The scale of SUSY breaking in models of inflation driven by the volume modulus

Marcin Badziak

16th June 2009

Institute of Theoretical Physics, University of Warsaw

based on JCAP 0807 (2008) 021 and JCAP 0902 (2009) 010

in collaboration with Marek Olechowski
Outline

- Motivations
- Constraints for the Kähler potential
- Model building
- Conclusions
**KKLT moduli stabilization**

F-term potential in 4D SUGRA:

\[ V = e^K \left( K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3 |W|^2 \right) \]

Kähler potential for the volume modulus:

\[ K = -3 \ln(T + \bar{T}) \]

For fixed dilaton and CSM fluxes contribute constant term to the superpotential:

\[ W = A \]

Introducing non-perturbative correction (e.g. gaugino condensation) to the superpotential:

\[ W = A + C e^{-cT} \]

volume modulus can be stabilized at AdS SUSY minimum.

We live in dS space \( \Rightarrow \) D3-branes introduced to uplift minimum to dS space:

\[ \Delta V = \frac{E}{(T + \bar{T})^2} \]
Hubble scale vs SUSY breaking scale

KKLT stabilization allow for constructing models of inflation within string theory e.g. racetrack inflation with 2 non-perturbative terms in the superpotential:

\[ W = A + Ce^{-cT} + De^{-dT} \]

In generic inflationary model based on KKLT moduli stabilization Hubble scale during inflation is related to the gravitino mass (Kallosh, Linde '04):

\[ H \lesssim m_{3/2} \]

If we insist on a low energy SUSY breaking, the relation \( H \lesssim m_{3/2} \) forces us to construct low scale inflationary models (\( H \sim \mathcal{O}(1\text{TeV}) \)) \( \Rightarrow \) very hard to find such models (no moduli inflation model of this type constructed so far)

Large \( m_{3/2} \) originates from a deep SUSY AdS minimum before uplifting \( \Rightarrow \) for the SUSY Minkowski minimum (\( m_{3/2} = 0 \)) there is no relation between \( H \) and \( m_{3/2} \)

SUSY "almost"-Minkowski minimum + small uplifting \( \Rightarrow m_{3/2} \ll H \)
Constraints on Kähler potential

Slow-roll inflation requires slow-roll parameter $|\eta| \ll 1$

The maximal value of $\eta_{\text{max}}$ is related to the curvature of the Kähler manifold spanned by the scalar fields appearing in the theory. (MB, Olechowski '08; Covi et al. '08)

The necessary condition for $|\eta| \ll 1$ (i.e. $\eta_{\text{max}} \gtrsim 0$):

$$R(f^i) < \frac{2}{\hat{G}^2} < \frac{2}{3}$$

where $G = K + \log |W|^2$ and $\hat{G}^2 \equiv \sqrt{G^i G_i} = 3 + e^{-G V}$

$R(f^i) \equiv R_{i j p q} f^i f^j f^p f^q$ is the sectional curvature along the direction of the SUSY breaking ($f_i \equiv G_i / \hat{G}^2$ is the unit vector defining that direction).

Note: $\hat{G}^2 = 3$ for Minkowski, $\hat{G}^2 > 3$ for de Sitter.

The above condition can be used to eliminate some models even without specifying the superpotential!

16th June 2009
Volume modulus as the inflaton

Kähler potential for the volume modulus:

\[ K = -3 \ln(T + \bar{T}) \]

The curvature scalar takes the form:

\[ R_T = \frac{2}{3} \]

The trace of the \( \eta \)-matrix is constant and negative:

\[ \eta_t^t + \eta_{\tau}^{\tau} = -\frac{4}{3} \]

where \( t = \text{Re} T \) and \( \tau = \text{Im} T \).

\[ \eta_{\text{max}} = -\frac{2}{3} \Rightarrow \text{No inflation for any superpotential} \Rightarrow \text{corrections to Kähler potential required} \]
Corrections to Kähler potential

The necessary condition for the positivity of the $\eta$-matrix trace:

$$R_T < 2/3$$

Sufficient condition:

$$R_T \leq 0$$

We consider Kähler potential with leading $\alpha'$-correction and string loop correction:

$$K = -3 \ln(T + \overline{T}) - \frac{\xi_{\alpha'}}{(T + \overline{T})^{3/2}} - \frac{\xi_{\text{loop}}}{(T + \overline{T})^2}$$

Curvature scalar for this setup reads:

$$R_T = \frac{2}{3} - \frac{35}{48} \frac{\xi_{\alpha'}}{(T + \overline{T})^{3/2}} - \frac{8}{3} \frac{\xi_{\text{loop}}}{(T + \overline{T})^2} + \ldots$$

Relatively small corrections could make trace of the $\eta$-matrix positive.
No inflation in Kallosh-Linde model

The superpotential in KL model reads:

\[ W = A + Ce^{-cT} + De^{-dT} \]

SUSY Minkowski minimum exists for fine-tuned value of \( A \):

\[ A = -C \left| \frac{cC}{dD} \right|^{\frac{c}{d-c}} - D \left| \frac{cC}{dD} \right|^{\frac{d}{d-c}} \]

SUSY Minkowski minimum occurs at:

\[ T_{\text{mink}} = t_{\text{mink}} = \frac{1}{c - d} \ln \left| \frac{cC}{dD} \right|, \quad \tau_{\text{mink}} = 0 \]

Inflation ending in SUSY Minkowski minimum cannot be realized in KL model even with the corrections to Kähler potential. (MB, Olechowski ’08)
Triple gaugino condensation model

The superpotential reads:

$$W = A + Be^{-bT} + Ce^{-cT} + De^{-dT}$$

Kähler potential with leading corrections:

$$K = -3 \ln(T + \bar{T}) - \frac{\bar{\xi}_{a'}}{(T + \bar{T})^{3/2}} - \frac{\bar{\xi}_{\text{loop}}}{(T + \bar{T})^2}$$

SUSY Minkowski conditions ($\partial_T W = W = 0$) cannot be solved analytically.
Solution is not unique. There are 2 types of solutions:

- $\tau_{\text{mink}} = 0$ and $A$ real $\rightarrow$ structure of the potential as in KL model $\rightarrow$ no inflation

- $\tau_{\text{mink}} \neq 0$ and $A$ complex $\rightarrow$ inflation can be realized only if $B$ ($C$ or $D$) complex
Inflationary potential

AdS minimum: $t_{\text{AdS}} = 104.646$, $\tau_{\text{AdS}} = -11.664$

SUSY Minkowski minimum: $t_{\text{mink}} = 104.473$, $\tau_{\text{mink}} = 10.885$

Inflationary saddle point: $t_{\text{saddle}} = 115.475$, $\tau_{\text{saddle}} = -0.183$
Fine-tuning

Triple gaugino condensation model is the first one that accommodates TeV-range gravitino mass and high scale of inflation but requires significant amount of fine-tuning:

Two parameters fine-tuned to (almost) cancel diagonal and off-diagonal entry of the $\eta$-matrix ⇒ one more fine-tuning than in typical models (e.g. racetrack inflation)

Is this additional tuning necessary in models with light gravitino?

NO, if parameters of $W$ are real and $\tau = 0$ during inflation ⇒ off-diagonal entry of the $\eta$-matrix vanishes automatically and one tuning is enough

Is it possible to construct such models?
Inflection point inflation

Inflation in the $t$-direction ($\tau = 0$) can occur in the vicinity of the inflection point.

Naturally realized with positive exponents in gaugino condensation terms.

Positive exponents may occur when gauge kinetic function takes the form:

$$f = w_S S + w_T T.$$  

Gaugino condensation generates:

$$W_{\text{np}} = B e^{-\frac{2\pi}{N} (w_S S + w_T T)}.$$

When $w_T < 0$ and dilaton $S$ is stabilized at higher scales, positive exponents appear in effective theory for the volume modulus:

$$W_{\text{np}}^{\text{eff}} = B^{\text{eff}} e^{b T}.$$  

Gauge kinetic functions of this type realized in string theory (Marchesano, Shiu '04; Cascales, Uranga '03; Lukas, Ovrut, Waldram '97)
Model building with positive exponents

We consider superpotential with 2 gaugino condensates:

\[ W = A + Ce^{cT} + De^{dT}. \]

- For \( c < 0, d < 0 \) (KL model):
  - SUSY AdS minimum
  - no inflation with light gravitino

- For \( c < 0, d > 0 \) and \( c > 0, d > 0 \):
  - nonSUSY AdS/dS minimum
  - inflection point inflation
What is the price for the working model?

Fine-tuning of one parameter (e.g. $C$) at the level of $10^{-5}$ (similar to racetrack inflation).

Stabilization of the $\tau$-direction through string corrections to tree-level Kähler potential.

Fine-tuning of the initial conditions for $t$ at the level of one percent (comparable to other models of small-field inflation).

**TeV-range gravitino mass** requires:

Fine-tuning of $A$ at the level of $10^{-5}$. 
Threshold corrections to gauge kinetic function

We consider superpotential with 1 gaugino condensate and threshold corrections:

\[ W = A + (C_0 + C_1 T)e^{cT}. \]

SUSY Minkowski minimum exists for fine-tuned value of \( A \):

\[ A = \frac{C_1}{c} \exp \left( -\frac{cC_0}{C_1} - 1 \right). \]

SUSY Minkowski minimum occurs at:

\[ T_{\text{Mink}} = -\frac{1}{c} - \frac{C_0}{C_1}. \]

For positive \( c \) inflection point inflation with light gravitino can be realized.

The same conditions for successful inflation as in double gaugino condensation model.

Very modest model → only 4 parameters in \( W \)

→ 1 parameter in \( K \) (to stabilize the \( \tau \) direction)
Negative exponents and inflation with heavy gravitino

Without positive exponents inflation possible only with uplifting (heavy gravitino)

\[ \Delta V = \frac{E}{t^2}. \]

Examples: → KL model \textit{(Linde, Westphal '07)}
→ Single gaugino condensation with threshold corrections
Fine-tuning and the overshooting problem

Fine-tuning of the potential and the initial conditions is inevitably related to the height of the barrier that protects the inflaton from overshooting Minkowski vacuum.

Uplifting is decreasing function of $t \Rightarrow$ maximum (before uplifting) necessarily much below 0 if the barrier is to be high (after uplifting)

Fine-tuning of the parameters in models with negative exponents (heavy gravitino) at least $10^{-8} \Rightarrow$ substantially bigger than in models with positive exponents (light gravitino)
Supersymmetric uplifting from matter field

In models with negative exponents non-supersymmetric uplifting can be substituted by SUSY breaking matter field sector (MB, Olechowski, in preparation):

\[
W_{\text{matter}} = c_0 + \mu^2 \Phi \\
K_{\text{matter}} = |\Phi|^2 - \frac{|\Phi|^4}{\Lambda^2}
\]

- \( \Lambda \rightarrow \infty \) - Polonyi model

\[m_\Phi \sim m_T \Rightarrow \text{the inflaton is a mixture of } T \text{ and } \Phi\]

**Advantage**: fine-tuning no longer related to the height of the barrier

- \( \Lambda \ll 1 \) - O’KKLT model

\[m_\Phi \sim \Lambda^{-1} \Rightarrow m_\Phi \gg m_T - \text{matter field decoupled from the inflationary dynamics but provides spontaneous SUSY breaking and uplifting} \]
Conclusions

• For the volume modulus, parameter $\eta$ necessarily smaller than $-2/3$ and inflation with TeV-range gravitino mass cannot be realized unless corrections to the leading Kähler potential are included.

• Even for corrected Kähler potential inflation cannot be realized in KL model with only two non-perturbative terms in the superpotential.

• Adding third non-perturbative term to the superpotential makes inflation possible but significant amount of fine-tuning is necessary.

• Positive exponents in non-perturbative terms help in realizing inflection point inflation with light gravitino. Double gaugino condensation or single one with threshold corrections are enough to realize successful models.
• Inflection point models with all exponents negative can be realized only with uplifting (heavy gravitino) and suffer from overshooting problem.

• To overcome overshooting problem in models with all exponents negative parameters has to be much more fine-tuned than in models with positive exponents where overshooting problem is absent.

• Fine-tuning in models with all exponents negative can be relaxed if SUSY is broken by the matter field with a mass comparable to the volume modulus (e.g. Polonyi model). In such case fine-tuning is not related to the height of the barrier and is comparable to the fine-tuning in models with positive exponents.