Absence of quantum criticality in imbalanced Fermi mixtures at the mean-field level

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1. Introduction

Understanding non-Fermi liquids is among the most interesting challenges to quantum many-body physics [1, 2]. Such systems may occur in a variety of physical contexts such as for example superfluidity in imbalanced Fermi gases. Non-Fermi liquid behaviour could appear in the proximity of a quantum critical point (QCP) [3, 4]. We study the possibility of generating a QCP for imbalanced Fermi gases [5–8] at the mean field level in two and three spatial dimensions. Such systems, containing a mixture of fermionic atoms, nowadays are experimentally available [9–14].

2. Theoretical Model

We study the model defined by the hamiltonian [15]:

$$\hat{H}-\sum_{\sigma}\mu_{\sigma}\hat{n}_{\sigma}=\sum_{\vec{k},\sigma}\xi_{\vec{k},\sigma}c^{\dagger}_{\vec{k},\sigma}c_{\vec{k},\sigma}+\frac{g}{V}\sum_{\vec{k},\vec{k}',\vec{q}}c^{\dagger}_{\vec{k}+\vec{q}/2,\uparrow}c^{\dagger}_{-\vec{k}+\vec{q}/2,\downarrow}c_{\vec{k}'+\vec{q}/2,\downarrow}c_{-\vec{k}'+\vec{q}/2,\uparrow} \tag{1}$$

where $\xi_{\vec{k},\sigma}=\vec{k}^2/2m_\sigma-\mu_\sigma$, with $\sigma=\uparrow,\downarrow$, g is an attractive interspecies contact interaction constant and V is the volume of the system. In general, the masses m_σ and chemical potentials μ_σ can be different.

3. Grand Potential

The grand potential density at the mean-field level can be expressed as:

$$\tilde{\Omega}=\frac{\Omega}{V}=\min_{\Delta}\left\{-\frac{|\Delta|^2}{g}-\frac{1}{\beta}\int\frac{d^d\vec{k}}{(2\pi)^d}\sum_{i\in\{+,-\}}\ln\left(1+e^{-\beta E_i}\right)\right\}=\min_{\Delta}\left\{\tilde{\Omega}_L[\Delta]\right\}, \tag{2}$$

where $\min_\Delta$ leads to the global minimum with respect to the order parameter Δ. In this case, there are two possible phases: a normal phase with $\Delta=0$ and a superfluid phase with $\Delta\neq 0$ [16, 17]. We are denoting the Landau functional as $\tilde{\Omega}_L[\Delta]$. Moreover, the elementary excitations energies are given by:

$$E_{\pm}=\frac{\xi_{\vec{k},\uparrow}-\xi_{\vec{k},\downarrow}}{2}\pm\sqrt{|\Delta|^2+\left(\frac{\xi_{\vec{k},\uparrow}+\xi_{\vec{k},\downarrow}}{2}\right)^2}. \tag{3}$$

We use $\mu=\frac{\mu_\uparrow+\mu_\downarrow}{2}$ as the average chemical potential and $h=\frac{\mu_\uparrow-\mu_\downarrow}{2}$ as the average "Zeeman" field. We are considering a case in which $\mu_\sigma>0$.

4. Landau Coefficients

Setting $\xi=\frac{\xi_{\vec{k},\uparrow}+\xi_{\vec{k},\downarrow}}{2}$ we express Landau functional $\tilde{\Omega}_L[\Delta]$ as a Taylor expansion in the order parameter Δ:

$$\tilde{\Omega}_L[\Delta]=\tilde{\Omega}_L(\Delta=0)+a_2(T)|\Delta|^2+a_4(T)|\Delta|^4+O\left(|\Delta|^6\right), \tag{4}$$

where the quadratic Landau coefficient a_2 is given by

$$a_2(T)=\left(\frac{\partial\tilde{\Omega}_L}{\partial|\Delta|^2}\right)_{|\Delta|^2=0}=-\frac{1}{g}-\frac{1}{4}\int\frac{d^d\vec{k}}{(2\pi)^d}\frac{1}{\xi}\sum_{\sigma}\tanh\left(\frac{\beta\xi_{\vec{k},\sigma}}{2}\right) \tag{5}$$

and the quartic Landau coefficient a_4 is given by

$$a_4(T)=\frac{1}{2}\left(\frac{\partial^2\tilde{\Omega}_L}{\partial\left(|\Delta|^2\right)^2}\right)_{|\Delta|^2=0}=\frac{1}{16}\int\frac{d^d\vec{k}}{(2\pi)^d}\frac{1}{\xi^3}\sum_{\sigma}\left[\tanh\left(\frac{\beta\xi_{\vec{k},\sigma}}{2}\right)-\frac{\beta\xi}{2}\text{sech}^2\left(\frac{\beta\xi_{\vec{k},\sigma}}{2}\right)\right]. \tag{6}$$

The QCP correspond to continuous phase transition at zero absolute temperature, so we compute Landau coefficients for $T\to 0$.

5. Coefficient a_2 for $T=0$

We use the fact that $\tanh\left(\frac{\beta\xi_{\vec{k},\sigma}}{2}\right)=1-2f(\xi_{\vec{k},\sigma})$, where $f(\cdot)$ is the Fermi function, which leads us to the expression below:

$$a_2(T=0)=-\frac{1}{g}-\frac{1}{2}\int\frac{d^d\vec{k}}{(2\pi)^d}\frac{1}{\xi}\left[1-\sum_{\sigma}\theta\left(-\xi_{\vec{k},\sigma}\right)\right], \tag{7}$$

where $\theta(\cdot)$ is the Heaviside step function. The first condition of the existence of the QCP requires the disappearance of the coefficient a_2 for that point of the phase diagram.

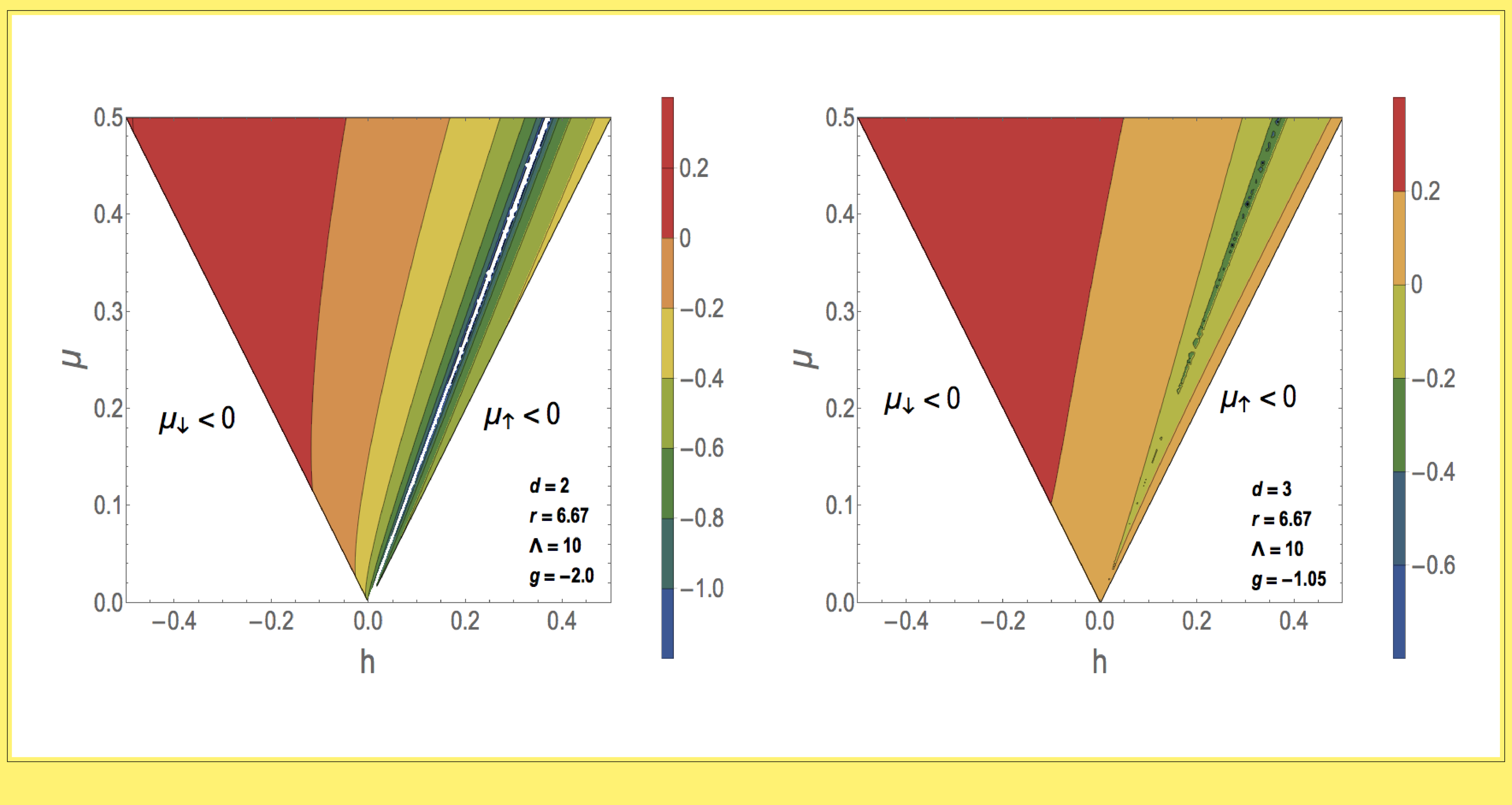

Figure 1: *Coefficient a_2 plotted as a function of the Zeeman field (h) and average chemical potential (μ). On the left side we show the plot for $d=2$, on the right side we show the plot for $d=3$. In addition, mass ratio r correspond to the case of ^6Li and ^{40}K mixture.*

First we will perform an analysis of this expression in two dimensions:

$$a_2^{2D}=-\frac{1}{g}-\frac{m_r}{4\pi}\ln\left[\frac{|\Lambda^2-2\mu m_r|\cdot 2\mu m_r}{|\lambda_\uparrow^2-2\mu m_r|\cdot|\lambda_\downarrow^2-2\mu m_r|}\right], \tag{8}$$

where $\frac{2}{m_r}=\frac{1}{m_\uparrow}+\frac{1}{m_\downarrow}$ is the reduced mass, Λ is the UV cutoff and $\lambda_\sigma^2=2\mu_\sigma m_\sigma$. The strength of the interaction can be adjusted using the Feshbach resonance [5–7], so we can always tune the coefficient a_2^{2D} to zero.

Analogously, in three dimensions

$$a_2^{3D}=-\frac{1}{g}-\frac{m_r}{2\pi^2}\left[\Lambda+\frac{\sqrt{2\mu m_r}}{2}\ln\left(\frac{|\Lambda-\sqrt{2\mu m_r}|}{|\Lambda+\sqrt{2\mu m_r}|}\right)-\sum_{\sigma}\left\{\lambda_\sigma+\frac{\sqrt{2\mu m_r}}{2}\ln\left(\frac{|\lambda_\sigma-\sqrt{2\mu m_r}|}{|\lambda_\sigma+\sqrt{2\mu m_r}|}\right)\right\}\right]. \tag{9}$$

As before, we can tune the a_2^{3D} value to zero. As an example, see figure 1, where we introduce $r=m_\downarrow/m_\uparrow$.

6. Coefficient a_4 for $T=0$

Existence of a QCP requires that $a_4>0$ in addition to $a_2=0$. As before we consider the zero-temperature case:

$$a_4(T=0)=\frac{1}{8}\int\frac{d^d\vec{k}}{(2\pi)^d}\frac{1}{\xi^3}\left[1-\sum_{\sigma}\left\{\theta\left(-\xi_{\vec{k},\sigma}\right)+\xi\delta\left(-\xi_{\vec{k},\sigma}\right)\right\}\right]. \tag{10}$$

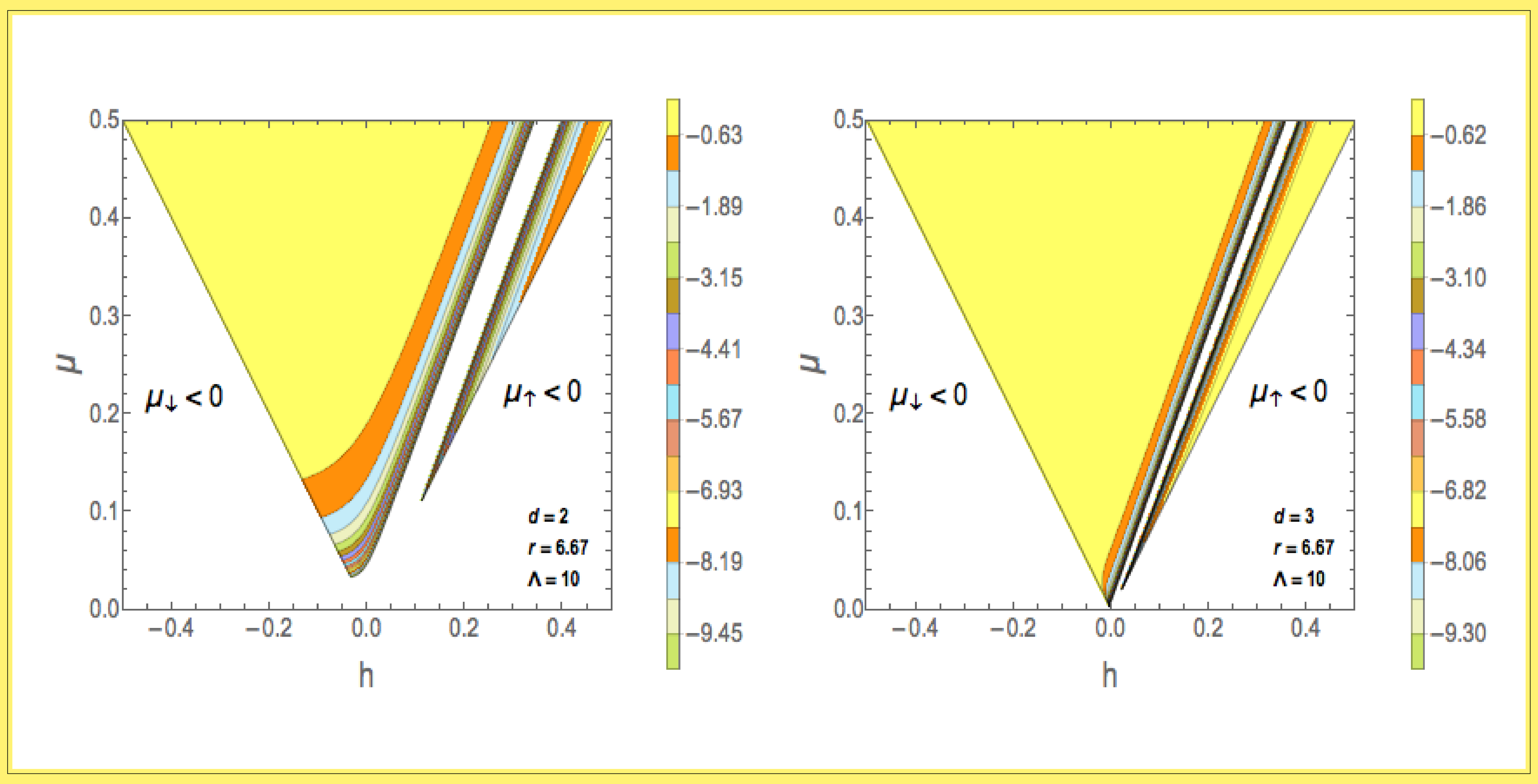

Figure 2: *Coefficient a_4 plotted as a function of the Zeeman field (h) and average chemical potential (μ). On the left side we show the plot for $d=2$, on the right side we show the plot for $d=3$. Also, in this case, mass ratio corresponds to the lithium and potassium mixture.*

In two dimensions we get

$$a_4^{2D}=-\frac{m_r^3}{16\pi}\left[\frac{1}{\left(\Lambda^2-2\mu m_r\right)^2}+\frac{1}{(2\mu m_r)^2}+\frac{1}{r\left(\lambda_\uparrow^2-2\mu m_r\right)^2}+\frac{r}{\left(\lambda_\downarrow^2-2\mu m_r\right)^2}\right], \tag{11}$$

which is always negative, so in two dimensions there is no QCP at the mean-field level.

Furthermore, the coefficient a_4^{3D} in three dimensions is given by

$$a_4^{3D}=-\frac{m_r^2}{64\pi^2}\left[\frac{\Lambda\left(\Lambda^2+2\mu m_r\right)}{\mu\left(\Lambda^2-2\mu m_r\right)^2}+\frac{m_r}{(2\mu m_r)^{3/2}}\ln\left(\frac{|\Lambda-\sqrt{2\mu m_r}|}{|\Lambda+\sqrt{2\mu m_r}|}\right)+\right.$$
$$\left.-\sum_{\sigma}\left(\frac{\lambda_\sigma\left(\lambda_\sigma^2+2\mu m_r\right)}{\mu\left(\lambda_\sigma^2-2\mu m_r\right)^2}+\frac{m_r}{(2\mu m_r)^{3/2}}\ln\left(\frac{|\lambda_\sigma-\sqrt{2\mu m_r}|}{|\lambda_\sigma+\sqrt{2\mu m_r}|}\right)-\frac{8\lambda_\sigma}{\left(\lambda_\sigma^2-2\mu m_r\right)^2}\right)\right], \tag{12}$$

which as it turns out is also negative, therefore in three dimensions as well there is no QCP at the mean-field level. Figure 2 shows the corresponding graphs.

7. Summary

- We perform a detailed analysis of the conditions necessary for the existence of the QCP in two and three spatial dimensions at the mean-field level.
- In both cases, it appears that no QCP can be generated at the mean-field level.
- Landau coefficients for $T=0$ are not specified for some parameter choices, such as those corresponding to the BCS theory ($r=1$, $h=0$).
- The inclusion of the fluctuations and use of the renormalization group approach allows to modify that result and to generate the QCP in two dimensions [15].

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