Effective Action and Phase Diagram of a Model of Superconductivity with Population Imbalance

Piotr Zdybel and Paweł Jakubczyk

Institute of Theoretical Physics, Faculty of Physics, University of Warsaw
Pasteura 5, 02-093 Warsaw, Poland
`Piotr.Zdybel@fuw.edu.pl, Pawel.Jakubczyk@fuw.edu.pl`

1. Introduction

Understanding non-Fermi liquids is among the most interesting challenge to quantum many-body physics [1, 2]. Such systems occur in a variety of physical contexts such as for example superconductivity. Of particular focus are systems of interacting particles in two spatial dimensions [3, 4]. We develop a theoretical approach based on an extension of the BCS model and compute its mean-field phase diagram.

2. Theoretical Model

We study the model defined by the following action [5]:

$$\mathcal{S}[\bar{\psi},\psi] = \sum_{\vec{k},\sigma} \xi_{\vec{k},\sigma}\bar{\psi}_{\vec{k},\sigma}\psi_{\vec{k},\sigma} + \frac{g}{V}\sum_{\vec{k},\vec{k}',\vec{q}} \bar{\psi}_{\vec{k}+\vec{q}/2,\uparrow}\bar{\psi}_{-\vec{k}+\vec{q}/2,\downarrow}\psi_{\vec{k}'+\vec{q}/2,\downarrow}\psi_{-\vec{k}'+\vec{q}/2,\uparrow} \tag{1}$$

where $\xi_{\vec{k},\sigma} = \vec{k}^2/2m_\sigma - \mu_\sigma$, with $\sigma = \uparrow, \downarrow$, g is an attractive contact interaction constant and V is the volume of the system. In general, the masses m_σ and chemical potentials μ_σ can be different.

3. Effective Action

The problem of interacting fermions may be reinterpreted in the language of a bosonic field theory. We obtain the effective action, which is the uniform contribution to the Landau free energy functional:

$$\mathcal{S}_{\text{eff}}[\Delta^*,\Delta] = -\frac{|\Delta|^2}{2g} - \frac{1}{\beta}\int \frac{d^d\vec{k}}{(2\pi)^d}\left[\ln\left(1+e^{-\beta E_+}\right) + \ln\left(1+e^{-\beta E_-}\right)\right], \tag{2}$$

where

$$E_\pm = \frac{\xi_{\vec{k},\uparrow} - \xi_{\vec{k},\downarrow}}{2} \pm \sqrt{\frac{|\Delta|^2}{2} + \left(\frac{\xi_{\vec{k},\uparrow} + \xi_{\vec{k},\downarrow}}{2}\right)^2}. \tag{3}$$

We use $\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}$ as the average chemical potential and $h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$ as the average "Zeeman" field. Note that Δ is an order parameter.

4. Mean-field Phase Diagram in 2D

We compute the mean-field phase diagram for the model using an expression for the attractive interaction in two dimensions [6, 7]:

$$g = -\frac{2\pi}{m_{\text{red}}\ln(a_{2D}\Lambda_{UV})}, \tag{4}$$

where a_{2D} is the 2D scattering length, Λ_{UV} is the ultraviolet cutoff and $m_{\text{red}} = \frac{2m_\uparrow m_\downarrow}{m_\uparrow + m_\downarrow}$.

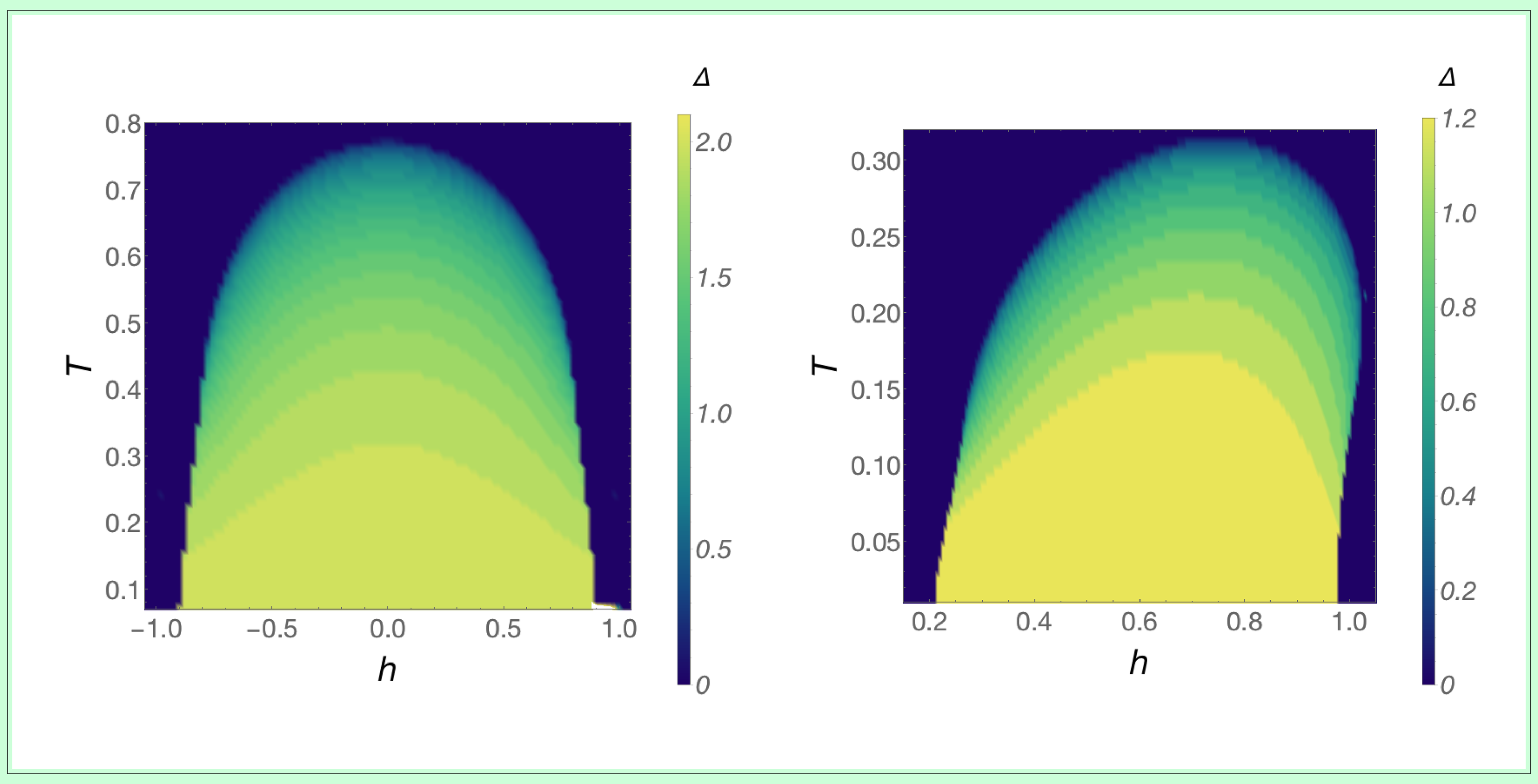

Figure 1: *Mean-field phase diagram in two spatial dimensions of the considered model plotted as a function of the Zeeman field (h) and temperature (T). On the left side we show the phase diagram for the equal masses of particles ($m_\downarrow/m_\uparrow = 1$), on the right side we show the phase diagram for the different masses of particles ($m_\downarrow/m_\uparrow = 20$).*

Numerical analysis suggests that there are no quantum critical points in two dimensions at the mean-field level. Inclusion of the order parameter fluctuations [5] allows for the emergence of the Kosterlitz-Thouless phase [8], which is consistent with the Mermin-Wagner theorem [9].

5. Mean-field Phase Diagram in 3D

We also compute the mean-field phase diagram in three spatial dimensions for the model using an expression for the attractive interaction [10, 11]:

$$g = -\frac{2\pi^2}{m_{\text{red}}\Lambda_{UV}}. \tag{5}$$

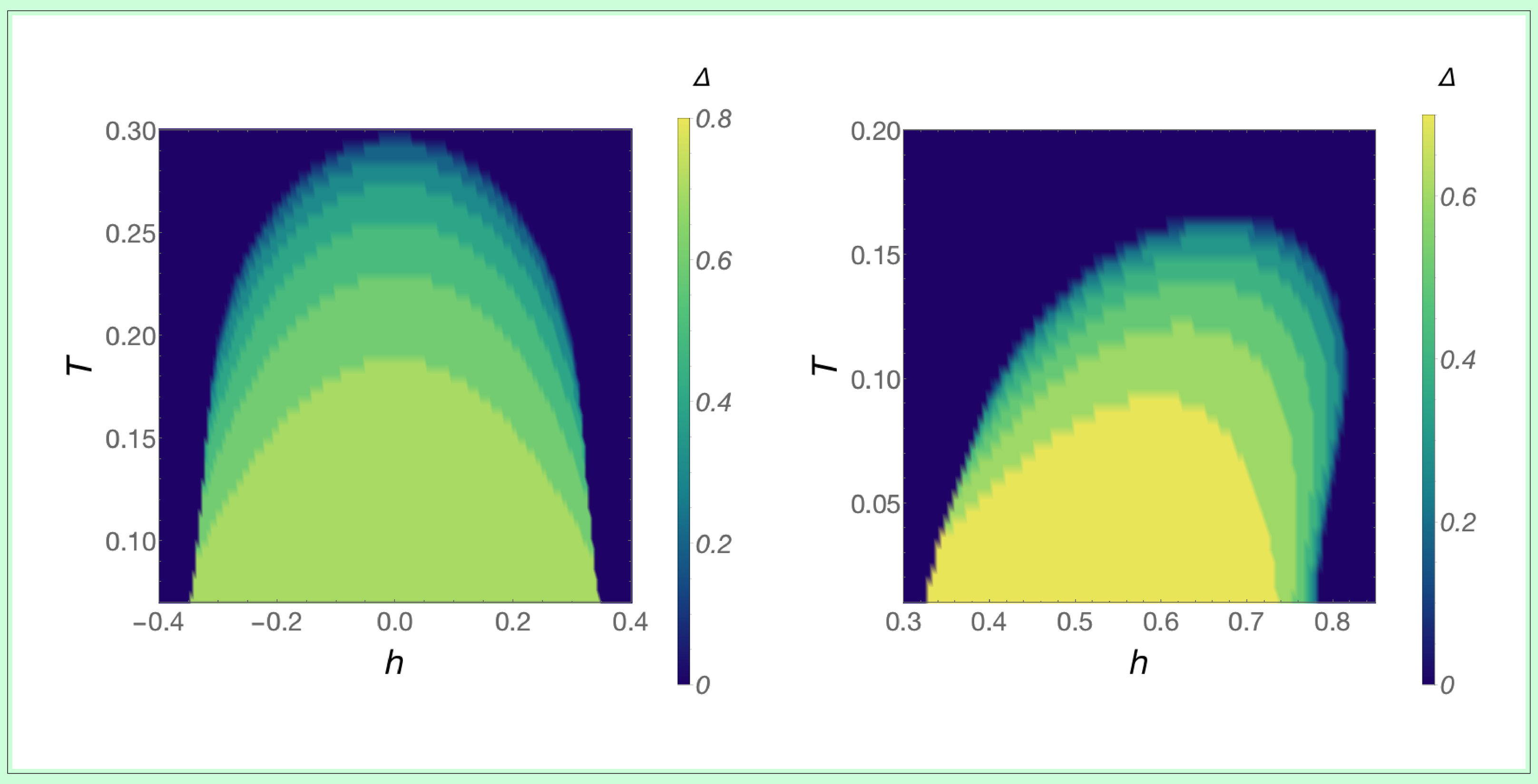

Figure 2: *Mean-field phase diagram in three spatial dimensions of the considered model plotted as a function of the Zeeman field (h) and temperature (T). On the left side we show the phase diagram for the equal masses of particles ($m_\downarrow/m_\uparrow = 1$), on the right side we show the phase diagram for the different masses of particles ($m_\downarrow/m_\uparrow = 20$).*

Numerical computations in three spatial dimensions show the possibility of generating a quantum critical point of the model for sufficiently large mass ratios [12].

6. Maximal Critical Temperature

We have investigated the behavior of the maximal value of the critical temperature as a function of the mass ratio of particles in the system. For large mass ratio the maximal critical temperature tends to constant value in two and three spatial dimensions. This behavior is a consequence of the pairing mechanism for the Sarma-Liu-Wilczek phase [13, 14].

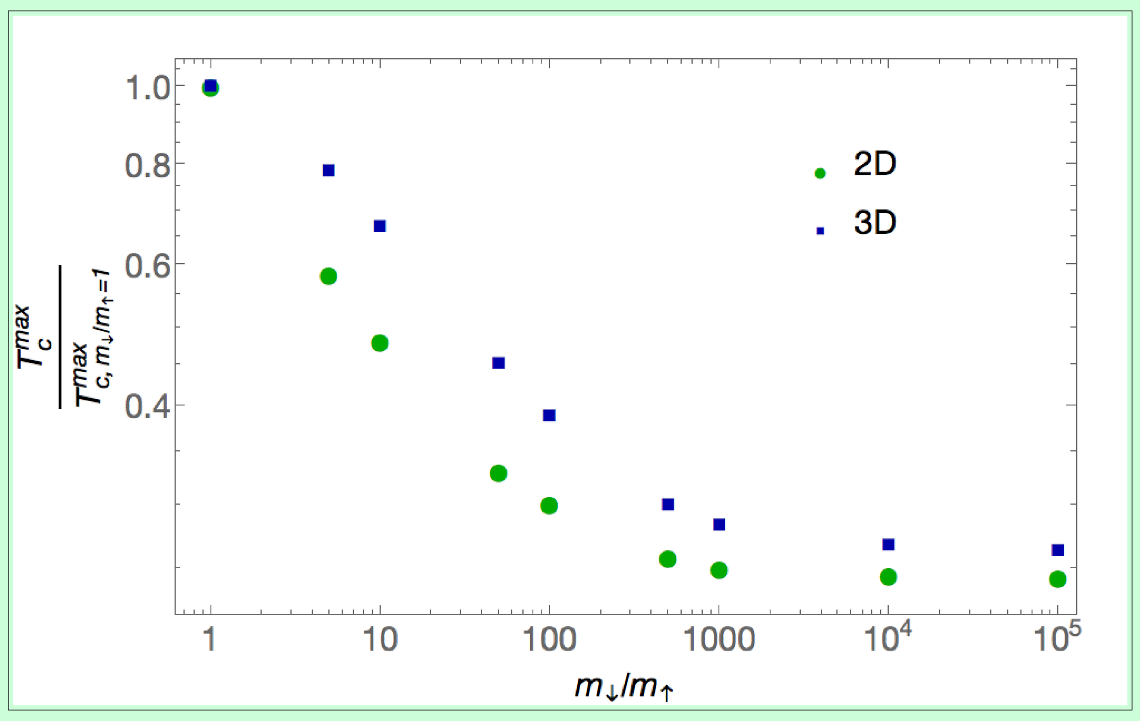

Figure 3: *The maximum value of the critical temperature ratio ($T_c^{\max}/T_c^{\max}(m_\downarrow/m_\uparrow = 1)$) plotted as a function of the mass ratio ($m_\downarrow/m_\uparrow$). We use logarithmic scales on both the horizontal and vertical axes.*

7. Summary

- We perform a detailed analysis of the phase phase diagram in two and three spatial dimensions at the mean-field level.
- Shape of the phase diagram depends on masses of each spin species of particles.
- Mean-field theory in two spatial dimensions predicts a superconducting dome with long-range order and first-order transitions on its wings.
- Unlike the two dimensional case in three dimensions is the possibility of generating a quantum critical point of the model.
- Inclusion of the fluctuations and use of the renormalization group approach allows for the emergence of the Kosterlitz-Thouless phase in two spatial dimensions.

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