Natural Dark Matter: A Window on the GUT scale

Jonathan Roberts

February 18, 2007

The Lightest Supersymmetric Particle

In R-parity conserving supersymmetry the lightest particle is stable.

The Lightest Supersymmetric Particle

In R-parity conserving supersymmetry the lightest particle is stable.
Due to R-parity, decays of the form:

$$\textbf{sparticle} \rightarrow \textbf{particle} + \textbf{particle}$$

are not allowed.

The Lightest Supersymmetric Particle

In R-parity conserving supersymmetry the lightest particle is stable.
Due to R-parity, decays of the form:

$$\textbf{sparticle} \rightarrow \textbf{particle} + \textbf{particle}$$

are not allowed.

R-parity conserving supersymmetry requires a relic density of sparticles.

The Lightest Supersymmetric Particle

In R-parity conserving supersymmetry the lightest particle is stable. Due to R-parity, decays of the form:

$$\textbf{sparticle} \rightarrow \textbf{particle} + \textbf{particle}$$

are not allowed.

R-parity conserving supersymmetry requires a relic density of sparticles.

Is this a natural explanation for dark matter?

Neutralino spectrum

Neutralinos, $\tilde{\chi}_i^0$, are formed from a mixture of the neutral Wino, neutral Bino and two neutral higgsinos

Neutralino spectrum

Neutralinos, $\tilde{\chi}_i^0$, are formed from a mixture of the neutral Wino, neutral Bino and two neutral higgsinos via the mass matrix:

Neutralino spectrum

Neutralinos, $\tilde{\chi}_i^0$, are formed from a mixture of the neutral Wino, neutral Bino and two neutral higgsinos via the mass matrix:

$$\begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_w & m_Z\sin\beta\sin\theta_w \\ 0 & M_2 & m_Z\cos\beta\cos\theta_w & -m_Z\sin\beta\cos\theta_w \\ -m_Z\cos\beta\sin\theta_w & m_Z\cos\beta\cos\theta_w & 0 & -\mu \\ m_Z\sin\beta\sin\theta_w & -m_Z\sin\beta\cos\theta_w & -\mu & 0 \end{pmatrix}$$

Neutralino spectrum

Neutralinos, $\tilde{\chi}_i^0$, are formed from a mixture of the neutral Wino, neutral Bino and two neutral higgsinos via the mass matrix:

$$\begin{pmatrix} M_1 & 0 & -m_Z \cos\beta \sin\theta_w & m_Z \sin\beta \sin\theta_w \\ 0 & M_2 & m_Z \cos\beta \cos\theta_w & -m_Z \sin\beta \cos\theta_w \\ -m_Z \cos\beta \sin\theta_w & m_Z \cos\beta \cos\theta_w & 0 & -\mu \\ m_Z \sin\beta \sin\theta_w & -m_Z \sin\beta \cos\theta_w & -\mu & 0 \end{pmatrix}$$

- If one of M_1, M_2 or μ is much lighter than the others, the LSP will be predominantly of this form.

Neutralino spectrum

Neutralinos, $\tilde{\chi}_i^0$, are formed from a mixture of the neutral Wino, neutral Bino and two neutral higgsinos via the mass matrix:

$$\begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_w & m_Z\sin\beta\sin\theta_w \\ 0 & M_2 & m_Z\cos\beta\cos\theta_w & -m_Z\sin\beta\cos\theta_w \\ -m_Z\cos\beta\sin\theta_w & m_Z\cos\beta\cos\theta_w & 0 & -\mu \\ m_Z\sin\beta\sin\theta_w & -m_Z\sin\beta\cos\theta_w & -\mu & 0 \end{pmatrix}$$

- If one of M_1, M_2 or μ is much lighter than the others, the LSP will be predominantly of this form.
- Therefore we would expect the neutralino to be either Bino, Wino or Higgsino.

Naturalness

We have a problem.

Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM} h^2 \gg \Omega_{CDM}^{WMAP} h^2$

Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$
- Wino/Higgsino Dark Matter: Generally gives $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$

Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$
- Wino/Higgsino Dark Matter: Generally gives $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$

Therefore SUSY naturally gives the wrong dark matter density!

Resonant Annihilation

To account for the observed dark matter density, we need different annihilation channels.

Resonant Annihilation

To account for the observed dark matter density, we need different annihilation channels. Firstly if:

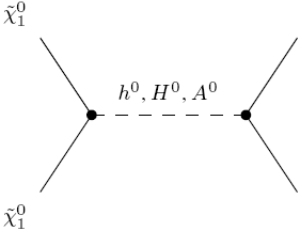
$$2m_{\tilde{\chi}_1^0} \approx m_{h^0,H^0,A^0,Z^0}$$

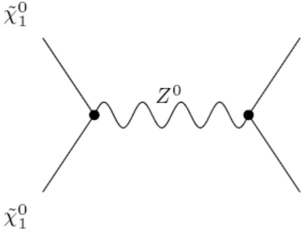
Resonant Annihilation

To account for the observed dark matter density, we need different annihilation channels. Firstly if:

$$2m_{\tilde{\chi}_1^0} \approx m_{h^0,H^0,A^0,Z^0}$$

then we get a significant boost to the annihilation cross-section through processes of the form:



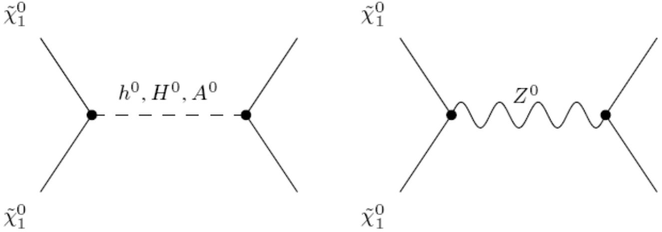


Resonant Annihilation

To account for the observed dark matter density, we need different annihilation channels. Firstly if:

$$2m_{\tilde{\chi}_1^0} \approx m_{h^0,H^0,A^0,Z^0}$$

then we get a significant boost to the annihilation cross-section through processes of the form:



In this case, annihilation is usually **too efficient**: $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$ except on the edges of the resonance.

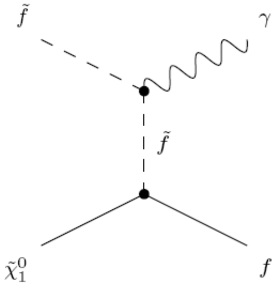
Coannihilation

A second exception is if there is another sparticle close in mass to the LSP, then we must adapt our calculation of the dark matter density as there will be a significant number density of these particles at freeze-out.

Coannihilation

A second exception is if there is another sparticle close in mass to the LSP, then we must adapt our calculation of the dark matter density as there will be a significant number density of these particles at freeze-out.

In this case we must take into account processes of the form:



Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$
- Wino/Higgsino Dark Matter: Generally gives $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$

Therefore SUSY naturally gives the wrong dark matter density!

There are 2 possible solutions:

Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$
- Wino/Higgsino Dark Matter: Generally gives $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$

Therefore SUSY naturally gives the wrong dark matter density!

There are 2 possible solutions:

1. Add **just enough** Wino or Higgsino into Bino dark matter.

Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM} h^2 \gg \Omega_{CDM}^{WMAP} h^2$
- Wino/Higgsino Dark Matter: Generally gives $\Omega_{CDM} h^2 \ll \Omega_{CDM}^{WMAP} h^2$

Therefore SUSY naturally gives the wrong dark matter density!

There are 2 possible solutions:

1. Add **just enough** Wino or Higgsino into Bino dark matter.
2. Enhance an annihilation channel **just enough** to allow Bino dark matter to account for the observed relic density.

Naturalness

We have a problem.

- Bino Dark Matter: Generally gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$
- Wino/Higgsino Dark Matter: Generally gives $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$

Therefore SUSY naturally gives the wrong dark matter density!

There are 2 possible solutions:

1. Add **just enough** Wino or Higgsino into Bino dark matter.
2. Enhance an annihilation channel **just enough** to allow Bino dark matter to account for the observed relic density.

This sounds like fine-tuning.

Fine-tuning

Is SUSY Dark Matter fine-tuned?

Fine-tuning

Is SUSY Dark Matter fine-tuned?

We need a quantitative measure of fine-tuning.

Fine-tuning

Is SUSY Dark Matter fine-tuned?

We need a quantitative measure of fine-tuning.

Ellis and Olive introduced an analagous measure to the one used to measure the fine-tuning required for electroweak symmetry breaking:

$$\Delta_a^{\Omega} = \left| \frac{\partial \ln \left(\Omega_{CDM} h^2 \right)}{\partial \ln \left(a \right)} \right|$$

Fine-tuning

Is SUSY Dark Matter fine-tuned?

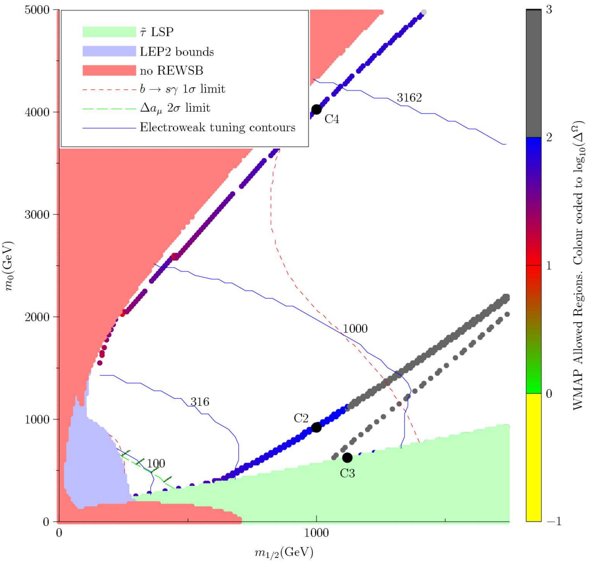
We need a quantitative measure of fine-tuning.

Ellis and Olive introduced an analagous measure to the one used to measure the fine-tuning required for electroweak symmetry breaking:

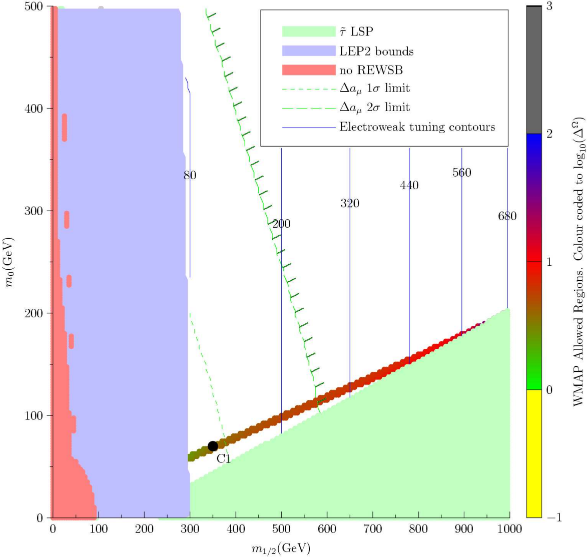
$$\Delta_a^\Omega = \left| \frac{\partial \ln\left(\Omega_{CDM} h^2\right)}{\partial \ln(a)} \right|$$

With these tools we can quantify the naturalness of SUSY dark matter.

The CMSSM Parameter Space



The CMSSM - Take 2



Studying the full MSSM

By relaxing our constraints we can find typical tuning scales for all dark matter annihilation channels.

Studying the full MSSM

By relaxing our constraints we can find typical tuning scales for all dark matter annihilation channels.

Region	Typical Δ^{Ω}
Mixed bino/wino	~ 30
Mixed bino/higgsino	$30 - 60$
maximally mixed bino/wino/higgsino	$4 - 60$
Bulk region (t-channel $\tilde{f}$ exchange)	< 1
slepton coannihilation (low M_1, m_0)	$3 - 15$
slepton coannihilation (large M_1, m_0)	~ 50
Z-resonant annihilation	~ 10
h^0-resonant annihilation	$10 - 1000$
A^0-resonant annihilation	$80 - 300$

Studying the full MSSM

By relaxing our constraints we can find typical tuning scales for all dark matter annihilation channels.

Region	Typical Δ^{Ω}
Mixed bino/wino	~ 30
Mixed bino/higgsino	$30 - 60$
maximally mixed bino/wino/higgsino	$4 - 60$
Bulk region (t-channel $\tilde{f}$ exchange)	< 1
slepton coannihilation (low M_1, m_0)	$3 - 15$
slepton coannihilation (large M_1, m_0)	~ 50
Z-resonant annihilation	~ 10
h^0-resonant annihilation	$10 - 1000$
A^0-resonant annihilation	$80 - 300$

Therefore the MSSM allows for **natural dark matter**.

Changing variables

When dealing with the MSSM we have the inputs:

$$a_{MSSM} \in \{m_i, M_i, A_i, \tan\beta\}$$

Changing variables

When dealing with the MSSM we have the inputs:

$$a_{MSSM} \in \{m_i, M_i, A_i, \tan\beta\}$$

In some explicit model of SUSY breaking we will have a smaller set of parameters that determine the SUSY breaking masses:

$$a_{string}$$

Changing variables

When dealing with the MSSM we have the inputs:

$$a_{MSSM} \in \{m_i, M_i, A_i, \tan\beta\}$$

In some explicit model of SUSY breaking we will have a smaller set of parameters that determine the SUSY breaking masses:

$$a_{string}$$

The dark matter tuning with respect to a_{string}, $\Delta^{\Omega}_{a_{string}}$ is directly related to $\Delta^{\Omega}_{a_{MSSM}}$ via the relation:

$$\Delta^{\Omega}_{a_{string}} = \sum_{a_{MSSM}} \frac{a_{string}}{a_{MSSM}} \frac{\partial a_{MSSM}}{\partial a_{string}} \Delta^{\Omega}_{a_{MSSM}}$$

Changing variables

When dealing with the MSSM we have the inputs:

$$a_{MSSM} \in \{m_i, M_i, A_i, \tan\beta\}$$

In some explicit model of SUSY breaking we will have a smaller set of parameters that determine the SUSY breaking masses:

$$a_{string}$$

The dark matter tuning with respect to a_{string}, $\Delta^{\Omega}_{a_{string}}$ is directly related to $\Delta^{\Omega}_{a_{MSSM}}$ via the relation:

$$\Delta^{\Omega}_{a_{string}} = \sum_{a_{MSSM}} \frac{a_{string}}{a_{MSSM}} \frac{\partial a_{MSSM}}{\partial a_{string}} \Delta^{\Omega}_{a_{MSSM}}$$

Therefore if we minimise the coefficients, we minimise the dark matter tuning.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.
- By combining this with other data, we will get an idea of the GUT scale values.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.
- By combining this with other data, we will get an idea of the GUT scale values.
- A large number of models will fit this data.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.
- By combining this with other data, we will get an idea of the GUT scale values.
- A large number of models will fit this data.

Via fine-tuning considerations we can go further.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.
- By combining this with other data, we will get an idea of the GUT scale values.
- A large number of models will fit this data.

Via fine-tuning considerations we can go further.

- Dark matter fine-tuning allows us to weigh up the different GUT models.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.
- By combining this with other data, we will get an idea of the GUT scale values.
- A large number of models will fit this data.

Via fine-tuning considerations we can go further.

- Dark matter fine-tuning allows us to weigh up the different GUT models.
- We can identify models that provide the most natural explanation of the observed phenomena.

Conclusions: In favour of fine-tuning

- If SUSY exists in some form, the LHC should be able to pin down a large number of the parameters at the low energy scale.
- By combining this with other data, we will get an idea of the GUT scale values.
- A large number of models will fit this data.

Via fine-tuning considerations we can go further.

- Dark matter fine-tuning allows us to weigh up the different GUT models.
- We can identify models that provide the most natural explanation of the observed phenomena.
- We can then make novel predictions for both the LHC, ILC and dark matter detection experiments to test the theory.