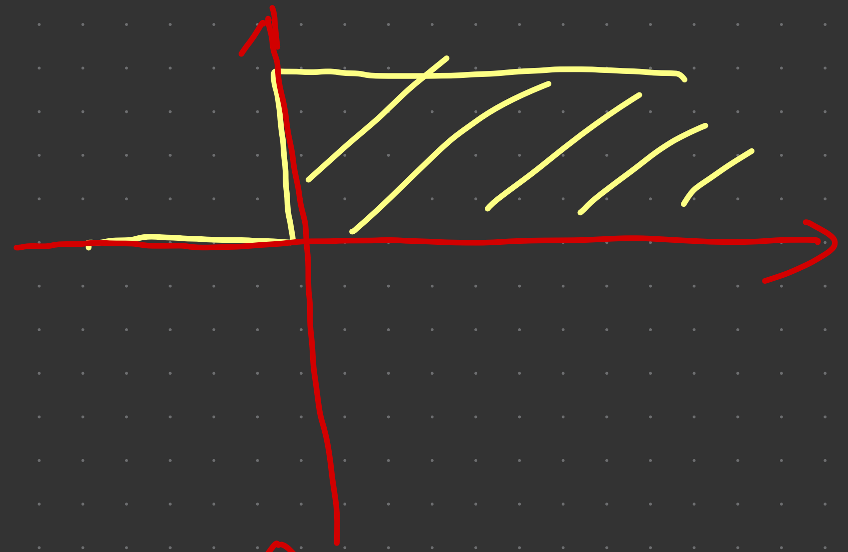
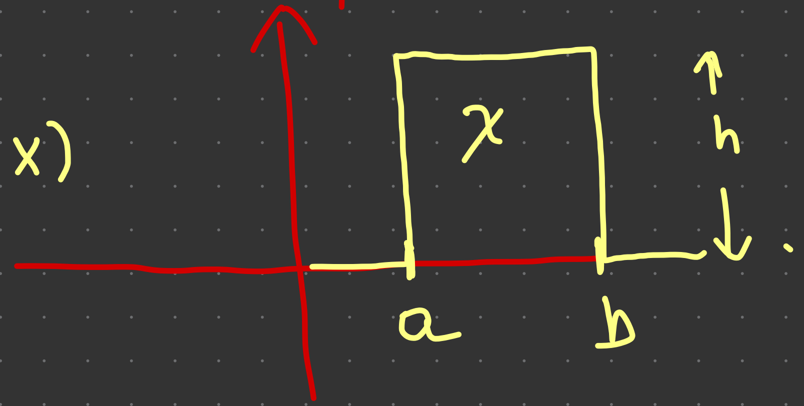
7a

* $\Theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$



$\Theta'(x) = \delta(x)$

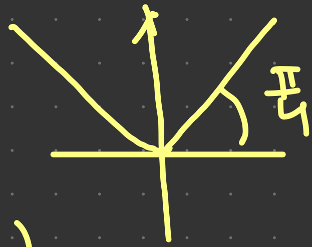


* $\chi'(x) = h \cdot \delta_a(x) - h\,\delta_b(x)$

* Reguła Leibniza

$(f \cdot T)' = f' \cdot T + f \cdot T'$

7a: $f(x) = |x| = x\cdot\chi_1 - x\cdot\chi_2$



$\chi_1 = \Theta(x)$ $\chi_2(x) = \Theta(-x)$

$$f'(x) = (x\cdot\chi_1 - x\cdot\chi_2)' =$$

$$(x\cdot\chi_1)' - (x\cdot\chi_2)' =$$

$$1\cdot\chi_1 + \underbrace{x\cdot\delta_0(x)}_{=0} - 1\cdot\chi_2 - \underbrace{x\cdot(-\delta_0(x))}_{=0}.$$

$$(\underbrace{x\cdot\delta_0}_{})(f) = \delta_0(x\cdot f) = 0\cdot f(0) = 0.$$

$$f' = \chi_1 - \chi_2.$$

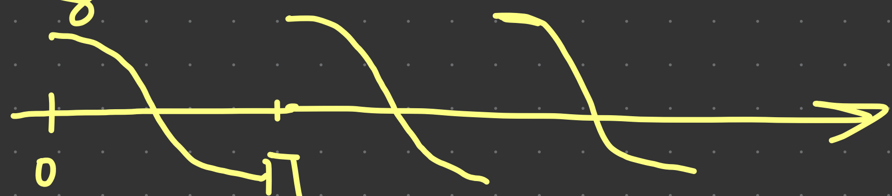
$$f'' = \chi_1' - \chi_2' = \delta_0 - (-\delta_0) = 2\delta_0 .$$

$$f''' = 2\,\delta_0' \quad \text{itd.}$$

$$f^{(4)}(x) = 2\,\delta_0''(x) .$$

$$g(x) = \chi(x)\sin(x)$$

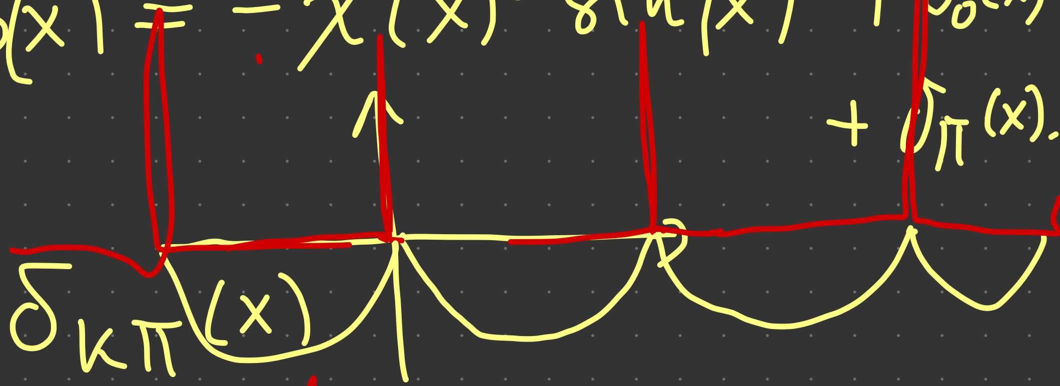
$$g' = \chi(x)\cdot\cos(x) + \underbrace{\big(\delta_0(x) - \delta_\pi(x)\big)\cdot\sin(x)}_{0}$$



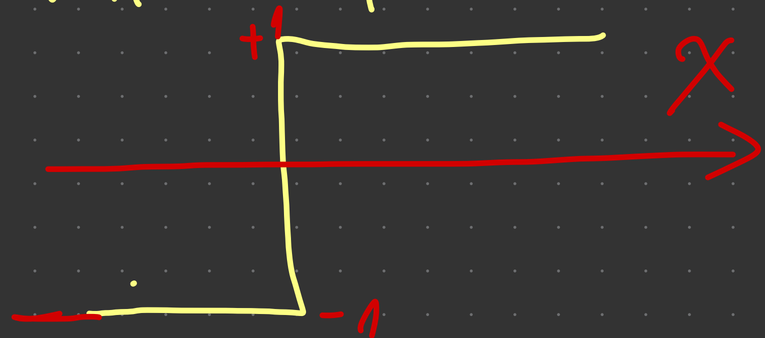
$$= \chi(x)\cdot\cos(x)$$

$$f'(x) = \begin{cases} \cos(x) : & x \in [2k\pi, (2k+1)\pi] \\ -\cos(x). & [(2k+1)\pi, (2k+2)\pi] \end{cases}$$

$$g''(x) = \left(\chi(x)\cdot\cos(x)\right)' = -\chi(x)\cdot\sin(x) + \left(\delta_0(x) - \delta_\pi(x)\right)\cos(x) = -\chi(x)\cdot\sin(x) + \delta_0(x) + \delta_\pi(x).$$



$$f''(x) = -|\sin(x)| + \sum_{k=-\infty}^{+\infty} 2\cdot\delta_{k\pi}(x)$$



$$f(x) = |x|\cdot\sin(x).$$

$$f' = \chi(x)\cdot\sin(x) + |x|\cdot\cos x.$$

$$f'' = \chi(x)\cdot\cos(x) + \chi(x)\cos x - |x|\sin(x).$$

$$= 2\chi(x)\cdot\cos(x) - |x|\cdot\sin(x).$$

$$f''' = 4\delta_0(x) + 2\chi\cdot(-\sin x) - \chi(x)\sin x - |x|\cos(x)$$

$$= 4\delta_0(x) - 3\chi\cdot\sin(x) - |x|\cos(x).$$

$$f^{(IV)} = 4\delta_0'(x) - \underline{3\chi}(x)\cos(x) - \underline{\chi(x)}\cos x + |x|\sin(x).$$

$$f^{(IV)} - f = 4\delta_0'(x) - 4\cdot\chi(x)\cos x.$$

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(8) $x^2 T' = 1$.

$$-x^2 P\left(\frac{1}{x}\right)' \overset{?}{=} 1$$

$$-x^2 \cdot P\left(\frac{1}{x}\right)'(f) =$$

$$P\left(\frac{1}{x}\right)'(-x^2 f) = P\left(\frac{1}{x}\right)\left((x^2 f)'\right) =$$

$$= P\left(\frac{1}{x}\right)\left(2xf + x^2 f'\right) = \int f\,dx = T_1(f).$$

$$\lim_{\varepsilon \to 0^+} \int_{|x| \geq \varepsilon} \frac{2x \cdot f + x^2 f'}{x}\,dx =$$

$$\lim_{\varepsilon \to 0^+} \int_{|x| \geq \varepsilon} (2f + xf')\,dx = \int_{\mathbb{R}} (\underbrace{xf' + 2f}) dx = \int f(x).$$

$$\underline{\underline{\int x f'}} = \underbrace{x \cdot f \Big|_{-\infty}^{+\infty}}_{=0} - \int x' \cdot f = \underline{\underline{-\int f(x)}}.$$

$$\boxed{-x^2 P'\left(\frac{1}{x}\right) = 1}$$

RSRNJ: $x^2 T' = 1$: $-P'\left(\frac{1}{x}\right)$.

RORJ: $x^2 T' = 0$.

$$x^2 \cdot S = 0 \overset{\text{cw}}{\rightsquigarrow} S = a_0 \delta_0 + a_1 \delta_0'.$$

$$T' = a_0 \delta_0 + a_1 \delta_0'$$

$$T = a_0 \Theta(x) + a_1 \delta_0 + a_2.$$

$$\mathrm{RORNY} = \mathrm{RORY} + \mathrm{RSRNY} =$$

$$\left\{ a_0 \cdot \Theta(x) + a_1 \delta_0(x) + a_2 - P\left(\frac{1}{x}\right) : a_i \in \mathbb{R} \right\}$$