Grand F-theory Uplifts

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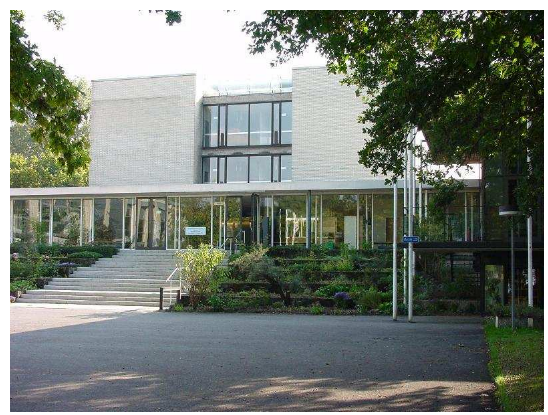


based on: (Bhg. , B. Jurke, T. Grimm and T. Weigand, arXiv:0906.0013)
substantial overlap with: (A. Collinucci, arXiv:0906.0003)

Grand Unification from String Theory

Grand Unification from String Theory

Realizations of GUTs from String Theory

- Weakly coupled $E_8 \times E_8$ Heterotic String (heterotic orbifolds)
  - $SU(N)$ bundles embedded into E_8
  - hierarchy $M_X/M_{\rm pl} \simeq 10^{-3} \Rightarrow$ "large" threshold corrections at M_X, anisotropic backgrounds
  - GUT breaking via discrete Wilson lines
- F-theory/Type IIB with 7-branes:
  - no exceptional groups via perturbative string
  - F-theory: non-perturbative states (string junctions) realize E_8 and its subgroups
  - hierarchy $M_X/M_{\rm pl} \simeq 10^{-3} \Rightarrow$ GUT brane wraps del Pezzo surface (exist a limit where gravity decouples)
  - GUT breaking via $U(1)_Y$ flux

Basic idea of F-Theory

Basic idea of F-Theory

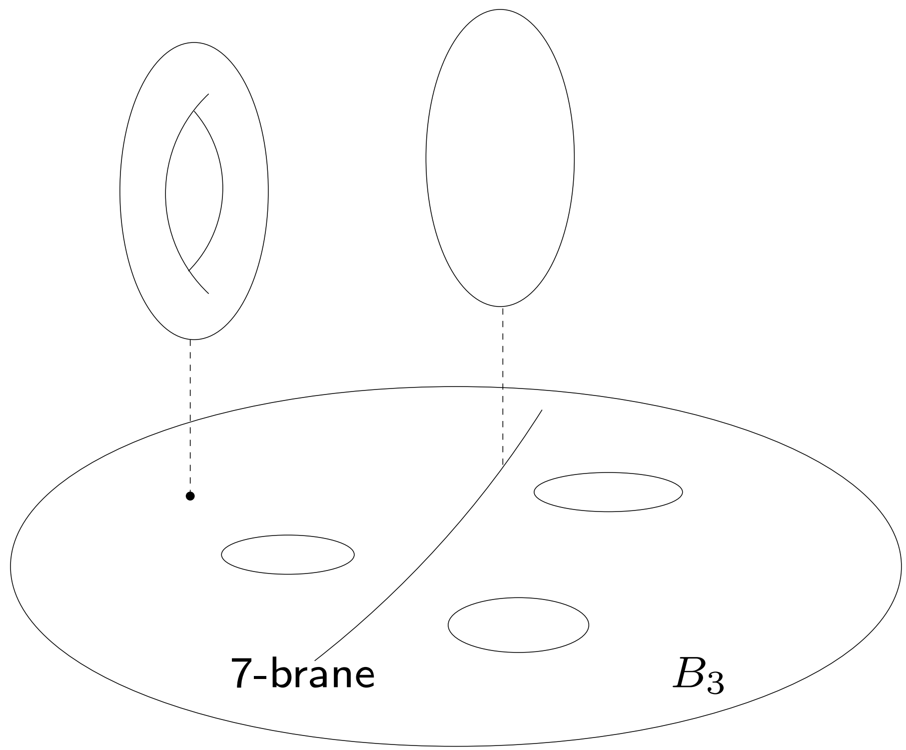
F-theory is a way of book-keeping of the positions of more general (p,q)-7-branes in Type IIB $\mathcal{N}=1$ compactifications

$$\text{elliptic fibration}: \qquad Y \to B_3$$

Susy $\to$ Y Calabi-Yau 4-fold. Elliptic curve $y^2 = x^3 + f(u)\,x + g(u)$ with complex structure:

$$\tau = C_0 + i\,e^{-\varphi}$$

with $j(\tau) = \frac{4(24)f^3}{4f^3+27\,g^2}$



(Vafa,Nucl.Phys.B469:403,1996), (Beasley, Heckman, Vafa, arXiv:0802.3391 & 0806.0102), (Donagi, Wijnholt, arXiv:0802.2969)

Grand Unification from F-theory

Grand Unification from F-theory

Working hypothesis: decoupling of GUT scale from Planck scale $\longrightarrow$ localization of GUT physics on del Pezzo surfaces (Beasley, Heckman, Vafa, arXiv:0806.0102)

Shortcomings of a purely local approach:

- Missing stringy global consistency conditions: landscape vs. swampland
- Physics of abelian gauge symmetries: Green-Schwarz mechanism, trivial cycles in B_3.
- closed string moduli stabilization: need to explain why susy breaking is subleading to gauge mediation (see F. Quevedo's talk)

series of recent papers: (Tatar, Watari, Donagi, Wijnholt, Beasley, Heckman, Vafa, Marsano, Saulina, Schäfer-Nameki, Plauschinn, Kane et. al.)

Grand Unification from F-theory

Grand Unification from F-theory

Program:

- Embed the local ideas into a global framework: F-theory on compact elliptically fibered four-folds
- Derivation of the global consistency conditions
- First step: Formulate the consistency conditions on much better understood global Type IIB orientifolds (T. Weigand's talk)
- Up-lift and generalize them to genuine F-theory models (this talk)
- Study of consequences of $U(1)_Y$ flux (Bhg, arXiv:0812.0248)
- Moduli stabilization $\longrightarrow$ gravity/moduli/gauge mediated supersymmetry breaking and "low" energy signatures at LHC

Creating del Pezzos

Creating del Pezzos

Identify geometries X that contain del Pezzo surfaces with $h_2(\mathrm{dP}_n, \mathbb{Z}) > h_2(X, \mathbb{Z})$. (Grimm, Klemm, 2008)

- Start with the Quintic $Q = \mathbb{P}^4[5]$ and choose the quintic constraint as

  $$x_5^2\, P_3(x_1, x_2, x_3, x_4) + x_5 P_4(x_1, \ldots, x_4) + P_5(x_1, \ldots, x_4) = 0$$

  At $(x_1, x_2, x_3, x_4, x_5) = (0, 0, 0, 0, 1)$ a del Pezzo singularity of the form $\mathrm{dP}_6 = \mathbb{P}^3[3]$ is generated.
- blow up singularity by pasting in a $\mathrm{dP}_6 \to Q_1^{\mathrm{dP}_6}$ with $(h^{1,1}, h^{2,1}) = (2, 90)$
- only a single $f \in H_2(\mathrm{dP}_6, \mathbb{Z})$ is non-trivial in $H_2(Q_1, \mathbb{Z})$ $\Rightarrow$ essential ingredient for massless $U(1)_Y$

Toric description

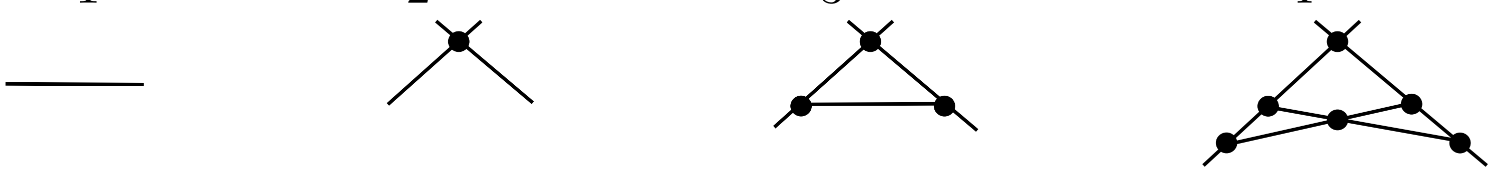
Toric description

Such geometries are naturally described in toric geometry:
scaling relations: $\{x_i\} \simeq \{\lambda^{q_a(x_i)} x_i\}$

	u_1	u_2	u_3	u_4	v	w	p
q_1	1	1	1	1	1	0	5
q_2	0	0	0	0	1	1	2
class	H	H	H	H	H +X	X	5H+2X

- Sequence of transitions:

$Q_1^{\mathrm{dP}_6}$ $\quad Q_2^{(\mathrm{dP}_7)^2}$ $\quad Q_3^{(\mathrm{dP}_8)^3}$ $\quad Q_4^{(\mathrm{dP}_9)^4}$



- new del Pezzos intersect in $\mathbb{P}^1$
- E_6 sublattice of each higher dP_n is trivial on Calabi-Yau

Toric description

Toric description

Toric data for $Q_2^{(\mathrm{dP}_7)^2}$:

	u_1	u_2	u_3	v_1	v_2	w_1	w_2	p
q_1	1	1	1	1	1	0	0	5
q_2	0	0	0	0	1	1	0	2
q_3	0	0	0	1	0	0	1	2
class	H	H	H	H+Y	H +X	X	Y	5H+2X+2Y

Orientifold projection

Orientifold projection

Specify orientifold projection $\Omega\sigma(-1)^{F_L}$: (Bhg. , V. Braun, T. Grimm and T. Weigand, arXiv:0811.2936)

- Type 1: reflection symmetry. For $Q_1^{\mathrm{dP}_6}$ consider

$$\sigma : v \longrightarrow -v$$

  requires that def. polynomial contains even powers of v, w, i.e. $P_{5,2} = p(u_i)_{3,0}\, v^2 + q(u_i)_{5,0}\, w^2 \Rightarrow$ O7-planes: $O7 = D_v + D_w$

- Type 2: exchange symmetry. For $Q_2^{(\mathrm{dP}_7)^2}$ consider

$$\sigma : v_1 \leftrightarrow v_2, \qquad w_1 \leftrightarrow w_2$$

  which exchanges the two dP_7. Using Q^2 and Q^3 the O7-plane wraps the surface $v_1\, w_1 = v_2\, w_2$ in the homol. class $[H + X + Y]$.

F-Theory uplift

F-Theory uplift

F-Theory: elliptic fibration Y over base B with degenerations over divisors $D_i \subset B$.

$$\sum_i n_i\, D_i = 12\, c_1(B)$$

with n_i: degree of zeroes of $\Delta = 4f^3 + 27g^2$.

- Y is smooth $\Leftrightarrow$ B is Fano, ($-K_B|_C > 0 \quad \forall$ curves C)
  $\Rightarrow$ only I_1 degenerations of fiber and
  $\chi^*(Y) = 12 \int_B c_1(B)\, c_2(B) + 360 \int_B c_1^3(B)$
- Non-abelian enhancement for proper singularities in Y
  $\Rightarrow \chi(Y) = \chi^*(Y) - \delta$
  For a singularity of type G along D:
  $\chi(Y) = \chi^*(Y) - r_G\, c_G\, (c_G + 1) \int_D c_1(D)^2$
  (Andreas, Curio, arXiv:0902.4143)

Sen limit

Sen limit

Connection to IIB orientifold on CY 3-fold X by Sen limit:

- General ansatz:
  $f = -3h^2 + \epsilon\eta, \qquad g = -2h^3 + \epsilon h\eta - \frac{\epsilon^2}{12}\chi$
- IIB limit: $\epsilon \to 0 \Rightarrow \Delta = -9\epsilon^2 h^2(\eta^2 - h\chi) + \mathcal{O}(\epsilon^3)$
  $O7: h = 0, \qquad D7: \eta^2 - h\chi = 0$
- X: double cover of base B branched over $h = 0$, simplest case: X given by equation $h = \xi^2$, orientifold $\xi \to -\xi$.

Uplift: reversal of Sen limit

- take $B = X/\sigma$ and consider Weierstrass fibration over B
- Check: $\chi(Y)/24 = \frac{\chi(D_{O7})}{12} + \sum_a N_a \frac{\chi(D_a)}{24}$
  for configuration of D7-branes on top of O7-plane

F-Theory uplift of Q_1

F-Theory uplift of Q_1

Construction of $B = X/\sigma$: (Collinucci, arXiv:0812.0175)
2-1 map $X \to B$: $(u_i, v, w) \mapsto (u_i, v^2, w^2) \equiv (u_i, \tilde{v}, \tilde{w})$

	u_1	u_2	u_3	u_4	$\tilde{v}$	$\tilde{w}$	p
Q_1	1	1	1	1	2	0	5
Q_2	0	0	0	0	1	1	1
class	P	P	P	P	2P +X	X	5P+X

- B is not Calabi-Yau: $K_B^{-1} = P + X$
- B can be analyzed with toric methods: $\chi(2P + X) = 55$, $\chi(X) = 9$, i.e. topology of O7-plane unchanged
- 4-fold Y: Weierstrass model: $y^2 = x^3 + x\, z^4\, f(u_i, \tilde{v}, \tilde{w}) + z^6\, g(u_i, \tilde{v}, \tilde{w})$

F-Theory uplift of Q_1

F-Theory uplift of Q_1

B is not Fano $\leftrightarrow$ generic appearance of singularities!

- Type IIB picture:
  Naively: cancel O7-tadpole by $1 \times [8D_v] + 1 \times [8D_v]$
  $\Rightarrow \chi^*(Y) = \frac{1}{2}(\chi_o(8\,D_v) + \chi_o(8\,D_w)) + 2\chi(O7) = 1728$
  But: dP_6 along $[D_w]$ is rigid $\Rightarrow \exists$ no single brane of charge $[8D_w] \Rightarrow$ minimal gauge group: $SO(1) \times SO(8)$
  $\Rightarrow$ branes $1 \times [8D_v] + 8 \times [D_w]$ with $\chi(Y) = 1224$
- F-theory picture: Naively:
  $\chi^*(Y) = 12 \int_B c_1(B)\, c_2(B) + 360 \int_B c_1^3(B) = 1728$ ✓
  But gauge enhancement $SO(8)$ along $D_{\tilde{w}}$:
  $\chi(Y) = \chi^*(Y) - r_{SO(8)}\, c_{SO(8)}\, (c_{SO(8)} + 1) \int_{D_{\tilde{v}}} c_1(D_{\tilde{v}})^2$
  $= 1228$ ✓

Tate algorithm

Tate algorithm

Gauge enhancements from Tate's algorithm:

$$y^2 + x\,y\,z\,a_1 + y\,z^3\,a_3 = x^3 + x^2\,z^2\,a_2 + x\,z^4\,a_4 + z^6\,a_6,$$

with $a_n \in H^0(B, K_B^{-n})$.

- Gauge group along $D \leftrightarrow$ order of zeroes of a_n and Δ
- O7-plane in Sen limit: $h = a_1^2 + 4a_2 \Rightarrow$ F-theory with orientifold limit $h = \tilde{v}\tilde{w} = 0 \Longleftrightarrow$
  $a_1 = p_1(\mathbf{u})\,\tilde{w}, \qquad a_2 = c_0\,\tilde{v}\,\tilde{w} - \frac{1}{4}p_1^2(\mathbf{u})\,\tilde{w}^2$
- generic choice of $a_1, a_2 \rightarrow$ inherently non-pert. vacua

Questions:

- Do general Type IIB configurations survive uplift?
- Does uplift of IIB geometry allow for interesting non-pert. gauge groups?

Tate algorithm

Tate algorithm

sing. type	discr. $\deg(\Delta)$	group enhancement	coefficient vanishing degrees a_1	a_2	a_3	a_4	a_6
I_0	0	—	0	0	0	0	0
I_1	1	—	0	0	1	1	1
I_2	2	$SU(2)$	0	0	1	1	2
$\mathrm{I}_3^{\mathrm{ns}}$	3	[unconv.]	0	0	2	2	3
$\mathrm{I}_3^{\mathrm{s}}$	3	[unconv.]	0	1	1	2	3
$\mathrm{I}_{2k}^{\mathrm{ns}}$	$2k$	$SP(2k)$	0	0	k	k	$2k$
$\mathrm{I}_{2k}^{\mathrm{s}}$	$2k$	$SU(2k)$	0	1	k	k	$2k$
$\mathrm{I}_{2k+1}^{\mathrm{ns}}$	$2k+1$	[unconv.]	0	0	$k+1$	$k+1$	$2k+1$
$\mathrm{I}_{2k+1}^{\mathrm{s}}$	$2k+1$	$SU(2k+1)$	0	1	k	$k+1$	$2k+1$
II	2	—	1	1	1	1	1
III	3	$SU(2)$	1	1	1	1	2
$\mathrm{IV}^{\mathrm{ns}}$	4	[unconv.]	1	1	1	2	2
IV^{s}	4	$SU(3)$	1	1	1	2	3
$\mathrm{I}_0^{*\,\mathrm{ns}}$	6	G_2	1	1	2	2	3

Tate algorithm

Tate algorithm

sing. type	discr. $\deg(\Delta)$	group enhancement	coefficient vanishing degrees				
			a_1	a_2	a_3	a_4	a_6
$\mathrm{I}_0^{*\,\mathrm{ss}}$	6	$SO(7)$	1	1	2	2	4
$\mathrm{I}_0^{*\,\mathrm{s}}$	6	$SO(8)^*$	1	1	2	2	4
$\mathrm{I}_1^{*\,\mathrm{ns}}$	7	$SO(9)$	1	1	2	3	4
$\mathrm{I}_1^{*\,\mathrm{s}}$	7	$SO(10)$	1	1	2	3	5
$\mathrm{I}_2^{*\,\mathrm{ns}}$	8	$SO(11)$	1	1	3	3	5
$\mathrm{I}_2^{*\,\mathrm{s}}$	8	$SO(12)^*$	1	1	3	3	5
$\mathrm{I}_{2k-3}^{*\,\mathrm{ns}}$	$2k+3$	$SO(4k+1)$	1	1	k	$k+1$	$2k$
$\mathrm{I}_{2k-3}^{*\,\mathrm{s}}$	$2k+3$	$SO(4k+2)$	1	1	k	$k+1$	$2k+1$
$\mathrm{I}_{2k-2}^{*\,\mathrm{ns}}$	$2k+4$	$SO(4k+3)$	1	1	$k+1$	$k+1$	$2k+1$
$\mathrm{I}_{2k-2}^{*\,\mathrm{s}}$	$2k+4$	$SO(4k+4)^*$	1	1	$k+1$	$k+1$	$2k+1$
$\mathrm{IV}^{*\,\mathrm{ns}}$	8	F_4	1	2	2	3	4
$\mathrm{IV}^{*\,\mathrm{s}}$	8	E_6	1	2	2	3	5
III^*	9	E_7	1	2	3	3	5
II^*	10	E_8	1	2	3	4	5

Observations

Observations

- non-local cancellation:
  IIB: $8 \times [D_{u_1}] + 16 \times [D_w] \Rightarrow G = SO(8) \times SP(16)$
  F-theory: $a_1 = p_1(\mathbf{u})\,\tilde{w}$, $a_2 = \tilde{v}\,\tilde{w} - \frac{1}{4}\big(p_1(\mathbf{u})\,\tilde{w}\big)^2$,
  $$a_3 = 0, \quad a_4 = c\,u_1^4 \tilde{w}^4, \quad a_6 = 0$$
  discriminants: $\Delta_\epsilon \simeq \epsilon^2\,u_1^8\,\tilde{w}^{10}\,\tilde{v}^2$,
  $$\Delta_F \simeq \epsilon^2 u_1^8\,\tilde{w}^{10}\,(\tilde{v}^2 - \epsilon 4 c u_1^4 \tilde{w}^2)$$
  non-pert. splitting of part of the O7-plane along $D_{\tilde{v}}$
- minimal gauge group:
  IIB: $1 \times [8D_w] + 8 \times [D_v] \Rightarrow G = SO(8)$
  F-theory : all $a_n = \tilde{w}^{d_n}(\ldots)$ with
  $$(d_1, d_2, d_3, d_4, d_6) = (1, 1, 2, 2, 3)$$
  $\Delta_F = \tilde{w}^6 \Rightarrow G = G_2$
  Sen limit: discard $a_6/a_i \to 0 \Rightarrow G_2 \to SO(8)$

Exceptional enhancements

Exceptional enhancements

Off the orientifold locus: exceptional gauge enhancements possible!

Example: E_6 along dP_6 $D_{\tilde{w}}$:

- E_6 along surface $\tilde{w} = 0$:

$$\begin{aligned} a_1 &= p_{(1,0)}\,\tilde{w}, \quad a_2 = p_{(2,0)}\,\tilde{w}^2, \quad a_3 = p_{(3,1)}\tilde{w}^2, \\ a_4 &= p_{(4,1)}\,\tilde{w}^3, \quad a_6 = p_{(6,1)}\,\tilde{w}^5 \end{aligned}$$

- E_7 on curve $\tilde{w} = p_{3,1} = 0$: **27** matter
- E_8 at point $\tilde{w} = p_{3,1} = p_{4,1} = 0$: $\mathbf{27}^3$ Yukawa

Q: Can one also realize $\mathbf{16}$ of $SO(10)$ and Yukawa $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ of $SU(5)$?

A: No, as $\tilde{w} = p_{2,1} = 0$ is in the Stanley-Reisner ideal!

F-Theory uplift of Q_2

F-Theory uplift of Q_2

Consider now: $Q_2^{(\mathrm{dP}_7)^2}$ with exchange involution ($h^-_{1,1} = 1$):
$\sigma : v_1 \leftrightarrow v_2, w_1 \leftrightarrow w_2$ and O7-plane: $v_1 v_2 - w_1 w_2 = 0$

Define $2-1$ map:
$(u_i, v_1, v_2, w_1, w_2) \mapsto (u_i, v_1\, v_2, w_1 w_2, v_1 w_1 + v_2 w_2) \equiv (u_i, v, w, h)$

toric data:

	u_1	u_2	u_3	v	h	w	p
Q_1	1	1	1	2	1	0	5
Q_2	0	0	0	1	1	1	2
class	P	P	P	2P+X	P+X	X	5P+2X

We have lost one divisor and one equivalence relation.

Spinors of $SO(10)$

Spinors of $SO(10)$

- B is non-Fano, as there exists a singular curve on O7
- In the Tate form, a_1 and a_2 can have more terms
  $a_1 = c_h\, h + p_1(\mathbf{u})\, w$
  $a_2 = c_0\, v\, w + c_{h^2}\, h^2 + q_1(\mathbf{u})\, h\, w + p_2(\mathbf{u})\, w^2$.
  Orientifold uplift: $b_2 = \eta(h^2 - 4v\, w) \Rightarrow$ restricts a_2.
- $SO(10)$ enhancement on divisor D_w:
  $$a_1 = p_{(1,0)}\,\tilde{w}, \quad a_2 = p_{(2,1)}\,\tilde{w}, \quad a_3 = p_{(3,1)}\,\tilde{w}^2,$$
  $$a_4 = p_{(4,1)}\,\tilde{w}^3, \quad a_6 = p_{(6,1)}\,\tilde{w}^5,$$
- E_6 enhancement over $g = 1$ curve $w = p_{(2,1)} = 0$: spinors $\mathbf{16}$ of $SO(10)$
- $SO(12)$ enhancement on $g = 4$ curve $\{w = p_{(3,1)} = 0\}$: Higgs fields in the $\mathbf{10}$ representation
- E_7 enhancement over the 6 points $\{w = p_{(2,1)} = p_{(3,1)} = 0\}$: $\mathbf{16}\,\mathbf{16}\,\mathbf{10}$ Yukawa coupling

SU(5) enhancement

SU(5) enhancement

SU(5) GUTs (away from IIB limit):

- $SU(5)$ enhancement over $w = 0$:

$$a_1 = p_{(1,1)}, \quad a_2 = p_{(2,1)}\, w, \quad a_3 = p_{(3,1)}, w^2,$$

$$a_4 = p_{(4,1)} w^3, \quad a_6 = p_{(5,0)}\, w^6$$

- $SO(10)$ enhancement over curve $\{w = p_{(1,1)} = 0\}$: matter in $\mathbf{10}$
- $SU(6)$ enhancement over curve $\{w = p_{(3,1)} = 0\}$: matter in $\overline{\mathbf{5}}$
- $SO(12)$ enhancement over the point $\{w = p_{(1,1)} = p_{(3,1)} = 0\}$: Yukawa $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}$
- E_6 enhancement over $\{w = p_{(1,1)} = p_{(2,1)} = 0\}$: Yukawa $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$, but: $X(P+X)(2P+X) = 0$

Conclusions

Conclusions

F-theory/IIB orientifold compactifications provide new and very promising string theory realizations of $SU(5)$ GUTs:

- Better understanding of CY 4-folds and its degenerations $\longrightarrow$ F-theory uplift of orientifold models
- Uplifted orientifold geometries can lead to E_8 structures on general F-theory moduli space. Challenge: $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ Yukawa
- Systematic searches for global GUTs, include G_4 flux in F-theory
- Include moduli stabilization in the construction, susy breaking (see F. Quevedo's talk)
- Implications for the so-called landscape of string vacua $\longrightarrow$ deconstruction of landscape philosophy?

Reminder

Reminder

You are all welcome to apply for (deadline June 15!)

- KITP workshop: Strings at the LHC and in the early Universe: R. Blumenhagen, M. Cvetic, P. Langacker, H. Verlinde, Santa Barbara, March 8 - May 14, 2010