Towards a theory of gravitational radiation or What is a gravitational wave?

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King's College London,
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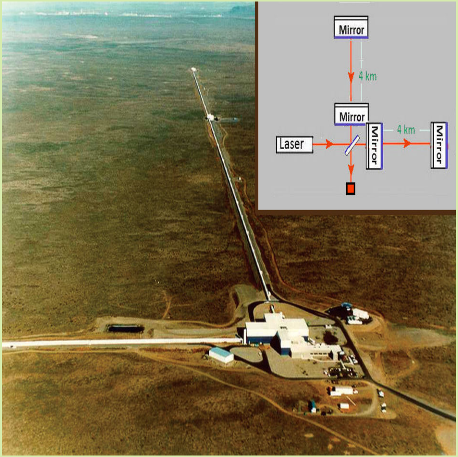
Plan

1. Recent LIGO announcement
2. Gravitational radiation theory: summary
3. Prehistory: 1916-1956
4. History: 1957-1962

LIGO detection: Its relevance



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- the first detection of a black hole; of a binary black-hole; of a merging process of black holes creating a new one; Kerr black holes exist; black holes with up to 60 Solar masses exist;
- the most energetic process ever observed
- important test of Einstein's General Theory of Relativity
- new window: a birth of gravitational wave astronomy

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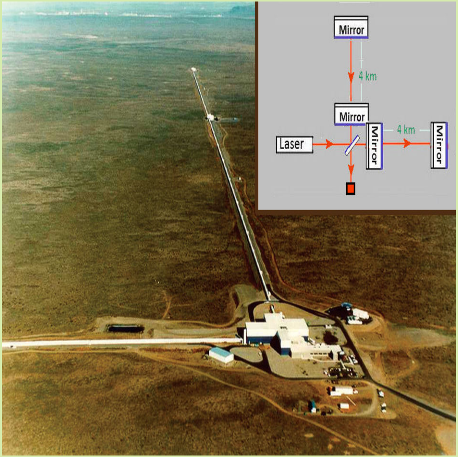

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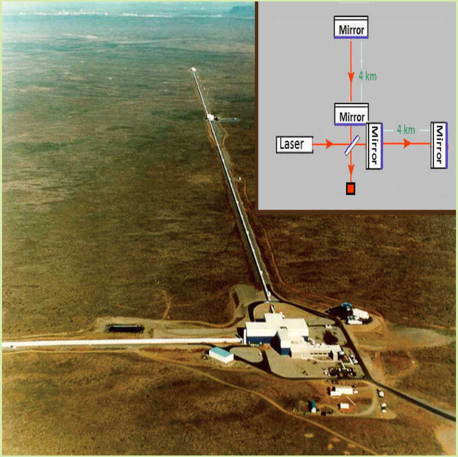

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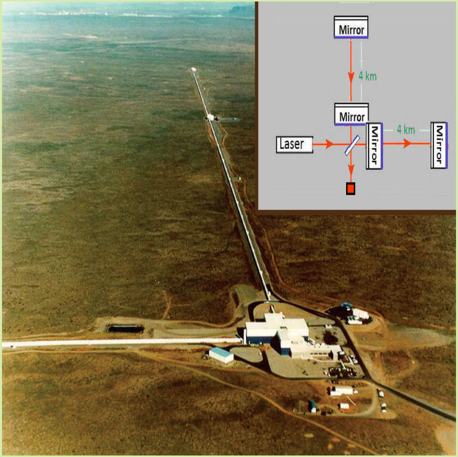

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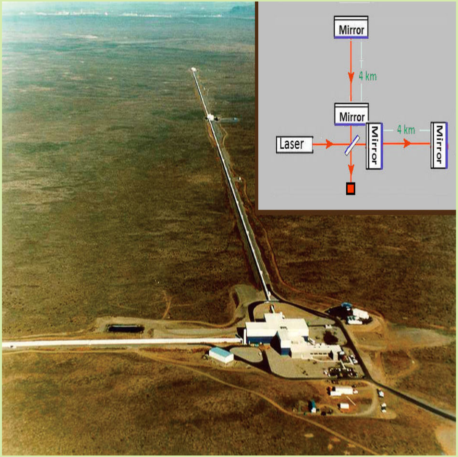

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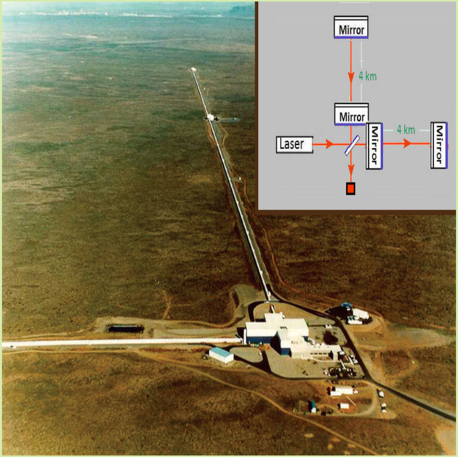

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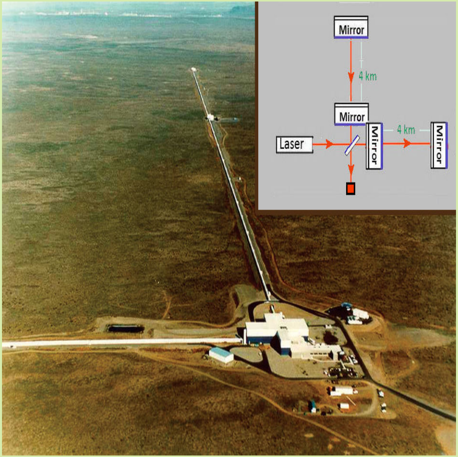

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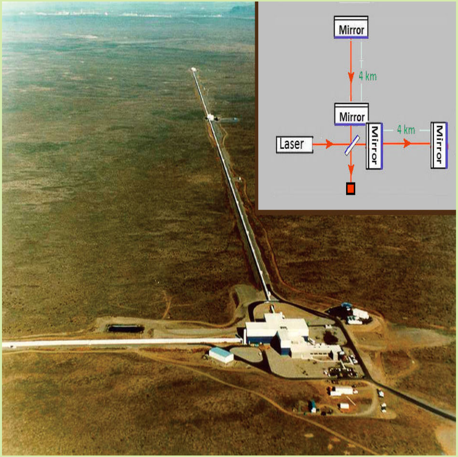

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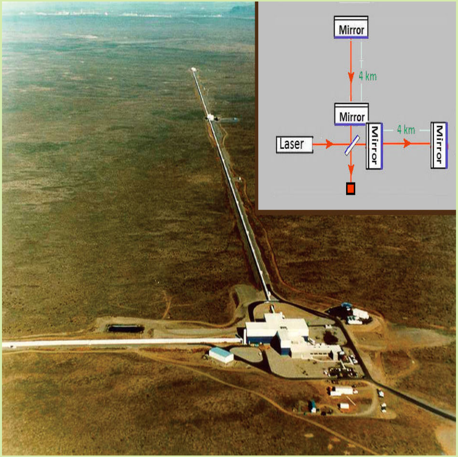

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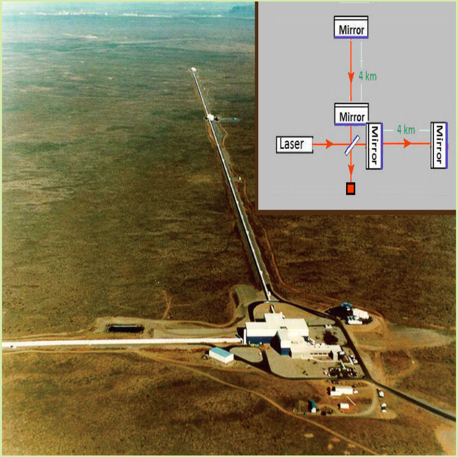

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The main ideas

- **Linearize** Einstein's field equations and look for waves in the linearized theory (**A. Einstein** 1916, 1918)
- Define what a **plane wave** is in the full theory (**N. Rosen**, **A. Einstein** 1937, **I Robinson** 1956(?), **H Bondi** 1957, H Bondi, **F Pirani**, I Robinson 1959)
- Define what a **radiative spacetime** is in the full theory:
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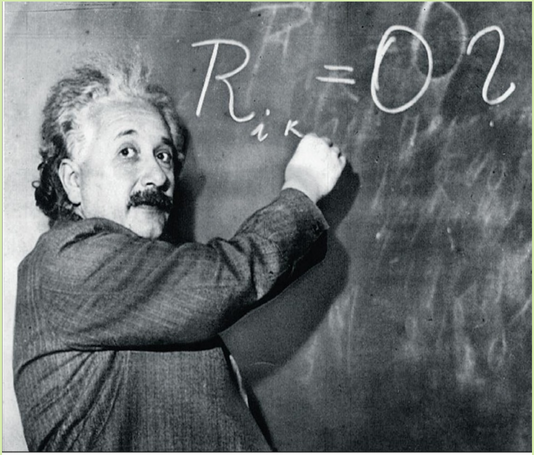
People: Albert Einstein (14.3.1879-18.04.1955)



People: Nathan Rosen (22.3.1909-18.12.1995)

People: Hermann Bondi (1.11.1919-10.9.2005)

H. Bondi with P. Bergman
(Warsaw 1962)

H. Bondi with L. Infeld
(Warsaw 1962)

People: Ivor Robinson

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H. Bondi with I. Robinson (Warsaw 1962)

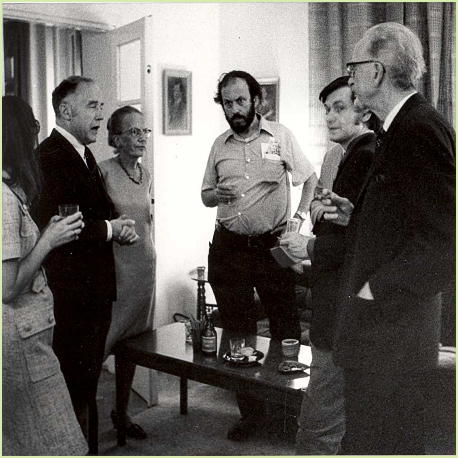
I. Robinson with A. Trautman (Trieste 1985)

People: Felix Pirani (2.2.1928-31.12.2015)

People: Andrzej Trautman

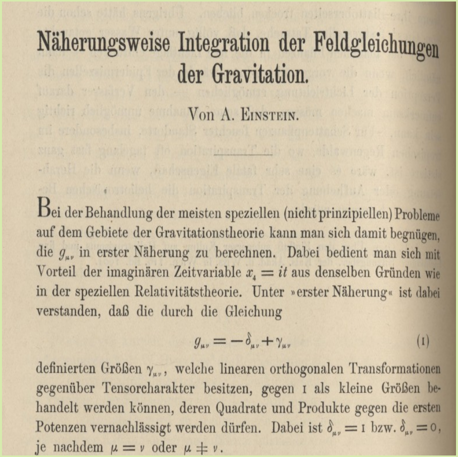
A. Trautman with S. Chandrasekhar (Warsaw 1973)

People: Roger Penrose



R Penrose (second from the right) with E T Newman (in the center); J. A. Wheeler on the left and C. Møller on the right (Warsaw 1973)

Gravitational waves: Einstein 1916



Näherungsweise Integration der Feldgleichungen der Gravitation.

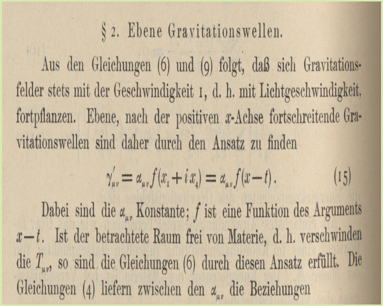
Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Einstein, Albert,
Näherungsweise Integration der Feldgleichungen der Gravitation, 22.6.1916



§ 2. Ebene Gravitationswellen.

Aus den Gleichungen (6) und (9) folgt, daß sich Gravitationsfelder stets mit der Geschwindigkeit 1, d. h. mit Lichtgeschwindigkeit, fortpflanzen. Ebene, nach der positiven x-Achse fortschreitende Gravitationswellen sind daher durch den Ansatz zu finden

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + ix_4) = \alpha_{\mu\nu} f(x - t). \qquad (15)$$

Dabei sind die $\alpha_{\mu\nu}$ Konstante; f ist eine Funktion des Arguments $x - t$. Ist der betrachtete Raum frei von Materie, d. h. verschwinden die $T_{\mu\nu}$, so sind die Gleichungen (6) durch diesen Ansatz erfüllt. Die Gleichungen (4) liefern zwischen den $\alpha_{\mu\nu}$ die Beziehungen

Gravitational waves: Einstein 1916

- Einstein **linearized his field equations** $\boxed{G_{\mu\nu} = \kappa T_{\mu\nu}}$ for the metric $g_{\mu\nu}$ assuming that

$$\boxed{g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \qquad \gamma'_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\mathrm{trace}(\gamma_{\alpha\beta})},$$

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Gravitational waves: Einstein 1916 - the wave equation

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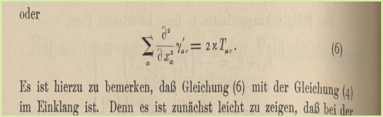
$$\boxed{\Box\gamma'_{\mu\nu} = 2\kappa T_{\mu\nu}}$$



oder

$$\sum_\alpha \frac{\partial^2}{\partial x_\alpha^2}\gamma'_{\mu\nu} = 2\varkappa T_{\mu\nu}. \qquad (6)$$

Es ist hierzu zu bemerken, daß Gleichung (6) mit der Gleichung (4) im Einklang ist. Denn es ist zunächst leicht zu zeigen, daß bei der

- this, outside the sources, is the **relativistic wave equation**

$$\Box\gamma'_{\mu\nu} = 0$$

for the perturbation, which justifies the claim that **in the linearized** Einstein's **theory gravitational waves** - perturbations of the spacetime metric traveling with speed of light - **do exist**.

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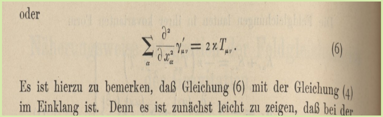

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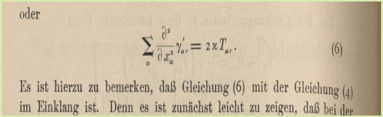

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Gravitational waves: Einstein 1916 - quadrupole formula

- Einstein has also shown that in the linearized theory his **waves carry energy**, and that the **power of the gravitational radiation** A is proportional to the **square of the third time derivative of the quadrupole moment** J **of the sources**

ein. Man erhält aus ihm also die Ausstrahlung A des Systems pro Zeiteinheit durch Multiplikation mit $4\pi R^2$:

$$A = \frac{\varkappa}{24\pi}\sum_{\alpha\beta}\left(\frac{\partial^3 J_{\alpha\beta}}{\partial t^3}\right)^2. \qquad (21)$$

Würde man die Zeit in Sekunden, die Energie in Erg messen, so würde zu diesem Ausdruck der Zahlenfaktor $\frac{1}{c^4}$ hinzutreten. Berücksichtigt man außerdem, daß $\varkappa = 1.87 \cdot 10^{-27}$, so sieht man, daß A in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß.

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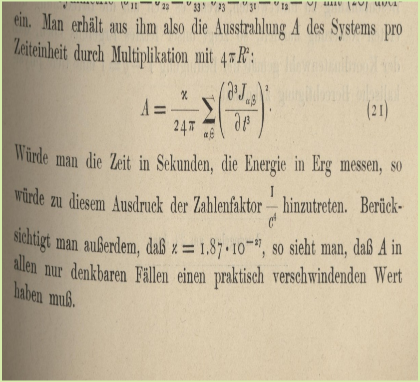

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Gravitational waves: Einstein and Rosen 1937 - plane waves are unphysical

ON GRAVITATIONAL WAVES.

BY

A. EINSTEIN and N. ROSEN.

ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of *rigorous* solutions for undulatory gravitational fields, we investigate rigorously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

Einstein A, Rosen N, *On gravitational waves*, Journ. of Franklin Institute, 223, (1937).

has the same sign. Progressive waves therefore produce a secular change in the metric.

This is related to the fact that the waves transport energy, which is bound up with a systematic change in time of a gravitating mass localized in the axis $x = 0$.

Note.—The second part of this paper was considerably altered by me after the departure of Mr. Rosen for Russia since we had originally interpreted our formula results erroneously. I wish to thank my colleague Professor Robertson for his friendly assistance in the clarification of the original error. I thank also Mr. Hoffmann for kind assistance in translation.

A. EINSTEIN.

Gravitational waves: Einstein and Rosen 1937

- Rosen's metric

$$g = \mathrm{e}^{2\phi}(\mathrm{d}\tau^2 - \mathrm{d}\xi^2) - u^2\left(\mathrm{e}^{2\beta}\mathrm{d}\eta^2 + \mathrm{e}^{-2\beta}\mathrm{d}\zeta^2\right)$$

  with $u = \tau - \xi$, $\beta = \beta(u)$, $\phi = \phi(u)$, $\phi' = u\beta'^2$ is a metric representing empty spacetime $Ric(g) = 0$ iff

$$u\beta'' + 2\beta' - u^2\beta'^3 = 0.$$

- Rosen wrongly concluded that this metric can not exist in reality as a spacetime because it contains certain physical singularities
- He confused **coordinate singularity** with a **true singularity**
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- He confused **coordinate singularity** with a **true singularity**
- after suitable coordinate change this can be interpreted as a **cylindrical wave**

Gravitational waves: Einstein and Rosen 1937

- Rosen's metric

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Even after the correct interpretation that this is a **cylindrical wave** the questions arrise:

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A naive definition

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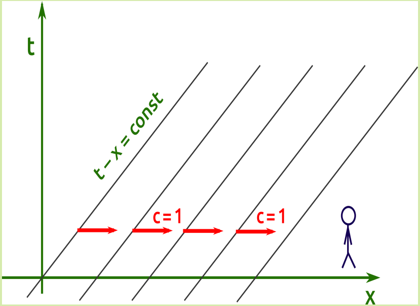

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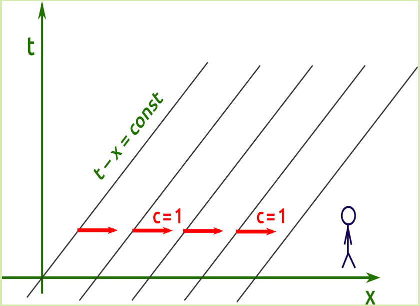

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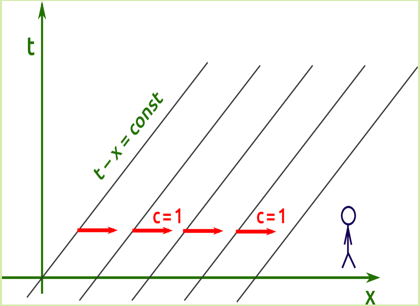

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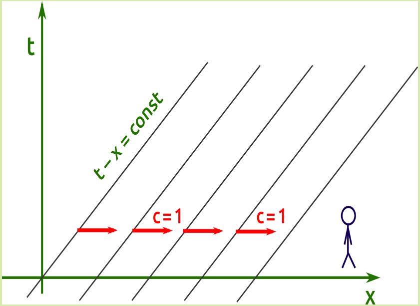

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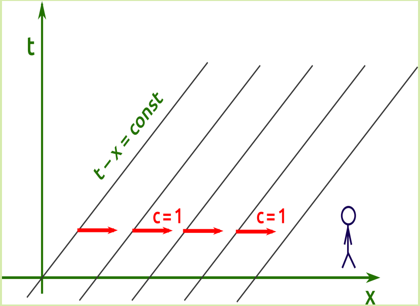

An (unfair) example

- take

$$g = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 + \\ \cos(t-x)(2+\cos(t-x))\mathrm{d}t^2 - 2\cos(t-x)\mathrm{d}t\mathrm{d}x - 2\cos^2(t-x)\mathrm{d}t\mathrm{d}x + \cos^2(t-x)\mathrm{d}x^2$$

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Bondi 1957 - Einstein and Rosen not right

Bondi H, *Plane gravitational waves in General relativity*, Nature, 179, (25.5.1957)

Plane Gravitational Waves in General Relativity

POLARIZED plane gravitational waves were first discovered by N. Rosen[1], who, however, came to the conclusion that such waves could not exist because the metric would have to contain certain physical singularities. More recent work by Taub[2] and McVittie[3] showed that there were no unpolarized plane waves, and this result has tended to confirm the view that true plane gravitational waves do not exist in empty space in general relativity. Partly owing to this, Scheidegger[4] and I[5] have both expressed the opinion that there might be no energy-carrying gravitational waves at all in the theory.

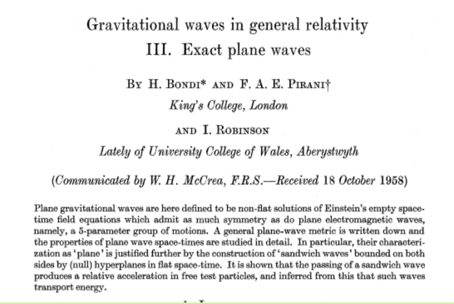
It is therefore of interest to point out, as was first shown by Robinson[6] and has now been independently proved by me, that Rosen's argument is invalid and that true gravitational waves do in fact exist. Moreover, it is shown here that these waves carry energy, although it has not yet been possible to relate the intensity of the wave to the amount of energy carried.

H. BONDI

King's College, Strand,
London, W.C.2. March 24.

[1] Rosen, N., *Phys. Z. Sowjet Union*, **12**, 366 (1937). See also, Einstein, A., and Rosen, N., *J. Franklin Inst.*, **223**, 43 (1937).
[2] Taub, A. H., *Ann. Math.*, **53**, 472 (1951).
[3] McVittie, G. C., *J. Rational Mech. and Analysis*, **4**, 201 (1955).
[4] Scheidegger, A. E., *Rev. Mod. Phys.*, **25**, 451 (1953). See also Brdička, M., *Proc. Roy. Irish Acad.*, **54**, 137 (1951).
[5] Bondi, H., various contributions to discussions at the International Conference on Gravitation, Chapel Hill, N.C., 1957.
[6] Robinson, I. (to be published shortly).
[7] Pirani, F. A. E., *Phys. Rev.*, **105**, 1089 (1957).
[8] Lichnerowicz, A., "Théories relativistes de la gravitation et de l'électromagnétisme" (Paris, 1955).

Bondi, Pirani, Robinson 1958



Gravitational waves in general relativity
III. Exact plane waves

By H. Bondi* and F. A. E. Pirani†
King's College, London

and I. Robinson
Lately of University College of Wales, Aberystwyth

(*Communicated by W. H. McCrea, F.R.S.—Received* 18 *October* 1958)

Plane gravitational waves are here defined to be non-flat solutions of Einstein's empty space-time field equations which admit as much symmetry as do plane electromagnetic waves, namely, a 5-parameter group of motions. A general plane-wave metric is written down and the properties of plane wave space-times are studied in detail. In particular, their characterization as 'plane' is justified further by the construction of 'sandwich waves' bounded on both sides by (null) hyperplanes in flat space-time. It is shown that the passing of a sandwich wave produces a relative acceleration in free test particles, and inferred from this that such waves transport energy.

Bondi H, Pirani F A E, Robinson I *Gravitational waves in General relativity III. Exact plane waves*, Proc. R. Soc. London, ser. A, 251, (18.10.1958)

Bondi, Pirani, Robinson 1958

- motivated by the **analogy with electromagnetism**, where plane waves have a 5-dimensional group of symmetries they **defined a plane wave in the full GR theory** as a **solution** to the equations $Ric(g) = 0$, which **has precisely 5-dimensional group of symmetries**
- inspecting **Petrov**'s list of solutions to $Ric(g) = 0$ with high symmetries they found a unique class of solutions that have 5 symmetries; the class is given in terms of **one free complex function** $f = f(u, \zeta)$, **holomorphic in variable** ζ, and has **remarkable property** which enables to **superpose** solutions from the class
- this enables to produce waves of a **sandwich** type; they have shown that a **sandwich wave falling on a system of test particles affects their motion**, concluding that **plane waves carry energy**.

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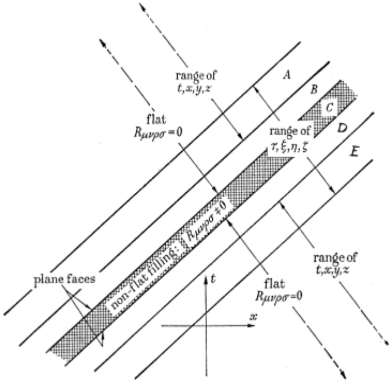

FIGURE 1. Arrangement of co-ordinate systems around sandwich wave.

Brinkmann 1924

It was a **mathematician** H. W. Brinkman who **first** had a general solution to Einstein's equations which now is called **plane wave**. It was in *Einstein spaces which are mapped conformally on each other*, Mathem. Annalen, 94, (1925). Totally **overlooked by the physicists!**

Einstein spaces which are mapped conformally on each other.

Von

H. W. Brinkmann in Cambridge (Mass., U. S. A.).

(47) $$ds^2 = 2dx\,dy + 2d\varphi\,d\theta + m\,d\varphi^2.$$

To this we apply the final equation (41 b) which gives us $\frac{\partial^2 m}{\partial x\,\partial y} = 0$ so that

$$m = X(x, \varphi) + Y(y, \varphi).$$

The only surviving components of the Riemann tensor are here

$$R_{x\varphi\varphi x} = \frac{1}{2}\frac{\partial^2 X}{\partial x^2}, \qquad R_{y\varphi\varphi y} = \frac{1}{2}\frac{\partial^2 Y}{\partial y^2}$$

so that the V_4 is Euclidean if and only if

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Thus we can go on constructing Einstein spaces that can be mapped upon Einstein spaces in more and more ways.

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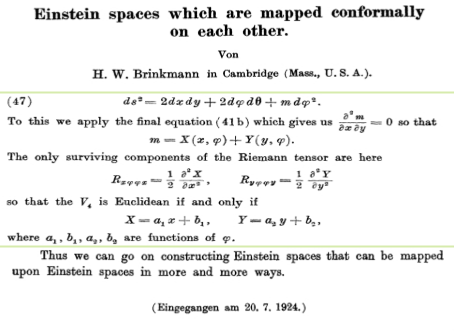

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Pirani 1957

Pirani F A E, *Invariant Formulation of Gravitational Radiation Theory*, Phys. Rev. 105, (18.10.1956)

PHYSICAL REVIEW VOLUME 105, NUMBER 3 FEBRUARY 1, 1957

Invariant Formulation of Gravitational Radiation Theory

F. A. E. Pirani
Department of Mathematics, King's College, Strand, London, England
(Received October 18, 1956)

In this paper, gravitational radiation is defined invariantly within the framework of general relativity theory. The definition is arrived at by assuming (*a*) that gravitational radiation is characterized by the Riemann tensor, and (*b*) that it is propagated with the fundamental velocity. Therefore a gravitational wave front should appear as a discontinuity in the Riemann tensor across a null 3-surface; the possible form of this discontinuity is here calculated from Lichnerowicz's continuity conditions.

The concept of an observer who follows the gravitational field is defined in terms of the eigenbivectors of the Riemann tensor. It is shown that the 4-velocity of this observer is timelike for one of Petrov's three canonical types of Riemann tensor, but null for the other two types. The first type is identified with the absence of radiation, the other two with its presence. This constitutes the definition. It is shown that the difference between the no-radiation type and one of the radiation types can be made to correspond to the discontinuity possible across a null 3-surface; this demonstrates the consistency of the wave front and following-the-field concepts.

A covariant approximation to the canonical energy-momentum pseudo-tensor is defined, using normal coordinates, which are given a physical interpretation. It is shown that when gravitational radiation is present, the approximate gravitational energy-flux cannot be removed by a local Lorentz transformation, which supports the definition of radiation.

It is proved that, as would be demanded of a sensible definition, there can be no gravitational radiation present in a region of empty space-time where the metric is static.

Pirani 1957

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- Pirani did not know all **Petrov types**, which were spelled out in full generality by **Penrose**, much later; he did not made his statement precise: it was **unclear which Petrov type he attributes to gravitational radiation** far from the sources
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Pirani 1957 - Petrov classification

Table 4.3. The roots of the algebraic equation (4.18) and their multiplicities

The corresponding multiplicities of the principal null directions are symbolically depicted on the right of this table.

Type	Roots E	Multiplicities	
I	$\dfrac{\sqrt{\lambda_2+2\lambda_1} \pm \sqrt{\lambda_1+2\lambda_2}}{\sqrt{\lambda_1-\lambda_2}}$	(1,1,1,1)	
D	$0,\ \infty$	(2,2)	
II	$0,\ \pm \mathrm{i}\sqrt{\frac{3}{2}\lambda}$	(2,1,1)	
III	$0,\ \infty$	(3,1)	
N	0	(4)	

Nowadays, due to our **next hero**, we know that far from the sources **gravitational wave is of Petrov type** N.

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Usefulness of Pirani's criterion

Rosen's metric is

$$ds^2 = (\exp 2\varphi)\,(d\tau^2 - d\xi^2) - u^2\{(\exp 2\beta)\,d\eta^2 + (\exp - 2\beta)\,d\zeta^2\} \quad (1)$$

where $u = \tau - \xi$, $\beta = \beta(u)$, $\varphi = \varphi(u)$ and $\varphi' = u\beta'^2$ (dashes denoting differentiation). This metric satisfies the empty space condition $R_{\mu\nu} = 0$, but is not flat unless $u\beta'' + 2\beta' = u^2\beta'^3$. Dr. Pirani has kindly informed me that, according to his criterion[7], this space-time contains radiation, but no sources.

Quote from Bondi's *Nature* paper

Trautman 1958

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THEORETICAL PHYSICS

Boundary Conditions at Infinity for Physical Theories

by
A. TRAUTMAN

Presented by L. INFELD on April 12, 1958

1. The Cauchy problem is the most natural for hyperbolic partial differential equations. When dealing with physical problems, we are, however, often interested in solutions of field equations with given sources when nothing is known about initial conditions. A whole set of fields corresponds, in general, to given sources and, in order to arrive at a unique solution of the problem, we must specify some additional condition. For linear field equations this condition may consist in prescribing the form of Green's function (e. g. retarded, advanced, etc.). If we investigate the field in the whole (unbounded) space-time we can ensure uniqueness by specifying some appropriate *boundary conditions* at spatial infinity. The latter approach has the advantage of being applicable to non-linear theories, such as the theory of general relativity. These boundary conditions, first formulated for a periodic scalar field by Sommerfeld [1], have a definite physical meaning. E. g., the "Ausstrahlungsbedingung" of Sommerfeld means that the system can lose its energy in the form of radiation and that no waves are falling on the system from the exterior.

The purpose of this paper is to formulate boundary conditions for scalar and Maxwell theories in a form which exhibits their physical meaning and is proper to a generalization for the gravitational case.

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Trautman 1958 - radiation is nonlocal

- a question from the border between Maxwell's theory and GR: **does a unit charge hang on a thread attached to the ceiling of an Einstein's lift radiates or not**? Viewed by the **observer in the lift** - NO!, as **it is at rest**; viewed by an **observer on the Earth** - YES!, as **it falls down** with an **acceleration** $\vec{g}$.
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Trautman 1958 - gravitational waves via boundary conditions

- **Trautman's idea**: from all solutions of vacuum Einstein equations **select those that satisfy suitable boundary conditions at infinity**
- apropriately **reformulate boundary conditions for radiative solutions of a scalar field** known as **Sommerfeld's radiation conditions** (Courant, Hilbert, Methods of Mathematical Physics, vol.2, p. 315)
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- the first of the two quoted papers is a preparation for the masterpiece, which is the next paper
- In the first one Trautman **reformulates Sommerfeld's RADIATION boundary conditions** for the scalar relativistic Poisson's equation **to be easily generalized to any filed theory**
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Trautman 1958: gravitational waves - DEFINITION

- In it Trautman defines the **boundary conditions for a radiative spacetime in full GR theory**. This is a **definition of gravitational radiation**. This is in [2], on p. 409, equations (9) and (10).

We generalize the conditions of Fock along the lines presented in the preceding paper. First, introduce a null vector field k, defined as follows. Let n^ν be a unit space-like vector lying in σ, perpendicular to the "sphere" $r = \text{const.}$, and pointing outside it. We put $k^\nu = n^\nu + t^\nu$, where t denotes a unit time-like vector normal to σ, such that $t^0 > 0$.

Now, we formulate the following boundary conditions to be imposed on gravitational fields due to isolated systems of matter: *there exist co-ordinate systems and functions* $h_{\mu\nu} = O(r^{-1})$ *such that*

(9) $$g_{\mu\nu} = \eta_{\mu\nu} + O(r^{-1}), \quad g_{\mu\nu,\varrho} = h_{\mu\nu} k_\varrho + O(r^{-2}),$$

(10) $$(h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}\eta^{\varrho\sigma}h_{\varrho\sigma})k^\nu = O(r^{-2}).$$

These conditions correspond to Sommerfeld's "Ausstrahlungsbedingung"; we obtain the "Einstrahlungsbedingung" assuming n^ν to be a normal pointing inward the sphere $r = \text{const.}$ Relations (9), (10) are

Trautman 1958 - reformulation of Einstein's equations

not depend on σ. As is known, in general relativity the energy-momentum tensor of matter $\mathfrak{T}_{\mu\nu}$ does not by itself lead to an integral conservation law. However, if we introduce an energy-momentum pseudotensor of the gravitational field $\mathfrak{t}_\mu^\nu = (\delta_\mu^\nu \mathfrak{G} + g^{\varrho\sigma}{}_{,\mu} \partial \mathfrak{G} / \partial g^{\varrho\sigma}{}_{,\nu})/2\varkappa$, then the sum $\mathfrak{T}_\mu^\nu + \mathfrak{t}_\mu^\nu$ is divergenceless by virtue of Einstein's equations *). Einstein's tensor density $\mathfrak{G}^\nu{}_\mu = \sqrt{-g}(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R)$ can namely be written in the form

(1) $$\mathfrak{G}_\mu^\nu \equiv \varkappa(\mathfrak{t}_\mu^\nu + \mathfrak{U}_\mu^{\lambda\nu}{}_{,\lambda}),$$

where the "superpotentials" $\mathfrak{U}_\mu^{\nu\lambda}$ are given in [1]

(2) $$2\varkappa \mathfrak{U}_\mu^{\nu\lambda} \equiv \sqrt{-g}\, g^{\varrho[\sigma} \delta_\mu^\nu g^{\lambda]\tau} g_{\varrho\sigma,\tau} \equiv -2\varkappa \mathfrak{U}_\mu^{\lambda\nu}.$$

If the Einstein equations

(3) $$G_{\mu\nu} = -\varkappa T_{\mu\nu}$$

are satisfied, then Eqs. (1) and (2) imply

(4) $$\mathfrak{T}_\mu^\nu + \mathfrak{t}_\mu^\nu = \mathfrak{U}_\mu^{\nu\lambda}{}_{,\lambda}, \quad \text{thus} \quad (\mathfrak{T}_\mu^\nu + \mathfrak{t}_\mu^\nu)_{,\nu} = 0.$$

The functions $\mathfrak{t}_\mu{}^\nu$ are not components of a tensor density (equivalence principle) and many physicists (e. g., Schrödinger [2]) have raised doubts

Trautman 1958 - energy-momentum of pure gravity

- uses **von Freud potential** 2-form $\mathcal{F}$, to split the Einstein tensor $E = Ric(g) - \frac{1}{2}Rg$ into $E = \mathrm{d}\mathcal{F} - 8\pi t$ so that **the Einstein equations** $E = 8\pi T$ **take the form**

$$\boxed{\mathrm{d}\mathcal{F} = 8\pi(T + t)}.$$

  Here T is the energy-momentum 3-form.
- Since t **is a 3-form totally determined by the geometry**, he **interprets it as an energy-momentum 3-form of a PURE GRAVITY**. This is in [2], on p. 407, equations (1) and (2).

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Trautman 1958 - 4-momentum of a gravitational system

- uses the closed 3-form $T + t$ to **define a 4-momentum** $P_\mu(\sigma)$ **of GRAVITATIONAL FIELD attributed to each space-like hypersurface** σ **of a space-time satisfying his radiative boundary conditions**, [2], p. 408, equation (5).

The functions $\mathfrak{t}_\mu{}^\nu$ are not components of a tensor density (equivalence principle) and many physicists (e. g., Schrödinger [2]) have raised doubts as to their physical meaning. Einstein [3] and F. Klein [4] formulated some conditions which enable us to consider the integrals

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- **calculates** precisely **how much of the gravitational energy** $p_\mu = P_\mu(\sigma_1) - P_\mu(\sigma_2)$ contained between the spacelike hypersurfaces σ_1 (initial one) and σ_2 (final one) **escapes to infinity** - or, in nowadays **Penrose's** terminology - **to scri**, [2], p.410-411, equations (16)-(17).
- shows that p_0 **is NON-negative**, [2], p. 411, remark after (17).

4. The total energy and momentum p_μ radiated between two hypersurfaces σ and σ' is given by (7), or by

$$p_\mu = P_\mu[\sigma] - P_\mu[\sigma'] = \int_\Sigma \mathfrak{t}_\mu^{\ \nu} dS_\nu$$

($T_{\mu\nu}$ vanishes on Σ). The boundary conditions enable the estimation of p_μ; we have, indeed,

(16) $$\mathfrak{t}_\mu^{\ \nu} = \tau k_\mu k^\nu + O(r^{-3}),$$

where

(17) $$4\varkappa\tau = h^{\mu\nu}(h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}\eta^{\varrho\sigma}h_{\varrho\sigma}).$$

τ is invariant with respect to transformation (15) and is *non-negative* by virtue of (10); therefore $p_0 \geqslant 0$. The existence of radiation is characterized by $p_\mu \neq 0$.

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Trautman 1958: PROOF that grav wave TRAVELS WITH SPEED OF LIGHT

- **shows** that the **Ricci tensor** of a spacetime satisfying his radiative conditions, **far from the sources, is of the form** $Ric_{\mu\nu} = \rho k_\mu k_\nu$, with k - **null vector**, [2], p. 411, eq. (20). This in particular means that the gravitational radiation in his radiative spacetimes travel with speed of light.

The terms proportional to $1/r$ in $R_{\mu\nu}$ cancel out by virtue of (10). Conversely, $R_{\mu\nu} \simeq 0$ and Eq. (18) imply $R_{\mu\nu\varrho\sigma} \simeq 0$ unless $k_\nu k^\nu = 0$. If we take into account the electromagnetic field, Einstein's equations can be written in the form

$$R_{\mu\nu} = \varrho k_\mu k_\nu + O(r^{-3}), \quad \varrho = O(r^{-2}). \tag{20}$$

Trautman 1958: PROOF that grav wave IS OF TYPE N

- **shows** that **the Riemann tensor** of his radiative spacetimes, **far from the sources, is of Petrov type** *N*, [2], p. 411, eq. (21). Since far from the sources *Riemann* = *Weyl*, this shows that **waves satisfying his boundary conditions satisfy the algebraic speciality criterion of Pirani**.

Moreover, it follows from (19) that

$$k_{[\mu} R_{\nu\varrho]\sigma\tau} \cong 0\,, \qquad k^{\mu} R_{\mu\nu\varrho\sigma} \cong 0\,. \tag{21}$$

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- Robinson-Trautman waves:

$$g = \frac{2r^2 \mathrm{d}\zeta \mathrm{d}\bar{\zeta}}{P^2(u,\zeta,\bar{\zeta})} - 2\mathrm{d}u\mathrm{d}r - \left(\triangle \log P - 2r(\log P)_u - \frac{2m(u)}{r}\right)\mathrm{d}u^2$$

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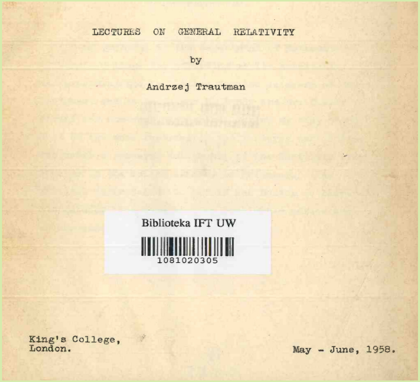

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by

Andrzej Trautman

King's College,
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THANK YOU!