II seria zadań domowych z Analizy I
18.11.2015

Zad. 1

(a) $\sum_{n=1}^{\infty} \frac{1}{nE(\sqrt{n})}$; (b) $\sum_{n=1}^{\infty}(\frac{1}{E(\sqrt{n})}-\frac{1}{\sqrt{n}})$; (c) $\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}}{E(n\sqrt{2})}$; (d) $\sum_{n=1}^{\infty} \sin\frac{n^2\pi}{n+1}$; (e) $\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt[4]{n^2+n+1})^p$; (h) $\sum_{n=1}^{\infty}(\frac{3-2n}{3+2n})^n$; (k) $\sum_{n=1}^{\infty}\sin\pi\sqrt{n^2+1}$; (n) $\sum_{n=1}^{\infty}\log\frac{n(n+1)}{n^2+1}$; (p) $\sum_{n=1}^{\infty}(1-\frac{1}{\sqrt{n}})^n$; (q) $\sum_{n=1}^{\infty}\frac{1}{n}\log(1+\frac{1}{n})$; (r) $\sum_{n=1}^{\infty}(\frac{\sqrt{n-1}+\sqrt{n+1}}{2}-\sqrt{n})$; (s) $\sum_{n=1}^{\infty}\frac{n^n+1}{n(n+1)^n}$; (t) $\sum_{n=1}^{\infty}\frac{\log(2n+1)}{n^p}$; (u) $\sum_{n=1}^{\infty}\frac{3^{\sqrt[3]{n^2+1}}}{2^n}$; (x) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[n]{(n+1)(n+2)\ldots(n+n)}}$; (y) $\sum_{n=1}^{\infty}(\frac{2+n}{1+n^2})^p$; (z) $\sum_{n=1}^{\infty}\frac{n^{n+1}}{(2n^2+n+1)^{\frac{n-1}{2}}}$; (aa) $\sum_{n=1}^{\infty}\frac{(n-\frac{1}{2n})^n}{n^{n-\frac{1}{2n}}}$; (ac) $\sum_{n=1}^{\infty}(\sqrt{1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}})$; (ad) $\sum_{n=1}^{\infty}\frac{n^3}{5^{\sqrt{2n}}}$; (ae) $\sum_{n=1}^{\infty}n^3 3^{-\sqrt{n}}$; (af) $\sum_{n=1}^{\infty}\log(\cos\frac{1}{n})$; (ag) $\sum_{n=1}^{\infty}(-1)^n\log(1+\frac{1}{n})$; (ai) $\sum_{n=1}^{\infty}n!\sin\frac{\pi}{2^n}$; (ak) $\sum_{n=1}^{\infty}\left(\begin{array}{c}3n\\ n\end{array}\right)11^{-n}$; (al) $\sum_{n=1}^{\infty}\binom{3n}{n}7^{-n}$; (an) $\sum_{n=1}^{\infty}(\frac{n-1}{n+1})^{n(n-1)}$; (ao) $\sum_{n=2}^{\infty}\frac{1}{n\log n}$; (ap) $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\log n}$; (aq) $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$; (ar) $\sum_{n=2}^{\infty}\frac{1}{(\log n)^{\log n}}$; (as) $\sum_{n=2}^{\infty}(\log n)^{-\log(\log n)}$; (at) $\sum_{n=2}^{\infty}(\log\log n)^{-\log n}$; (au) $\sum_{n=1}^{\infty}\frac{1}{n\sqrt[n]{n}}$; (aw) $\sum_{n=1}^{\infty}\frac{n^p}{\sqrt[n]{n!}}$; (ax) $\sum_{n=1}^{\infty}(\frac{1}{n}-\log\frac{n+1}{n})$; (ay) $\sum_{n=1}^{\infty}(\frac{(-1)^n}{\sqrt{n}}+\frac{1}{n})$; (az) $\sum_{n=3}^{\infty}\frac{(-1)^n}{E(\frac{n}{\sqrt{5}})}$; (ba) $\sum_{n=1}^{\infty}(2-\sqrt[n]{n})^n$; (bb) $\sum_{n=1}^{\infty}(1-\sqrt[n]{1-\frac{1}{n}})$; (bc) $\sum_{n=1}^{\infty}\sin(\pi\sqrt[n]{n^3+n})$;

Zad. 2

Dobrać parametry a, b, c tak, żeby funkcje $f, g : \mathbb{R} \to \mathbb{R}$ określone następująco:

$$a)\ f(x) = \begin{cases} \frac{\sin ax}{x} & \text{dla } x < 0 \\ \frac{x^3-1}{x^2+x-2} & \text{dla } 0 \le x < 1 \\ c & \text{dla } x = 1 \\ \frac{x^2+(b-1)x-b}{x-1} & \text{dla } x > 1 \end{cases} ; \qquad b)\ g(x) = \begin{cases} \frac{1}{1+e^{a/x}} & \text{dla } x \neq 0 \\ b & \text{dla } x = 0 \end{cases}.$$

były ciągłe na $\mathbb{R}$.

Zad. 3

Wyznaczyć pochodne następujących funkcji:

a) $f(b) = \frac{ax+b}{a+b}$, ą) $f(x) = (x-a)(x-b)$, b) $y(x) = (x+1)(x+2)^2(x+3)^3$,
c) $f(x) = (x\sin t + \cos t)(x\cos t - \sin t)$, ć) $f(t) = (x\sin t + \cos t)(x\cos t - \sin t)$,
d) $y(x) = (1+nx^m)(1+mx^n)$, e) $f(x) = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$, ę) $f(x) = \frac{\alpha x+\beta}{\gamma x+\delta}$,
f) $f(x) = \frac{(1-x)^p}{(1+x)^q}$, g) $f(x) = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}}$, h) $f(x) = \sqrt[3]{1+\sqrt[3]{1+\sqrt[3]{x}}}$, i) $f(x) = \sqrt[3]{\frac{1+x^3}{1-x^3}}$,
j) $f(x) = \sqrt[n+m]{(1-x)^m(1+x)^n}$, k) $f(x) = \frac{1}{\sqrt{1+x^2}(x+\sqrt{1+x^2})}$, l) $f(x) = \cos(2x) - 2\sin(x)$,
ł) $f(x) = \sin(\cos^2 x)\cos(\sin^2 x)$, m) $f(x) = \frac{1}{\cos^n x}$, n) $f(x) = \frac{\sin x - x\cos x}{\cos x + x\sin x}$,
ń) $y(x) = \mathrm{tg}(\frac{\sqrt{x}}{7})\,\mathrm{ctg}(\frac{\sqrt{x}}{7})$, o) $f(x) = \frac{1}{\sin^2(x/a)} + \frac{1}{\cos^2(x/a)}$, ó) $f(t) = e^{-t^2}$,
p) $y(x) = e^{\mathrm{tg}(1/x)}$, q) $z(t) = \left(\frac{1-t^2}{2}\sin t - \frac{(1+t)^2}{2}\cos t\right)e^{-t}$, r) $f(x) = \frac{a\sin(bx)-b\cos(bx)}{\sqrt{a^2+b^2}}e^{ax}$,
s) $\xi(t) = t^{a^a} + a^{t^a} + a^{a^t}$, $(a > 0)$, ś) $f(x) = \frac{1}{4}\ln\frac{x^2-1}{x^2+1}$, t) $y(x) = \frac{1}{2\sqrt{6}}\ln\frac{x\sqrt{3}-\sqrt{2}}{x\sqrt{3}+\sqrt{2}}$,
u) $f(x) = \frac{1}{1-k}\ln\frac{1+x}{1-x} - \frac{\sqrt{k}}{1-k}\ln\frac{1+x\sqrt{k}}{1-x\sqrt{k}}$, $(k > 0)$, v) $r(t) = \sqrt{t+1} - \ln(1+\sqrt{t+1})$,
w) $f(x) = \ln(x+\sqrt{x^2+1}) - \mathrm{arcsinh}\, x$, x) $\phi(t) = \ln\mathrm{tg}\left(\frac{t}{2}+\frac{\pi}{4}\right)$, y) $f(x) = \ln\frac{b+a\cos x+\sqrt{b^2-a^2}\sin x}{a+b\cos x}$, $(0 < a < b)$, z) $f(t) = t(\sin(\ln t) - \cos(\ln t))$, ź) $y(x) = x + \sqrt{1-x^2}\arccos x$,
ż) $f(x) = x\arcsin\sqrt{\frac{x}{1+x}} + \mathrm{arc\,tg}\sqrt{x} - \sqrt{x}$, α) $f(x) = \arcsin(\sin x)$, β) $f(x) = \mathrm{arccot}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$,
γ) $y(x) = \arcsin\left(\frac{1-x^2}{1+x^2}\right)$, δ) $y(t) = \arccos\left(\sin^2 t - \cos^2 t\right)$, ϵ) $f(x) = e^{m\arcsin x}(\cos(m\arcsin x) + \sin(m\arcsin x))$, ζ) $f(x) = (\log x)^{\log_x x}$, η) $y(x) = \mathrm{arc\,tg}(\mathrm{tgh}\, x)$, θ) $f(x) = \sqrt[x]{x}$,
ι) $f(x) = \sqrt[\ln x]{\ln x}$.