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SCALAR FLAVOUR CHANGING NEUTRAL CURRENTS IN MFV SUSY AT LARGE $\tan\beta$*

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We discuss the origin and phenomenological consequences of the flavour violating couplings of the neutral Higgs bosons to the down-type quarks generated in the minimal supersymmetric standard model (MSSM) for large values of $\tan\beta$. We concentrate on the scenario with minimal flavour and CP violation (MFV) and demonstrate the tight correlation of the B_s^0–$\bar{B}_s^0$ mass difference with $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ and $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$. While the first correlation holds also in the case of non-minimal flavour violation, the second one is specific for the minimal case and can potentially serve as a test of the sources of flavour violation in supersymmetry.

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1. Introduction

Supersymmetry (SUSY) continues to be the best motivated extension of the Standard Model (SM). Direct searches of superparticles, which are presumably much heavier than initially expected, will be again possible at future colliders (the LHC and linear ones). At present there is some hope to discover the effects of new physics in rare flavour changing neutral current (FCNC) and CP violating (CPV) processes. Among these, the ones involving the b-quark, which are being intensively studied in several experiments (Tevatron, Belle, BaBar), play a special role in the context of SUSY searches. This is because in the minimal supersymmetric extension (MSSM) of the SM the Yukawa couplings of the b-quark to some superpartners of the known particles and/or to the Higgs bosons can be rather strong. Therefore supersymmetric contributions to the radiative b decays could be substantial. An example is the radiative decay $\bar{B} \to X_s\gamma$ whose experimentally measured

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rate [1] agrees very well with the SM prediction [2] and, consequently, puts some constraints on the MSSM parameter space. These constraints become particularly stringent if the ratio $v_u/v_d \equiv \tan\beta$ of the vacuum expectation values of the two Higgs doublets is large, that is when the coupling of the right-chiral b-quark to charginos and the top squarks is enhanced: agreement with the experimental value can be then obtained either if the virtual chargino-stop contribution to the $b \to s\gamma$ amplitude cancels against the top-charged Higgs boson contribution — which requires the more fine tuning the lighter are these sparticles — or, more naturally, if chargino and/or stops as well as H^+ are sufficiently heavy.

SUSY scenario with large value of $\tan\beta$ has nevertheless some attractive features — it can explain large m_t/m_b ratio by the large ratio of the vacuum expectation values rather than by large ratio of the corresponding dimensionless Yukawa couplings, it can also lead to unification of the top and bottom Yukawa couplings which may be required by the SO(10) GUTs which in turn are very interesting in the context of the discovery of the neutrino masses — and it would be good if it could be tested experimentally even if sparticles are rather heavy. Here we show that this is indeed possible: for $\tan\beta \gg 1$ the rates of the $B_s^0 \to \mu^+\mu^-$ decay can be significantly bigger than in the SM, even for sparticle masses in the TeV range, provided the mass scale of the Higgs boson sector (set *e.g.* by the mass of the CP-odd scalar A^0) is not too high compared to the electroweak scale. In the same regime the B_s^0–$\bar{B}_s^0$ mass difference ΔM_s can differ significantly from the value predicted by the SM. The correlation of $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ with ΔM_s is therefore a testable prediction of the large $\tan\beta$ scenario. Moreover, if the CKM matrix remains the dominant source of flavour and CP violation in the MSSM, ΔM_s and $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ remain strongly correlated with $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ thus potentially allowing to disentangle possible mechanisms of flavour violation in supersymmetry.

2. Scalar penguins

In the most general version of the MSSM couplings of sfermions naturally violate flavour: if the mass matrices of squarks are not diagonal in the same basis as the quark mass matrices, the strong couplings quark–squark–gluino change the flavour leading to rates of the rare processes that are generically orders of magnitude bigger than the experimental limits or measurements. This imposes some restrictions on the off-diagonal entries of the sfermion mass squared matrices. The restrictions are usually quantified in terms of limits on the so-called mass insertions [3, 4]. Most stringent limits apply to mass insertions generating transitions between the first two generations of quarks whereas the ones responsible for transitions between the third and the first two generations (and in particular $b \to s$ transitions [5]) are substantially weaker. In general, however, the pattern of supersymmetric flavour violation can be much more complicated as the usually quoted limits [3,4,6] on mass insertions or their products do not take into account possible cancellations between different mass insertions. Such possible cancellations can be effectively studied only in specific models of soft terms [7].

In view of the complexity of the most general pattern of supersymmetric flavour violation it appears reasonable to contemplate a more predictive scenario, in which the effects of the mass insertions are negligible and the Cabbibo–Kobayashi–Maskawa (CKM) matrix remains the dominant source of flavour and CP violation. In this case the predictions for rates of radiatively induced rare process differ from the ones obtained in the SM due to additional contributions of sparticles circulating in loops; with the notable exception of the processes like the already mentioned decay $\bar{B} \to X_s\gamma$ the differences decrease with increasing value of $\tan\beta$ and usually rapidly vanish with increasing chargino and stop masses [4, 9].

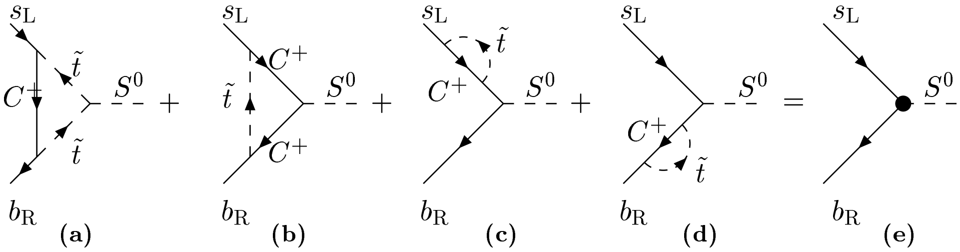

Fig. 1. Perturbative calculation of the flavour violating neutral Higgs boson $S^0 = H^0, h^0, A^0$ coupling to $s_{\rm L}$ and $b_{\rm R}$ quarks. Other flavour violating couplings of S^0 are given by similar diagrams.

For $\tan\beta \gg 1$ there is however a special class of contributions to the FCNC processes, which do not necessarily vanish with increasing scale of supersymmetry breaking. They are due to the flavour changing couplings of the neutral Higgs bosons generated at one loop by the chargino-stop Higgs boson penguin diagrams: 1PI vertex diagrams and diagrams with flavour changing on external quark lines (Fig. 1). For $\tan\beta \gg 1$ such couplings of h^0 are negligible compared to the ones of H^0 and A^0 to which the dominant contribution comes from the diagram shown in Fig. 1(d), and which in the case of *e.g.* the $s \to b$ transition are of order

$$y_b y_t^2 \varepsilon_{\rm Y} V_{tb}^* V_{ts} \tan\beta = \frac{g^3}{2\sqrt{2}} \frac{m_b}{M_W} \frac{m_t^2}{M_W^2} \varepsilon_{\rm Y} V_{tb}^* V_{ts} \tan^2\beta \,, \tag{1}$$

where $\varepsilon_{\rm Y} \sim \mathcal{O}(1/16\pi^2)$ is a dimensionless function of sparticle masses and, hence, does not vanish for $M_{\rm spart} \to \infty$. Since $\tan\beta$ can be as large 50–60 (if the bottom quark Yukawa coupling is to remain perturbative up to the GUT scale), the flavour violating couplings can be significantly enhanced. Thus, for $\tan\beta \gtrsim 30$ and not too high a mass scale of the Higgs sector, exchanges of H^0 and A^0 can give important contributions to the amplitudes of rare processes involving the b-quarks.

The potentially big terms containing extra powers of $\tan\beta$ (compared to the power of m_b) appear in all orders of the perturbation calculus [8]. Their origin is related to the two Higgs doublet nature of the MSSM and the necessary breaking of the so called Peccei–Quinn symmetry [10]. They can be understood by using the effective Lagrangian technique [11–13] which also allows to resumm them to all orders [8,13,14]. To this end consider first the general couplings of the down-type quarks to two Higgs doublets

$$\mathcal{L} = -H_{(d)}\bar{d}_{\rm R}\cdot \boldsymbol{Y}_d \cdot q_{\rm L} - H^*_{(d)}\bar{d}_{\rm R}\cdot \boldsymbol{\Delta}_u \boldsymbol{Y}_d \cdot q_{\rm L} + {\rm H.c.} \tag{2}$$

After the electroweak symmetry breaking we decompose

$$\begin{aligned} H^0_{(d)} &\to v_d + c_\alpha H^0 - s_\alpha h^0 + i s_\beta A^0 + \dots\,, \\ H^0_{(d)} &\to v_u + s_\alpha H^0 + c_\alpha h^0 - i c_\beta A^0 + \dots\,, \end{aligned} \tag{3}$$

where $v_u/v_d = \tan\beta$ and the rotation by the angle α diagonalizes the mass matrix of the neutral Higgs bosons. Inserting (3) into (2) one obtains the mass term $\mathcal{L}^{\rm mass}$ for the down-type quarks

$$\mathcal{L}^{\rm mass} = -\bar{d}_{\rm R}\cdot \left(v_d \boldsymbol{Y}_d + v_u \boldsymbol{\Delta}_u \boldsymbol{Y}_d\right)\cdot d_{\rm L} - {\rm H.c.} \tag{4}$$

and their Yukawa couplings $\mathcal{L}^{\rm Yuk}$ to the neutral Higgs bosons in the form:

$$\begin{aligned} \mathcal{L}^{\rm Yuk} \;=\; & -\bar{d}_{\rm R}\cdot \left[\left(c_\alpha \boldsymbol{Y}_d + s_\alpha \boldsymbol{\Delta}_u \boldsymbol{Y}_d\right) H^0 + \left(-s_\alpha \boldsymbol{Y}_d + c_\alpha \boldsymbol{\Delta}_u \boldsymbol{Y}_d\right) h^0 \right. \\ & \left. + i\left(s_\beta \boldsymbol{Y}_d - c_\beta \boldsymbol{\Delta}_u \boldsymbol{Y}_d\right) A^0 + \dots\right]\cdot d_{\rm L} + {\rm H.c.} \end{aligned} \tag{5}$$

The quark mass matrix is then diagonalized by the unitary rotations $d_{\rm L} \to \boldsymbol{D}_{\rm L}\cdot d_{\rm L}$, $\bar{d}_{\rm R} \to \bar{d}_{\rm R}\cdot \boldsymbol{D}^\dagger_{\rm R}$:

$$\boldsymbol{D}^\dagger_{\rm R}\cdot \left(v_d \boldsymbol{Y}_d + v_u \boldsymbol{\Delta}_u \boldsymbol{Y}_d\right)\cdot \boldsymbol{D}_{\rm L} = {\rm diag}\left(\bar{m}_d, \bar{m}_s, \bar{m}_b\right)\,. \tag{6}$$

The physical masses $\bar{m}_{d_I}$ must be distinguished from the eigenvalues m_{d_I} of the matrix $v_d\boldsymbol{Y}_d$. Since in general the coupling matrices $c_\alpha \boldsymbol{Y}_d + s_\alpha \boldsymbol{\Delta}_u \boldsymbol{Y}_d$, $-s_\alpha \boldsymbol{Y}_d + c_\alpha \boldsymbol{\Delta}_u \boldsymbol{Y}_d$ and $s_\beta \boldsymbol{Y}_d - c_\beta \boldsymbol{\Delta}_u \boldsymbol{Y}_d$ in (5) are not diagonalized by the rotations $\boldsymbol{D}_{\rm L}$, $\boldsymbol{D}_{\rm R}$, the quark flavour is not conserved in the neutral Higgs boson vertices.

Due to the holomorphicity of the superpotential $\boldsymbol{\Delta}_u \boldsymbol{Y}_d$ vanishes at the tree level in the MSSM. It is however easy to see that in the regime

$M_{\rm spart} \gg M_A \gtrsim M_W$, in which sparticles can be integrated out in the SU(2)×U(1) symmetric phase (that is, for $v_u = v_d = 0$), and in which the effective theory below the sparticle mass scale is the two Higgs doublet model, $\boldsymbol{\Delta}_u\boldsymbol{Y}_d$ arises from the threshold corrections represented by the diagrams shown in Fig. 2. If the CKM matrix is the only source of flavour violation $\boldsymbol{\Delta}_u\boldsymbol{Y}_d$ has the form

$$(\boldsymbol{\Delta}_u\boldsymbol{Y}_d)^{JI} = y_{d_J}\left(\varepsilon_0\delta^{JI} + y_t^2\varepsilon_{\rm Y}V_{tJ}^*V_{tI}\right), \tag{7}$$

$$\varepsilon_0 = -\frac{2\alpha_{\rm s}}{3\pi}\frac{\mu}{m_{\tilde g}}H_0\,, \quad \varepsilon_{\rm Y} = -\frac{1}{16\pi^2}\frac{A_t}{\mu}H_{\rm Y}\,, \tag{8}$$

where A_t is the parameter of the left- and right stop mixing, μ is the chargino mass parameter and H_0, $H_{\rm Y}$ are functions of the mass ratios. ε_0 arises from the flavour conserving gluino exchange diagram 2(a). (Of course, in the case of non-minimal flavour violation also the diagram 2(a) contributes to off-diagonal entries of $\boldsymbol{\Delta}_u\boldsymbol{Y}_d$ [15–17].)

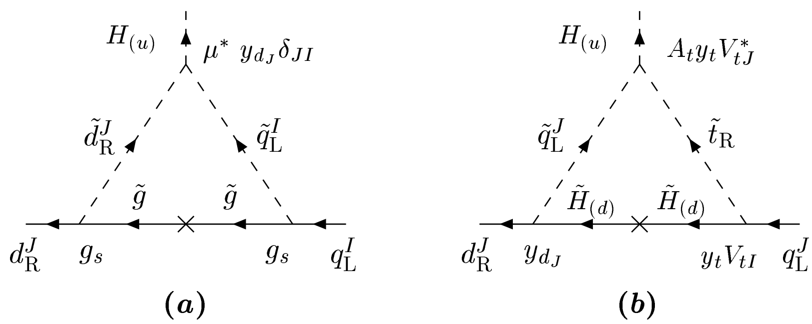

Fig. 2. Threshold corrections generating $(\Delta_u\boldsymbol{Y}_d)^{JI}$ in the SU(2)×U(1) symmetry limit and vanishing electroweak gauge couplings.

Nonzero couplings $\Delta_u\boldsymbol{Y}_d$ have the following consequences.

- With (7) one gets from (6)

$$\bar m_{d_J} \approx y_{d_J}v_d + (\Delta_u\boldsymbol{Y}_d)^{JJ}v_u = m_{d_J}\left(1+\tan\beta\tilde\varepsilon_J\right), \tag{9}$$

where $\tilde\varepsilon_{d,s} \approx \varepsilon_0$ and $\tilde\varepsilon_b \approx \varepsilon_0 + \varepsilon_{\rm Y}y_t^2$. The constants y_{d_J} entering the expressions for the flavour changing (and also flavour conserving) couplings of the neutral Higgs bosons to the down-type quarks are related to the "measured" quark masses[1] through

$$y_{d_J} \equiv \frac{m_{d_J}}{v_d} = \frac{g}{\sqrt2}\frac{\bar m_{d_J}}{M_W}\frac{\tan\beta}{(1+\tilde\varepsilon_J\tan\beta)}\,. \tag{10}$$

[1] By "measured" we mean here the $\overline{\rm MS}$ running quark masses at the scale of decoupling of heavy sparticles.

- Rediagonalization (6) of the mass matrix results in the change of the CKM matrix in the $W^{\pm}$ couplings: $\boldsymbol{V} \to \boldsymbol{V}^{\rm eff} = \boldsymbol{V} \cdot \boldsymbol{D}_{\rm L}$. This gives

$$\boldsymbol{V}^{\rm eff}_{JI} = \boldsymbol{V}_{JI} \times \left(\frac{1+\varepsilon_0 \tan\beta}{1+\tilde{\varepsilon}_b \tan\beta}\right) \quad \text{for } (JI) = (13), (23), (31), (32)\,. \tag{11}$$

  The remaining entries of the CKM matrix are practically unchanged.

- Rotations $\boldsymbol{D}_{\rm L}$ and $\boldsymbol{D}_{\rm R}$ induce the flavour violating couplings of $H^0_{(d)}$ of the form

$$\mathcal{L}^{\rm eff} = H^0_{(d)} \bar{d}^J_{\rm R} \cdot (\tan\beta \boldsymbol{\Delta}_u \boldsymbol{Y}_d)^{JI} \cdot d^I_{\rm L} + \text{H.c.} \tag{12}$$

  As follows from (7) and (10) these couplings are enhanced by the factor $\propto \tan^2\beta$. Note that both, the denominator in (10) and the $\tan\beta$-dependent factor in (11) are important for the correct resummation of the potentially big terms $\propto \bar{m}_{d_J} \tan^n\beta$ from all orders of perturbation calculus [13, 14, 18]. Because for $\tan\beta \gg 1$ $s_\alpha \approx 0$, it follows from (3) and (12) that only H^0 and A^0 couplings can violate flavour significantly.

- Threshold correction $\Delta_u \boldsymbol{Y}_d$ and the analogous correction $\Delta_d \boldsymbol{Y}_u$ to the couplings of the up-type quarks modify also the vertices of the charged Higgs boson which take the form [19, 20]

$$\begin{aligned}\mathcal{L}^{\rm eff} &= \frac{g}{\sqrt{2}M_W} \overline{u^J} \cdot V^{\rm eff}_{JI} \left[m_{u_J}(1-\varepsilon^{JI}_{\rm L}) \cot\beta\ P_{\rm L}\right. \\ &\quad \left. + \bar{m}_{d_I}(1-\varepsilon^{JI}_{\rm R}) \tan\beta\ P_{\rm R}\right] d^I H^+\,, \end{aligned} \tag{13}$$

  where the corrections $\varepsilon^{JI}_{\rm L,R} \propto \tan\beta$. Similar corrections to the charged Goldstone boson G^+ vanish in this approach [20].

The naive decoupling approach to the calculation of the FV H^0 and A^0 couplings described above allows for easy resummation of $\tan\beta$ enhanced terms to all orders. It has however some limitations. More complicated approach, which is not limited to the decoupling configuration $M_H \ll M_{\rm spart}$ and combines the resummation with the complete calculation of the one loop diagrams shown in Fig. 1 has been proposed in [20]. Its comparison with the simple approach described here shows that the latter is not very accurate. Firstly and most importantly, for split squarks belonging to different generations flavour dependence of ε_0 and $\varepsilon_{\rm Y}$ should be taken into account. Secondly, in some situations the approximation based on neglecting the electroweak couplings g and g' turns out to be a bad one. Nevertheless, the approach sketched here qualitatively captures the main features of radiative corrections for $\tan\beta \gg 1$.

Finally, it should be stressed that the generation of FV couplings of H^0 and A^0 is generic for SUSY with large $\tan\beta$ [15, 17]. However, in the case of non-minimal FV the correlation of such couplings responsible for $b \to s$, $b \to d$ and $s \to d$ transitions can be distinctly different from the one in the minimal FV.

3. Impact on $\bar{b} \to \bar{s}(\bar{d})l^+l^-$ transitions

For $\tan\beta \gg 1$ the most important for B-physics phenomenology is the generation of the H^0 and A^0 FV couplings:

$$\mathcal{L}^{\rm eff} = S^0 \bar{b}_{\rm R} \left[X_{\rm RL}\right]^{bs(d)} s_{\rm L}(d_{\rm L}) + S^0 \bar{b}_{\rm L} \left[X_{\rm LR}\right]^{bs(d)} s_{\rm R}(d_{\rm R}) \,, \tag{14}$$

where $S^0 = H^0$ or iA^0 and

$$\left[X_{\rm RL}\right]^{bs(d)} = V_{tb}^{\rm eff*} V_{ts(d)}^{\rm eff} \frac{\bar{m}_b \tan^2\beta}{(1+\tilde{\varepsilon}_b \tan\beta)(1+\varepsilon_0 \tan\beta)} \left(\frac{m_t}{M_W}\right)^2 \varepsilon_{\rm Y} \,. \tag{15}$$

The amplitudes $\left[X_{\rm LR}\right]^{bs(d)}$ of the transitions $s_{\rm R}(d_{\rm R}) \to b_{\rm L}$ are given by similar expressions but with $\bar{m}_b$ replaced by $\bar{m}_{s(d)}$ and are, therefore, suppressed. Despite this suppression, the amplitude of the transition $s_{\rm R} \to b_{\rm L}$ proves important for the B_s^0–$\bar{B}_s^0$ mixing [16, 18, 21]. Being $\propto \tan^2\beta$, the couplings $\left[X_{\rm RL}\right]^{bs(d)}$ are strongly enhanced for $\tan\beta \approx 50$. For $\mu > 0$, $\mu A_t > 0$ (when $\varepsilon_{\rm Y} > 0$, $\varepsilon_0 > 0$) this enhancement is partly compensated by the factors in the denominator [14]. In contrast, in the less plausible from the model building point of view scenario with $\mu < 0$ the factors in the denominator further increase the magnitude of $\left[X_{\rm RL}\right]^{bs(d)}$.

The first consequence of the couplings (15) is a substantial contribution of the neutral Higgs boson exchanges shown in Fig. 3 to the Wilson coefficients $C_{\rm S}$ (H^0 exchange) and $C_{\rm P}$ (A^0 exchange) of the operators

$$\mathcal{O}_S = \bar{m}_b(\bar{b}_{\rm R} s_{\rm L})(\bar{l}l) \,, \qquad \mathcal{O}_P = \bar{m}_b(\bar{b}_{\rm R} s_{\rm L})(\bar{l}\gamma^5 l) \tag{16}$$

of the effective Hamiltonian describing $\bar{b} \to \bar{s}ll$ transitions and similar operators of the effective Hamiltonian for the $\bar{b} \to \bar{d}ll$ transitions.

The most spectacular effects of the neutral Higgs boson exchange contributions to the Wilson coefficients $C_{\rm S}$ and $C_{\rm P}$ is the dramatic increase of the rates of the $B^0_{s,d} \to \mu^+\mu^-$ and $B^0_{s,d} \to \tau^+\tau^-$ decays [13, 15]. The contributions of $C_{\rm S}$ and $C_{\rm P}$ to the decay amplitudes behave as $\tan^3\beta/M_A^2$ and for $\tan\beta \gtrsim 30$ and not too heavy A^0 and H^0 dominate over the SM contributions of the Z^0 penguin and $W^\pm$ box. As a result, the branching

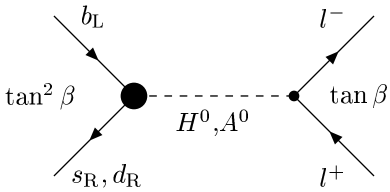

Fig. 3. Flavour changing neutral Higgs boson couplings contribution to the amplitude of the $B^0_{s,d} \to l^+l^-$ decays.

fractions for the experimentally investigated decays into muons, for which the SM predictions read (see *e.g.* [22])

$$\begin{aligned}
{\rm BR}(B^0_s \to \mu^+\mu^-) &= 3.8\times 10^{-9}\left(\frac{F_{B_s}}{238~{\rm MeV}}\right)^2\left(\frac{|V_{ts}|}{0.04}\right)^2 ,\\
{\rm BR}(B^0_d \to \mu^+\mu^-) &= 1.5\times 10^{-10}\left(\frac{F_{B_d}}{203~{\rm MeV}}\right)^2\left(\frac{|V_{td}|}{0.009}\right)^2
\end{aligned}$$

can be increased even by 3–4 orders of magnitude (depending on the magnitude of $\varepsilon_{\rm Y} \propto A_t$ and the sign of μ). This is illustrated in figure 4 for different values of the SUSY parameters.

The comparison of the complete calculation of [20] with naive resummation [14] based on the decoupling shows that for $\mu > 0$ the latter tends to underestimate the increase of ${\rm BR}(B^0_{s,d} \to \mu^+\mu^-)$ by a factor of ~ 1.5 while for $\mu < 0$ it leads to overestimation by a factor which can be as large as 5.

Similar conclusions apply also to the rate of the decay $B^0_d \to \mu^+\mu^-$. The only difference is that for each point of the MSSM parameter space the CKM matrix element $V^{\rm eff}_{td}$ has to be consistently determined by the standard unitarity triangle analysis (see *e.g.* [22]) taking into account the impact of the FV Higgs couplings on the B^0_s–$\bar{B}^0_s$ mass difference (see the next section). Using the naive scanning method as in [21] one finds $|V^{\rm eff}_{td}|$ in the range $(7\text{–}10)\times 10^{-3}$ with the smaller (bigger) values correlated with smaller (bigger) direct contribution of the scalar penguins to the $B^0_{d,s} \to \mu^+\mu^-$ amplitudes [16, 20].

The rate of the decay ${\rm BR}(B^0_s \to \mu^+\mu^-)$ turns out to be a powerful test allowing to discriminate different mechanisms of transmission of the supersymmetry breaking such as gravity mediation, anomaly mediation (AMSB), gugino mediation ($\tilde{g}$MSB) or gauge mediation (GMSB). It has been shown [23] that if $B^0_s \to \mu^+\mu^-$ is observed in the RunII of the Tevatron, that is if ${\rm BR}(B^0_s \to \mu^+\mu^-) \gtrsim 2\times 10^{-7}$, the scenarios AMSB, $\tilde{g}$MSB and GMSB with small number of the messenger fields will be disfavoured.

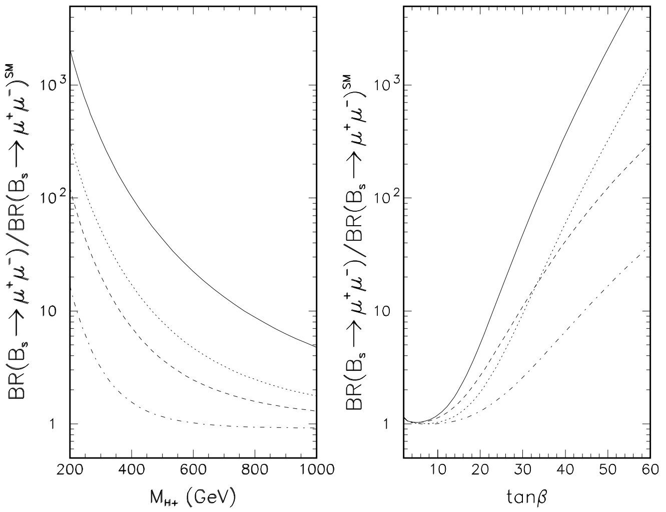

Fig. 4. Increase of the $B^0_s \to \mu^+\mu^-$ rate as a function of $M_{H^+} \approx M_{A^0}$ for $\tan\beta = 50$ and $\tan\beta$ for $M_{H^+} = 200$ GeV. The lighter chargino mass is 750 GeV, $M_2/|\mu| = 1$ and $m_{\tilde{g}} = 3M_2$. Solid and dashed (dotted and dash–dotted lines) correspond to $M_{\tilde{t}_1} = 500$ GeV and $M_{\tilde{t}_2} = 800$ GeV (600 and 700 GeV). $\mu < 0(> 0)$ for solid and dotted (dashed and dash–dotted) lines. The stop mixing angle ($\propto A_t$) is $+(-)10°$ for $\mu < 0(> 0)$.

In the general MFV MSSM (when the sparticle mass parameters are not correlated by some specific assumptions and the requirement of radiative electroweak symmetry breaking except for keeping masses of squarks of the first two generations equal) the well measured rate $\mathrm{BR}(\bar{B} \to X_s\gamma) = (3.23 \pm 0.42) \times 10^{-4}$ (world average, [2]) does not constrain[2] the possible increase of $\mathrm{BR}(B^0_{s,d} \to \mu^+\mu^-)$. The contributions of the neutral Higgs boson exchanges to the amplitude of the decay $B \to K\mu^+\mu^-$ can be non-negligible. However, in view of the uncertainties related to the theoretical calculations of rates of exclusive processes it is not clear whether the experimental result $\mathrm{BR}(B \to Kl^+l^-) = (7.6 \pm 1.8) \times 10^{-7}$ ($l = \mu, e$) [24] can be used to put interesting constrain on the strength of the FV H^0 and A^0 couplings [25]. More promising in this respect can be the inclusive mode $B \to X_s l^+l^-$ or the ratio $\mathrm{BR}(B \to X_s\mu^+\mu^-)/\mathrm{BR}(B \to X_s e^+e^-)$ (scalar penguin contributions to its denominator are negligible) [26] but the exper-

[2] In computing this rate we have also included the contribution

$$\propto \tan^3\beta (m_t/M_W)^2(\bar{m}_b/M_A)^2$$

generated by the couplings (15) $\bar{b}_{\mathrm{R}} s_{\mathrm{L}} H^0(A^0)$ [31].

imental uncertainty (BR($B \to X_s l^+ l^-$) = $(6 \pm 2) \times 10^{-6}$ for the dilepton invariant mass > 0.2 GeV [27]) is probably still too large. Thus, at present of all $|\Delta B| = 1$ processes the most stringent limits on the Wilson coefficients $C_{\rm S}$ and $C_{\rm P}$ and, hence, on the couplings (15) come from the upper limit BR($B^0_s \to \mu^+\mu^-$) $< 2.1 \times 10^{-6}$ [28] set by the CDF [25].

4. Impact on $\Delta B = 2$ transitions

As has been noted in [21] another important constraint on the magnitude of the FV couplings of neutral Higgs bosons comes from the $|\Delta B| = 2$ process — the B^0_s–$\bar{B}^0_s$ mixing. The effective Hamiltonian describing the mixing of neutral B mesons consists of eight operators [22]. In the SM as well as in the MSSM for $\tan\beta \lesssim 30$ only the Wilson coefficient of the operator $\mathcal{O}_{\rm VLL} = (\bar{b}_{\rm L}\gamma^\mu q_{\rm L})(\bar{q}_{\rm L}\gamma_\mu b_{\rm L})$ where $q = s$ or d is important. For large $\tan\beta$ potentially important become also the diagrams shown in Fig. 5, whose contribution to the Wilson coefficients of the operators $\mathcal{O}_{\rm SLL} = (\bar{b}_{\rm R} q_{\rm L})(\bar{q}_{\rm R} b_{\rm L})$, $\mathcal{O}_{\rm SRR} = (\bar{b}_{\rm L} q_{\rm R})(\bar{q}_{\rm L} b_{\rm R})$ and $\mathcal{O}_{\rm SLR} = (\bar{b}_{\rm R} q_{\rm L})(\bar{q}_{\rm L} b_{\rm R})$ is proportional to $\tan^4\beta$ [12]. However, as observed first in [13], $C_{\rm SLL}$ which is proportional to $\bar{m}^2_b$ is suppressed by the factor $1/M^2_{A^0} - 1/M^2_{h^0} \approx 0$ (the same suppression is present in $C_{\rm SRR}$ which is anyway negligible being proportional to $\bar{m}^2_s$ or $\bar{m}^2_d$). In turn $C_{\rm SLR}$ is proportional to $1/M^2_{A^0} + 1/M^2_{h^0}$ but it is suppressed by one light quark mass: $C_{\rm SLR} \propto \bar{m}_b \bar{m}_q$, where $q = s, d$. Despite this, it turns out it can be very important for the B^0_s–$\bar{B}^0_s$ mass difference [21]; numerically

$$C_{\rm SLR}(m_t) \approx -4.5 \times \left(\frac{200\ {\rm GeV}}{M^2_{A^0}}\right)^2 \left(\frac{\tan\beta}{50}\right)^4 \left(16\pi^2 \varepsilon_{\rm Y}\right)^2 . \tag{17}$$

This should be compared with the SM value of the $C_{\rm VLL}$ coefficient: $C_{\rm VLL}(m_t) \approx 9.5$. The importance of $C_{\rm SLR}$ is further increased by the QCD corrections: $C_{\rm SLR}(m_b) \approx 2.2 C_{\rm SLR}(m_t)$ whereas $C_{\rm VLL}(m_b) \approx 0.84 C_{\rm VLL}(m_t)$ [29].

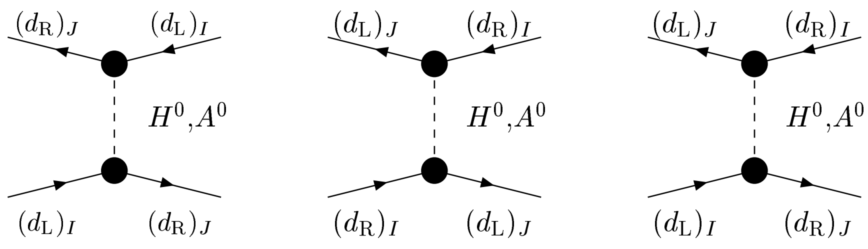

Fig. 5. Double penguin diagram contributing to $C_{\rm SLL}$, $C_{\rm SRR}$ and $C_{\rm SLR}$ Wilson coefficients, respectively in the MSSM with large $\tan\beta$.

If the CKM matrix is the only source of flavour violation the B_d^0–$\bar{B}_d^0$ and B_s^0–$\bar{B}_s^0$ mass differences can be written as [22]

$$\begin{aligned}\Delta M_{B_d} &\propto \hat{B}_{B_d} F_{B_d}^2 \left| V_{tb}^{\mathrm{eff}*} V_{td}^{\mathrm{eff}} \right|^2 |F_d| \\ \Delta M_{B_s} &\propto \hat{B}_{B_s} F_{B_s}^2 \left| V_{tb}^{\mathrm{eff}*} V_{ts}^{\mathrm{eff}} \right|^2 |F_s| \end{aligned} \tag{18}$$

where the factors $F_{d,s}$ depend on the Wilson coefficients C_{VLL}, C_{SLR}, *etc.* evaluated at the low energy scale. In the SM as well as in the MSSM for $\tan\beta < 20 - 30$, when only C_{VLL} Wilson coefficient is relevant, $F_d = F_s$ [9]. For $\tan\beta \gtrsim 30$ the value of F_s can be significantly modified by the contribution of C_{SLR}. An important feature of C_{SLL} is its always negative sign which results in $F_s < F_d \approx F_s^{\mathrm{SM}} = F_d^{\mathrm{SM}}$. This is illustrated in Fig. 6. For $\tan\beta \sim 50$, not too heavy Higgs sector and non-negligible stop mixing the scalar penguin contribution dominates over the SM and the charged Higgs boson box contributions and significantly decreases the value of F_s. Since $(\Delta M_s)^{\mathrm{exp}} > 14/\mathrm{ps}$ (and the factor $V_{tb}^{\mathrm{eff}*} V_{ts}^{\mathrm{eff}}$ is the same as in the SM), supersymmetric parameters leading to $|F_s/F_s^{\mathrm{SM}}| < 0.52$ are excluded [21] (a Bayesian treatment of uncertainties in (18) gives even stronger bound $|F_s/F_s^{\mathrm{SM}}| > 0.71$ [30]). For fixed masses of A^0 and H^0 this puts strong constraint on the FV coupling $[X_{\mathrm{RL}}]^{bs}$ (15). If all the FV is ruled by the CKM matrix the coupling $[X_{\mathrm{RL}}]^{bd}$ is also constrained because it differs from $[X_{\mathrm{RL}}]^{bs}$ only by $V_{ts}^{\mathrm{eff}} \to V_{td}^{\mathrm{eff}}$. As will be shown in the next section, this constraint eliminates part of the MSSM parameter space which is allowed by the CDF limit $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 2 \times 10^{-6}$.

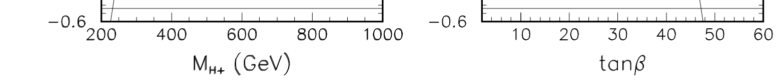

Fig. 6. F_s/F_s^{SM} as a function of $M_{H^+} \approx M_{A^0}$ for $\tan\beta = 50$ and $\tan\beta$ for $M_{H^+} =$ 200 GeV. Other supersymmetric parameters as in Fig. 4.

However, as the Fig. 6 shows, for large mass splitting of the two top squarks and large their mixing (that is large A_t) and/or $\mu < 0$ (when FV neutral Higgs boson couplings are enhanced by the denominator in the formula (15) rather than being suppressed) the contribution of the double penguin to $C_{\rm SLR}$ can reverse the sign of F_s yielding eventually ΔM_s compatible with the existing experimental limit. In this case the magnitude of the FV couplings is constrained only by the non-observation of the decay $B^0_s \to \mu^+\mu^-$.

5. Correlation of BR($B^0_{s,d} \to \mu^+\mu^-$) with ΔM_s

Since for $\tan\beta \gtrsim 30$ both, BR($B^0_s \to \mu^+\mu^-$) and ΔM_s receive important contributions depending on the FV coupling $[X_{\rm RL}]^{bs}$, these two observables must be strongly correlated in this regime. This correlation is clearly seen in Fig. 7(a) where we plot BR($B^0_{s,d} \to \mu^+\mu^-$) *versus* $\Delta M_s/(\Delta M_s)^{\rm SM}$ for $\tan\beta = 50$ and $M_A = M_H = 300$ GeV and a scan over the MSSM parameter space constrained by the requirement $M_{\rm spart} > 500$ GeV. Several comments should be made here. First of all, the two distinct bands of points correspond to $F_s > 0$ (the lower one) and $F_s < 0$ (the upper one). The upper band consists entirely of points with $\mu < 0$ so that the FV coupling (15) is enhanced by the denominator[3]. Most of these points are eliminated by the CDF limit [28] (marked in the figure) but this depends on the value of $\tan\beta$ and M_A. In general for $\tan\beta \gtrsim 30$ between the double penguin contribution $(\Delta M_s)^{\rm DP}$ to ΔM_s and the rate of the $B^0_s \to \mu^+\mu^-$ decay the following approximate relation holds [18, 20]:

$$\begin{aligned}
{\rm BR}(B^0_s \to \mu^+\mu^-) &\approx 10^{-6}\left(\frac{\tan\beta}{50}\right)^2\left(\frac{200~{\rm GeV}}{M_A}\right)^2\left|\frac{(\Delta M_s)^{\rm DP}}{12~{\rm ps}^{-1}}\right|,\\
(\Delta M_s)^{\rm DP} &\approx -\frac{12}{\rm ps}\left(\frac{\tan\beta}{50}\right)^4\left(\frac{m_t^4}{M_W^2 M_A^2}\right)\\
&\quad\times\left(\frac{16\pi^2\varepsilon_{\rm Y}}{(1+\tilde{\varepsilon}_b\tan\beta)(1+\varepsilon_0\tan\beta)}\right)^2. \qquad (19)
\end{aligned}$$

Therefore for the same $(\Delta M_s)^{\rm DP}$ the rate of $B^0_s \to \mu^+\mu^-$ is smaller for smaller $\tan\beta$ and/or heavier A^0.

For the more likely from the model building point of view points, for which $F_s > 0$ the lower limit $(\Delta M_s)^{\rm exp} > 14/{\rm ps}$ provides at present the

[3] We have rejected all points, for which BR($\bar{B} \to X_s\gamma$) is outside the experimentally allowed range. Therefore, for most of these points $A_t < 0$ (in our convention $A_t\mu > 0$ facilitates the cancellation of the chargino-stop and H^+-top contributions to the $b \to s\gamma$) amplitude). $A_t < 0$ may be difficult to obtain in the minimal SUGRA scenario unless $|A_t|$ at the GUT scale is not very large, but it is not excluded in general.

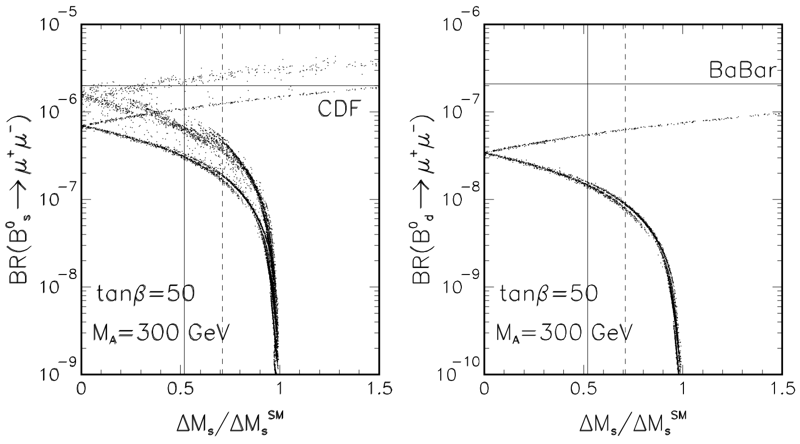

Fig. 7. Correlation of $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ (left panel) and $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ (right panel) with ΔM_s in the MFV MSSM for $\tan\beta = 50$ and $M_A = 300$ GeV.

strongest restriction on the FV violating coupling $[X_{\mathrm{RL}}]^{bs}$. For $\tan\beta < 50$ the bound $F_s/F^{\mathrm{SM}} > 0.52(0.7)$ then implies $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 1.2(0.8) \times 10^{-6}$. It has to be stressed, that for $\tan\beta \gtrsim 30$ the relation (19) depends essentially only on the existence of the FV coupling $[X_{\mathrm{RL}}]^{bs}$ and not on the particular source of FV in the MSSM. Since for small $\tan\beta$ no significant increase of $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ is possible [15] measuring its value much above the SM prediction and violating at the same time the above correlation would generally disfavour not only the large $\tan\beta$ scenario but the MSSM in general.

In contrast the approximate proportionality of $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ and $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ (as we have mentioned in Sec. 3, the element V_{td}^{eff} can depend weakly on the point in the MSSM parameters space and can deviate slightly from the value assumed in the SM whereas V_{ts}^{eff} is essentially fixed) and, consequently, also the correlation between $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ and ΔM_s shown in Fig. 7(b) is specific for the CKM matrix as the dominant source of flavour violation in the MSSM. In this scenario the experimental upper limit on $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ generally implies $\mathrm{BR}(B_d^0 \to \mu^+\mu^-) \lesssim 8 \times 10^{-8}$. For points, for which $F_s > 0$ the experimental lower bound $(\Delta M_s)^{\mathrm{exp}} > 14/ps$ sets even the stronger limit: $\mathrm{BR}(B_d^0 \to \mu^+\mu^-) \lesssim 3(2) \times 10^{-8}$. Breaking of the correlation of $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ with $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ and ΔM_s can be easily realized in the non-minimal flavour violation scenario, in which *e.g.* the chirality preserving mass insertions connecting the third generation with the first and the second ones are different. It has been checked [16, 17] that in such a case $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ can even exceed the present experimental upper limit $\mathrm{BR}(B_d^0 \to \mu^+\mu^-) < 2.1 \times 10^{-7}$ [32] respecting all other constraints on the magnitude of the (13) mass insertions. Therefore, finding the decay $B_d^0 \to \mu^+\mu^-$ with the branching fraction above 8×10^{-7} (and more likely, even above $3(2)\times 10^{-7}$) will be a clear sign of non-minimal flavour violation. The same conclusion will of course follow if both, $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ and $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$, are measured and their ratio differs significantly from the SM value of $|V_{td}^{\mathrm{eff}}/V_{ts}^{\mathrm{eff}}|^2$. The precise assessment will then depend, of course, on the statistical method used to determine how much the MSSM and the SM values of V_{td}^{eff} can differ.

Finally, the correlations of $\mathrm{BR}(B_{s,d}^0 \to \mu^+\mu^-)$ with ΔM_s may be also helpful in discriminating between the minimal and non-minimal flavour violation. The ideas of ref. [33] may be useful here but in general the possibility of more precise test of the minimal flavour violation in this way crucially depends on how much the uncertainties of the nonperturbative parameters F_{B_s}, B_{B_s}, *etc.* can be reduced in the nearest future.

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