LHCphysics4

Summary on the Higgs sector:

$SU_L(2) \times SU_R(2)$ global symmetry spontaneously broken to $SU_V(2)$

This is evident when we use $\phi = \begin{pmatrix} H^{0*} & H^+ \\ -H^- & H^0 \end{pmatrix}$

transforming like $(2, 2^*)$ repr

$$\phi' = U_L \phi U_R^{-1}$$

$$U_L = e^{i\theta_L T^a}, \quad U_R = e^{i\theta_R T^a}$$

Vector-like rotations: $\theta_L = \theta_R \equiv \theta_V$

Axial rotations: $\theta_L = -\theta_R \equiv \theta_5$

$$\left.\begin{aligned} \theta_V^a &= \frac{1}{2}(\theta_L^a + \theta_R^a) \\ \theta_5^a &= \frac{1}{2}(\theta_R^a - \theta_L^a) \end{aligned}\right\} \text{general change of the rotation parameters}$$

For infinitesimal transformations

$$\delta\Phi = -i\,\theta^a_L T^a_L \Phi + i\,\Phi\,\theta^a_R T^R_a$$

We can also write Φ as

$$\Phi = \sigma + i\,T^a \tilde{\pi}^a$$

One can check the following transformations

$$\delta\sigma = \frac{1}{2}\theta_5^a \pi^a$$

$$\delta\pi^a = -\epsilon_{abc}\,\theta_V^b \pi^c - \theta_5^a \sigma$$

$$\left(H^0 = \sigma + i\pi^3 \right)$$

$$(H^0)' = H^0 - \frac{i}{2}\left[\theta_5^3 H^0 + \theta_R^+ H^- + \theta_L^- H^+\right]$$

$$(H^+)' = H^+ - \frac{i}{2}\left[\theta^3 H^+ - \theta_R^+ H^0 + \theta_L^+ H^{0*}\right]$$

we can calculate to get

$$\mathcal{L} \sim \frac{1}{2}\partial_\mu \sigma' \partial^\mu \sigma' + \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a$$

$$- \frac{\lambda}{4}\left(\sigma'^2 + \pi_a^2 + 2v\sigma'\right)^2$$

π^a 's are massless (Goldstone bosons)

since $\phi = \sigma + i\pi^a T^a$, it is clear

that π^a transform as a triplet of

the unbroken $SU_V(2)$

Currents:

$$j^{\mu a}_{L,R} = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \pi^b)} \, \delta \pi^b_{L,R} / \delta \theta^a_{L,R} + \ldots$$

$$j^{\mu a}_{V,5} = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \pi^b)} \, \delta \pi^b_{V,5} / \delta \theta^a_{V,5} + \ldots$$

$$j^{\mu a}_{5} = - v \, \partial_\mu \pi^a(x) + \ldots$$

$$j^{\mu a}_{L,R} = \frac{v}{4} \partial_\mu \pi^a(x) \pm \ldots$$

We can do it in an even simpler way by using the non-linear parametrization for the field Φ:

$$\Phi = e^{i\frac{\pi_a T^a}{v}} \begin{pmatrix} \sigma + v & 0 \\ 0 & \sigma + v \end{pmatrix}$$

It is clear that the π^a's are 3 Goldstone bosons because they don't enter into the potential

$$V = m^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2, \quad v = -\frac{m^2}{\lambda}$$

Useful formulae to see the connection between different parametrization and to calculate transformation rules for the "pions":

$$e^{i\frac{\pi^a\sigma^a}{v}} = \cos\left(\frac{\pi}{v}\right)\mathbb{1} + i\,\pi^a\sigma^a\,\frac{\sin(\pi/v)}{\pi}$$

$$\pi = \sqrt{\pi^a\pi^a}$$

Campbell - Baker - Hausdorff formula for non-commuting matrices

$$e^A e^B = \exp\Big\{ A + B + \frac{1}{2!}[A,B] + \frac{1}{3!}\Big(\frac{1}{2}[[A,B],B] + \frac{1}{2}[A,[A,B]]\Big) + \frac{1}{4!}[[A,[A,B]],B] + \dots \Big\}$$

CBH formula can be used to derive quickly the following infinitesimal transformations:

for $\hat{\pi} = \pi^a T^a$ under vector rotations $\hat{\Theta}_V = \Theta_V^a T^a$

$$\delta \hat{\pi}(x) = \frac{i}{2}\left[\hat{\Theta}_V, \hat{\pi}(x)\right]$$

and under axial rotations $(\Theta_L^a = -\Theta_R^a)$

$$\delta \hat{\pi}(x) = \underbrace{\frac{v}{2}\hat{\Theta}_5}_{\text{inhomogeneous transformation}} - \frac{1}{v}\left[\hat{\pi}, \left[\hat{\pi}, \hat{\Theta}_5\right]\right] + \dots$$

Another view at the Higgs mechanism (as for U(1))

Gauge bosons couple to the currents of the $SU_L(2)\times U(1)$ symmetry which contain the fermionic and the Higgs part:

$$g W^a_\mu j_L^{a\mu} + g' B_\mu j_Y^\mu = g\left(W^+_\alpha j_L^- + W^-_\alpha j_L^+\right) +$$

$$+ e A_\mu j^\mu_{EM} + \frac{g}{\cos\theta_W} Z_\mu \left(j_L^{3\mu} - \sin^2\theta_W\, j^\mu_{EM}\right)$$

$$j^\mu_{L,EM,Z} = j^{\mu(f)}_{L,EM,Z} + j^{\mu(H)}_{L,EM,Z}$$

$$j_L^{\pm(H)} = v\,\partial_\mu \pi^\pm + \ldots$$

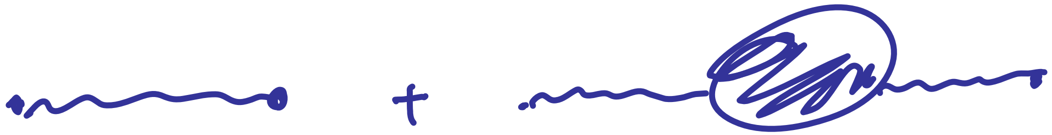
$$j_Z^{(H)} \sim v\,\partial_\mu \pi^3 + \ldots$$

and, therefore,

$$\langle 0 | j^{\pm\,\mu}_{L,Z} | \pi^{\pm,3} \rangle \sim v\, k_\mu\, e^{-ikx}$$

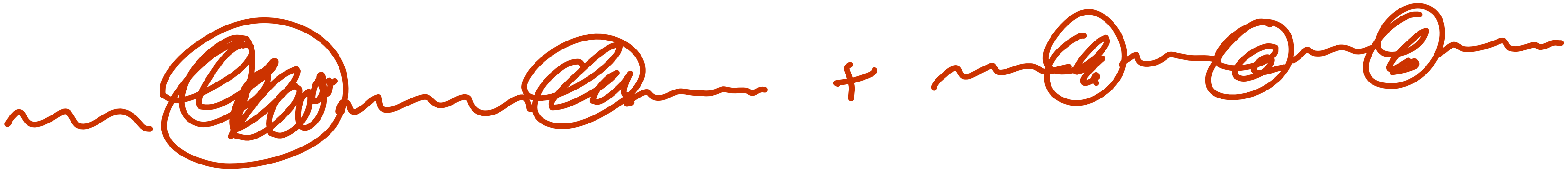
Similarly as for $U(1)$ case, we can discuss now the full propagators for the gauge bosons, starting with free propagators $1/k^2$ and including interactions

Diagrammatically



$i\Pi^{\mu\nu}(k)$

+ (in higher orders)



Take now the relevant part of the Fourier transform of the Lagrangian

$$\mathcal{L} = \frac{1}{2} A_\mu k^2 \left(g^{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) A_\nu - i g_e j_\mu A^\mu - i g_\phi j^\mu_\phi A_\mu + \ldots$$

(All objects are Fourier transforms; the Lagrangian is written in the Lorentz gauge $\partial_\mu A^\mu = 0$)

$$P_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$ is the projection operator on the transverse space.

The gauge field propagator then reads:

$$\langle 0 | (A_\mu^{(0)} + A_\mu^{(1)})(A_\nu^{(0)} + A_\nu^{(1)}) | 0 \rangle =$$

$$= \langle 0 | A_\mu^{(0)} A_\nu^{(0)} | 0 \rangle + \langle 0 | A_\mu^{(1)} A_\nu^{(1)} | 0 \rangle =$$

$$= D_{\mu\nu} + \frac{P_{\mu\rho}}{k^2} g^2 \underbrace{\langle 0 | j_\Phi^\rho \, j_\Phi^\sigma | 0 \rangle}_{\substack{\text{this is called a vacuum} \\ \text{polarization tensor} \\ \Pi^{\rho\sigma}(k)}} \frac{P_{\sigma\nu}}{k^2}$$

$$\Pi^{\rho\sigma}(k) = \left(g^{\rho\sigma} k^2 - k^\rho k^\sigma\right) \Pi(k^2)$$

This structure is fixed by Lorentz covariance and gauge invariance

We remember that the Goldstone boson contributes

$$g^2 \langle 0 | j^\rho_\Phi | X \rangle \langle X | j^\sigma_\Phi | 0 \rangle = k^\rho k^\sigma g^2 v^2 / k^2$$

Hence $\Pi_W(k^2) = \dfrac{g^2 v^2}{k^2}$ $\qquad \Pi_Z(k^2) = \dfrac{g^2 v^2}{\cos^2\theta_W \, k^2}$

The full gauge boson propagator is

$$G_{\mu\nu} = \frac{-i P_{\mu\nu}}{k^2} + \frac{(-i P_{\mu\rho})}{k^2} i\Pi^{\rho\sigma}(k) \frac{(-i P_{\sigma\nu})}{k^2} =$$

$$= \frac{-i P_{\mu\nu}}{k^2}\left(1 + \Pi(k^2)\right)$$ (in one loop)

Summing up the whole series we get

$$\boxed{G_{\mu\nu} = \frac{-i P_{\mu\nu}}{k^2(1-\Pi(k^2))}}$$

Tree level result :

$$\rho_0 = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Quantum corrections modify slightly this result because custodial symmetry is broken by

1) the coupling $g' B_\mu j^\mu_Y$ $\quad (j_Y \sim Y)$

remember $Y = T^3_R$ under $SU_L(2) \times SU_R(2)$ of the Higgs sector so, $SU_L(2) \times SU_R(2)$ is explicitly broken by the $g' B_\mu j^\mu_Y$ coupling

($g W^\alpha_a j^a_{\alpha L}$ is invariant)

2) by Yukawa couplings, responsible for the fermion masses

$$\mathcal{L}_{\text{yukawa}} = - Y_t \, \bar{t}_R \epsilon_{ij} H_i q^j_L - Y_b \, \bar{b}_R H^*_i q^i_L + \ldots$$

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \; ; \; \langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \Rightarrow m_t, m_b$$

$$H_i^* \, q_L^i = \left(H^\dagger q_L\right) \quad (SU_L(2) \text{ invariant})$$

For the top coupling, we used the fact that the two-dimensional representation of $SU(2)$ is real and $i\tau_2 H$ transforms as H^*, i.e. as 2^* of $SU_L(2)$; therefore

$$(i\tau_2 H)_i \, q_i = \epsilon_{ij} H_i q^j = \left[(H^*)^\dagger q_L\right] \text{ is}$$

also invariant under $SU_L(2)$

In general, let T_i form a representation of generators of G

$$[T_i, T_j] = i c_{ijk} T^k$$

c_{ijk} real if $T_i = T_i^+$

Taking the complex conjugate we see that

$$[(-T_i^*), (-T_j^*)] = i c_{ijk} (-T^{k*})$$

i.e. $(-T_i^*)$ also form a representation

If T_i are $(-T_i^*)$ are equivalent, i.e.

$$-T_i^* = u T_i u^+ \qquad (u - \text{unitary})$$

then the representation is said to be real ;

2 and 2^* of $SU(2)$ are real

because

$$-T_i^* = -T_i^T = -\frac{1}{2}\sigma_i^T$$

and

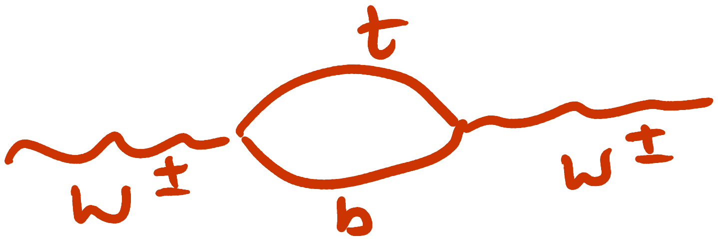
$$i\sigma_2\left(-\frac{1}{2}\sigma_i^T\right)(i\sigma_2)^\dagger = \frac{1}{2}\sigma_i$$

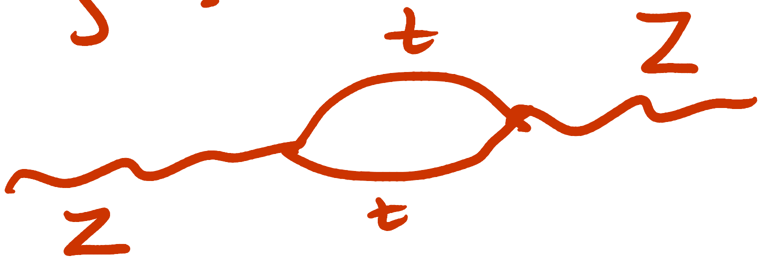
Yukawa couplings break $SU_L(2)$

because $m_t \cong 170$ GeV, $m_b \cong 5$ GeV

and $y_t \gg y_b$

An example of quantum corrections that contributes to S:





Anatomy of such corrections:

they modify the masses of W, Z

 $\rightarrow \dfrac{1}{p^2 - M^2_{\substack{W\\Z}}} \rightarrow \dfrac{1}{p^2 - \left(M^2_W + \Delta M^2_{\substack{W\\Z}}\right)}$

tree level effect

$$\Delta M^2_{W,Z} = \Pi_{W,Z}(p^2 = 0)$$

(we take $p^2 \to 0$ in the propagator)

Therefore $\qquad\qquad \tilde{M}$ is tree level value

$$\rho = \frac{\tilde{M}_W^2 + \Delta M_W^2}{(\tilde{M}_Z^2 + \Delta M_Z^2)\cos^2\vartheta_W} = \frac{\tilde{M}_W^2}{\tilde{M}_Z^2 \cos^2\vartheta_W}\left(1 + \frac{\Pi_W(0)}{M_W^2}\right) *$$

$$* \left(1 - \frac{\Pi_Z(0)}{M_Z^2}\right)$$

$$\Rightarrow \rho - 1 \approx \frac{\Pi_W(0)}{M_W^2} - \frac{\Pi_Z(0)}{M_Z^2}$$

To get the number we have to calculate the loop diagram

Important : the result is finite (renormalization scheme independent)

(we don't have at our disposal any counterterms to cancel potential divergences because the correction vanishes for $y_t = y_b$)

$$\Delta\rho \simeq 0.0096$$

$$\left(\Delta\rho \simeq 3 \frac{\alpha_{EM}}{16\pi s^2 c^2} \frac{m_t^2 - m_b^2}{M_Z^2} \right)$$

Corrections due to g' are smaller because g' is small ($g' \sim 0.1$, $Y_t \sim 1$)

At the two-loop level there is a correction due to the Higgs boson self-coupling (the Higgs mass):

$$\Delta\rho = - \frac{1}{c^2} \frac{\alpha_{EM}}{16\pi} \frac{11}{3} \ln \frac{M_h^2}{M_W^2}$$

$$\frac{1}{c^2} \alpha_{EM} \sim \frac{e^2}{4\pi} = g^2 \frac{\sin^2\theta}{\cos^2\theta} \frac{1}{4\pi} = g^2 \frac{g'^2}{g^2} = \frac{1}{4\pi} g'^2$$

Experiment: many precision data from LEP; $\Delta\rho_{Exp} = (\rho_{Exp} - 1) \sim 0.01 \pm 0.001$

This value is explained very well by the $m_t^2 - m_b^2$ mass difference; hence we should expect

$$\ln \frac{M_h^2}{M_W^2} \approx 0 \Rightarrow M_h^2 \approx M_W^2$$

Indeed, confirmed by a detailed fits

We do not need anything more but the heavy top quark to explain experimental data with precision ~ 1‰ !

In the SM, the higgs particle is "used" to make the electroweak theory unitary, renormalizable etc. But is not required by the data.

Beyond SM :

new contributions to $\Pi(k^2)$ ~

$$\sim \int e^{ikx} \langle 0 | J \, J | 0 \rangle$$

that is to $\Delta\rho$;

even a heavy higgs particle "destroys" $\Delta\rho$;

S, T parameters