Local Phenomenological Models in M-Theory and F-Theory

Jacob L. Bourjaily
Princeton University & IAS

[arXiv:0905.0142],
[arXiv:0901.3785],
[arXiv:0706.3364], [arXiv:0804.1132],
[arXiv:0704.0445], and [arXiv:0704.0444]

String Phenomenology 2009, Warsaw

Outline

An Engineer's Toolbox: the Basic Building Blocks

- Gauge symmetries arise via co-dim 4 'ADE'-type orbifold singularities

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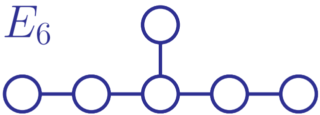

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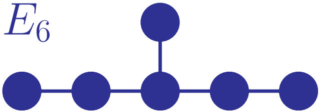

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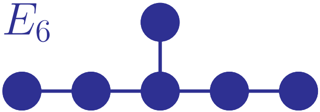

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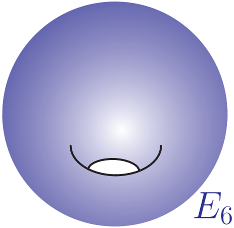
- M-theory: these are **real three-folds** in a G_2-manifold

F-Theory



E_6-gauge theory
via 'E_6'-singularities
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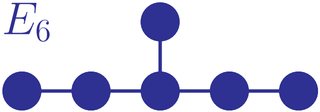

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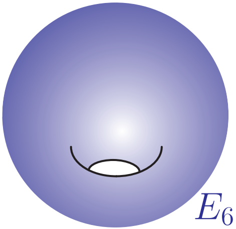

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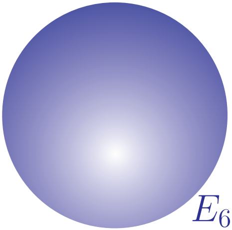

E_6-gauge theory via 'E_6'-singularities along a Poincaré-'S^3'

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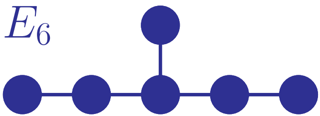

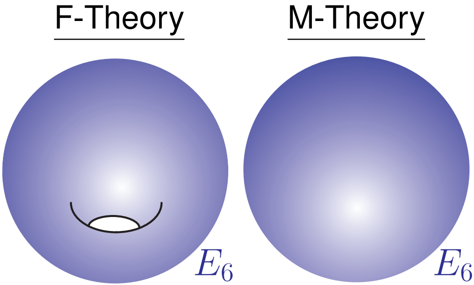

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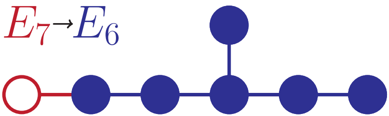

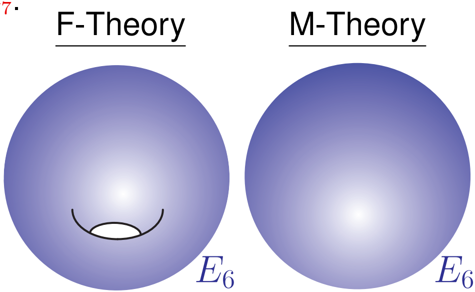

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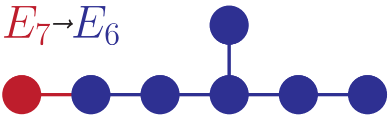

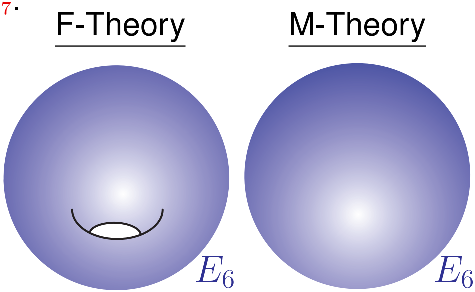

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- only curves with non-vanishing flux generate chiral matter, making 'exotic' matter curves relatively easy to ignore.
  - allows a single curve to support multiple 'generations'
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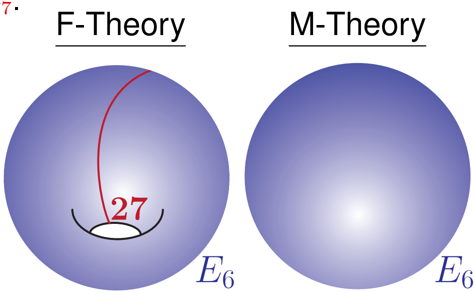

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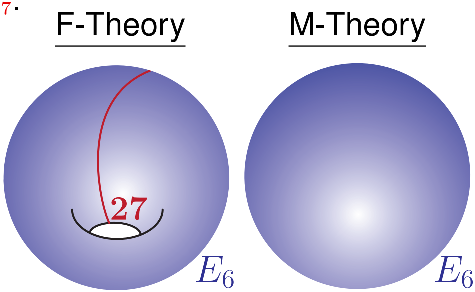

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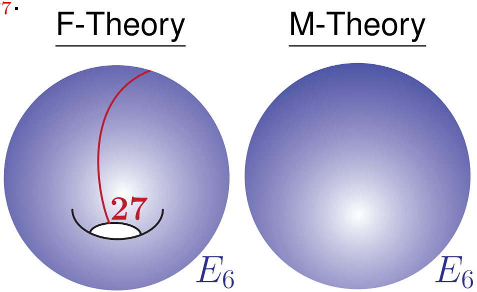

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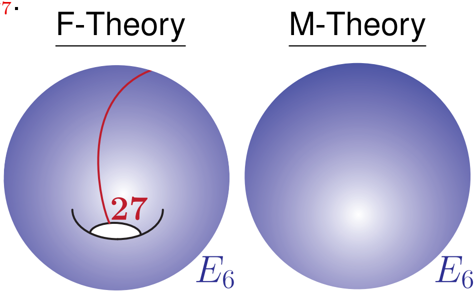

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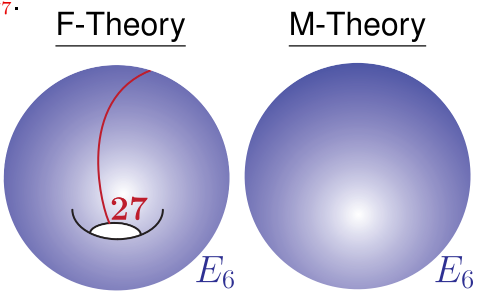

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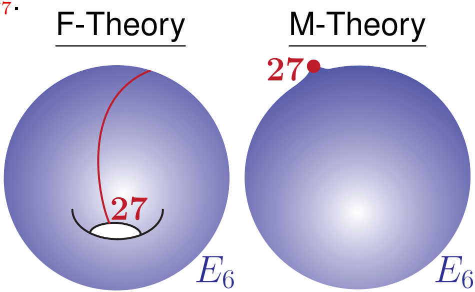

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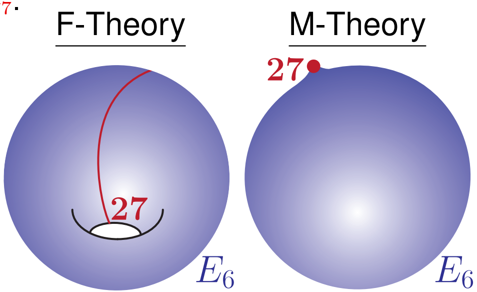

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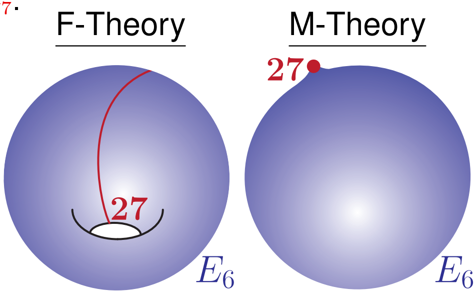

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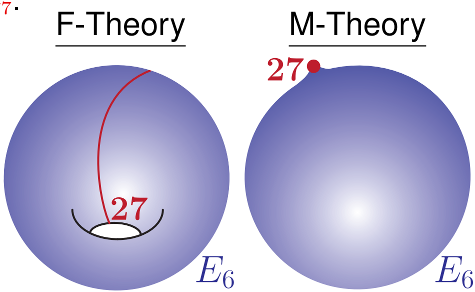

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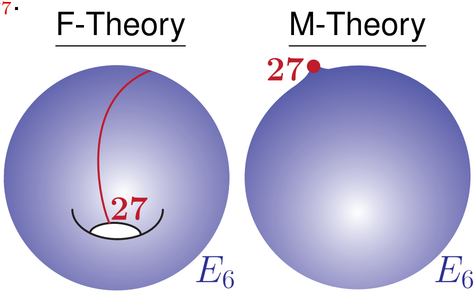

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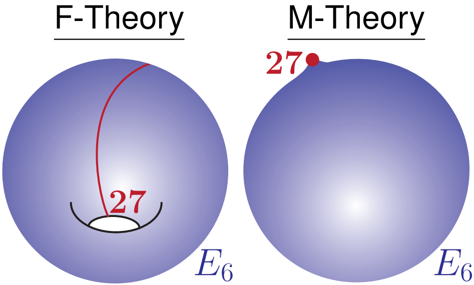

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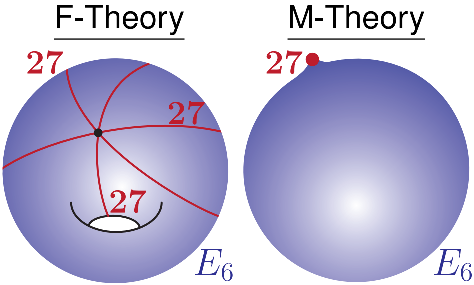

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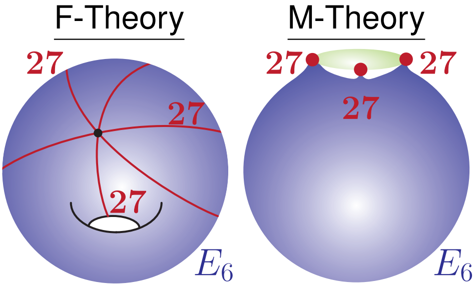

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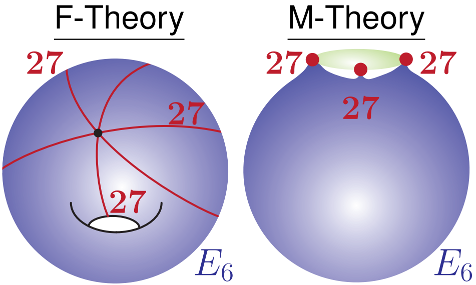

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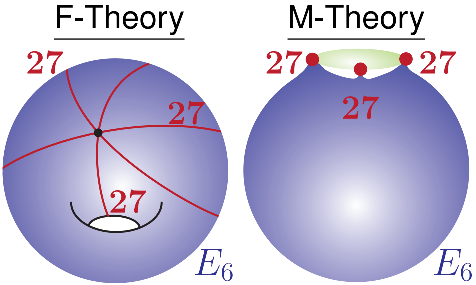

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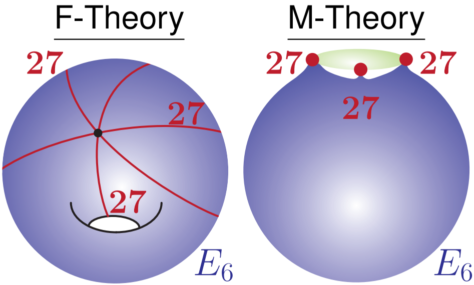

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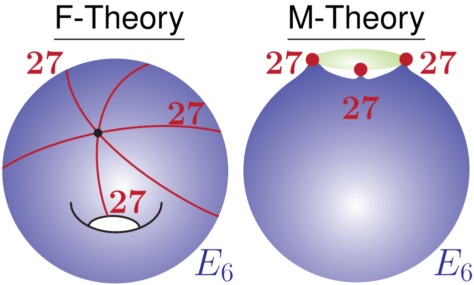
- sparse
  - ubiquitous U_1's: think of the extra 'flavour-branes' which must intersect along each matter-curve;
- hierarchical
  - coefficients are typically exponentially suppressed



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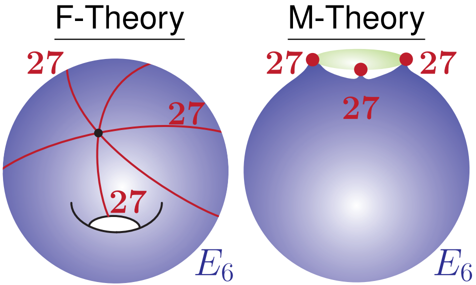

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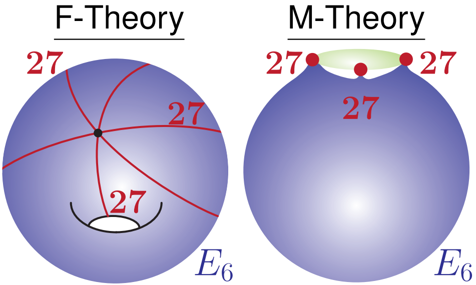
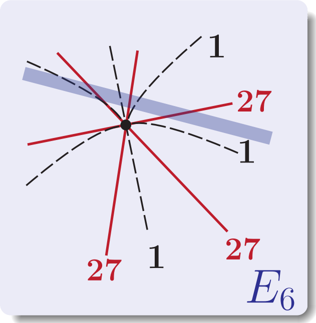


Correspondence of Local *Geometries* for F-Theory and M-Theory

- For any local model, the most important topological data to have is:
    - a list of all (potentially-massless-)matter-supporting singularities, and
    - the selection rules which determine how matter living along these singularities can interact in the theory
- This data can be encoded in F-theory by a cartoon-collection of mutually-intersecting matter-curves



- These models are constructed explicitly as ALE-fibrations over an appropriate base W, e.g. $\widehat{E_8}(a(W), b(W), 0, 0, 0, 0, 0, 0)$
- Any local geometry constructed in this way for F-theory can be immediately translated into a corresponding geometry for M-theory, with the same matter-singularities and interactions
- It is not hard to classify all possible 'cartoons' that can arise from a local ALE-fibration leading to an extremely small landscape of possibilities

Group-Theoretic Classification of Local Fibrations

- Recall that the new light degrees of freedom arising from an ALE-fibration are due to 2-branes wrapping vanishing cycles within the ALE-fibres.
- The correspondence between two-cycles in the ALE-space $\widehat{G}$ and the root lattice of the ADE-group G then leads to the following:
  - A generic* $\widehat{G}$-fibred Calabi-Yau four-fold or $\widehat{G}$-fibred G_2 manifold for which the typical fibre has a singularity of type $H \subset G$ will have one matter-singularity for each vector-like representation in the branching:

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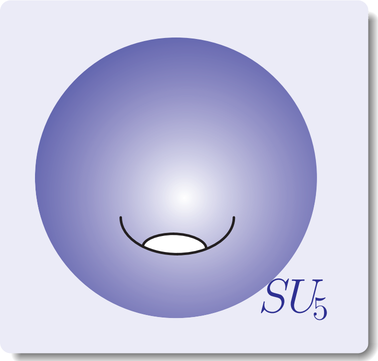
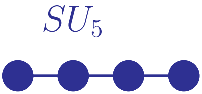


Spectrum of Matter-Curves



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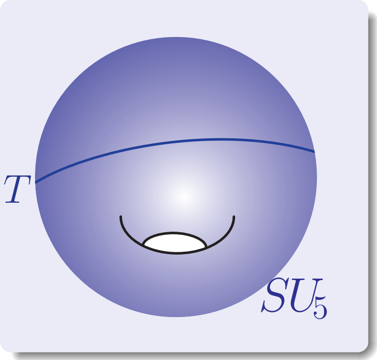
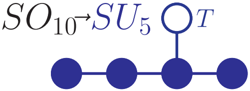


Spectrum of Matter-Curves

$\mathbf{10}$
T



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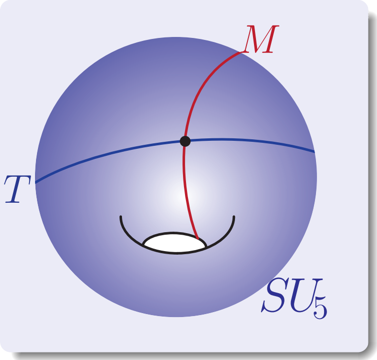
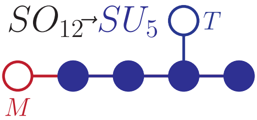


Spectrum of Matter-Curves

$\mathbf{10}$	$\mathbf{\overline{5}}$
T	M



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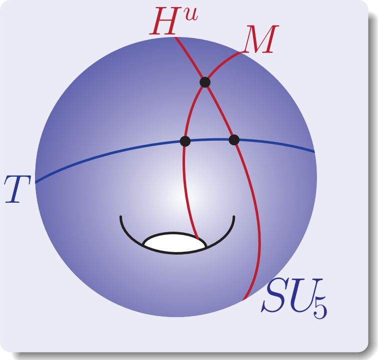

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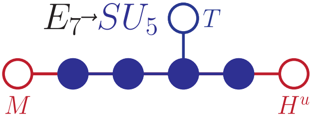
$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$
T	M	H^u



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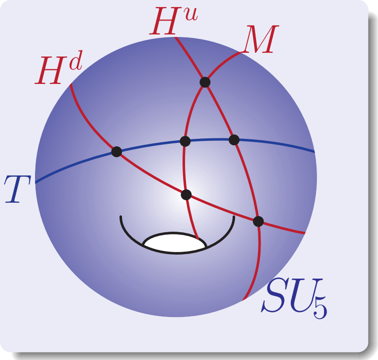

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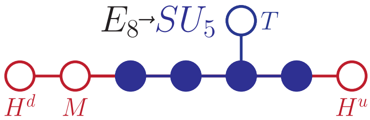
T	M	H^u	H^d



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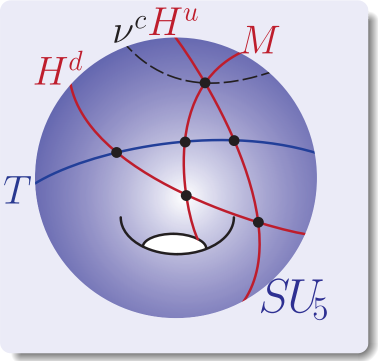

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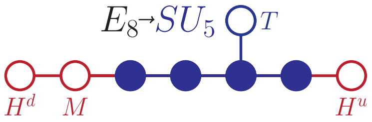
$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
T	M	H^u	H^d	ν^c



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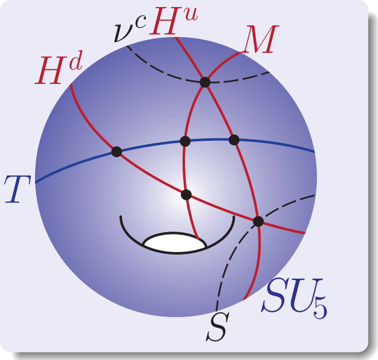

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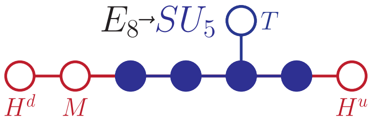
T	M	H^u	H^d	ν^c	S



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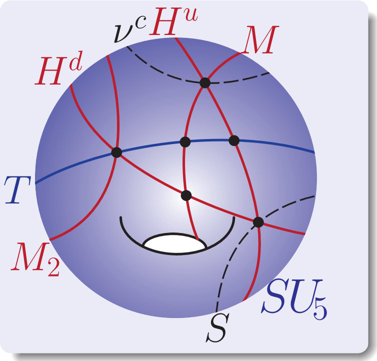

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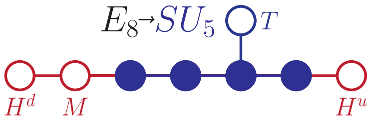
T	M	H^u	H^d	ν^c	S
	M_2				



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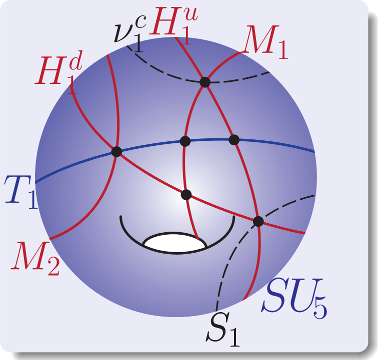

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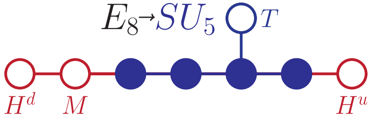
	M_2				



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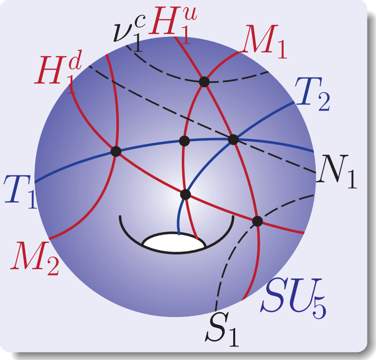

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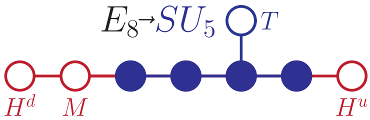
T_2	M_2					



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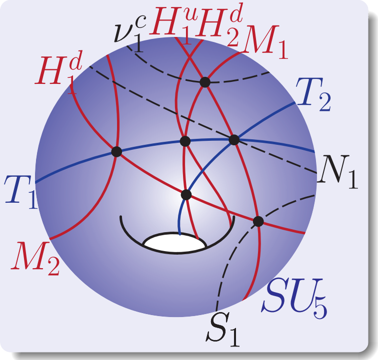
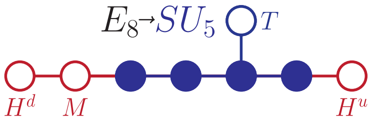


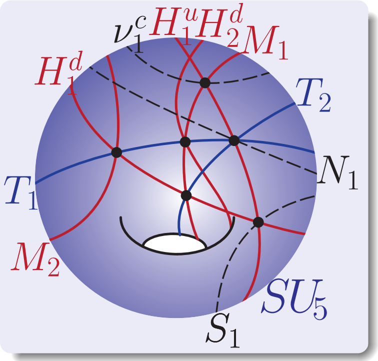
Spectrum of Matter-Curves

$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
T_1	M_1	H_1^u	H_1^d	ν_1^c	S_1	N_1
T_2	M_2		H_2^d			



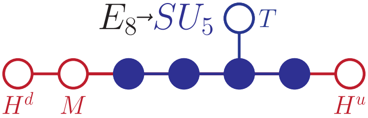
The *Minimality* of E_8 and the Origin of Three Families

- Because the structure of ALE-fibres plays such a critical role in phenomenology, we are naturally led to ask:
  - What is the lowest-rank ALE-fibrtation $\widehat{G}(f_1(W),\ldots,f_n(W))$ that is capable of (typically*) giving the interactions of even a single family?



Spectrum of Matter-Curves

$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
T_1	M_1	H_1^u	H_1^d	ν_1^c	S_1	N_1
T_2	M_2	H_2^u	H_2^d	ν_2^c	S_2	N_2
T_3	M_3					
T_X						



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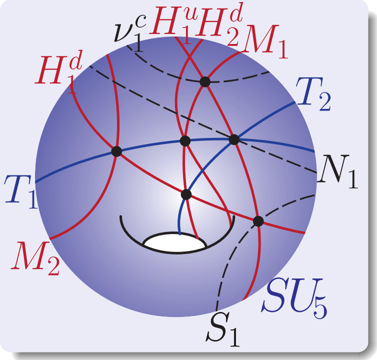

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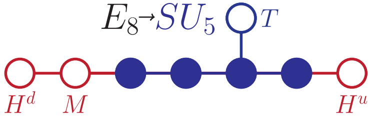
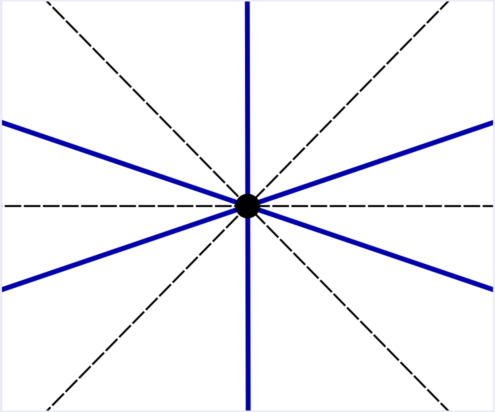
$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
T_1	M_1	H_1^u	H_1^d	ν_1^c	S_1	N_1
T_2	M_2	H_2^u	H_2^d	ν_2^c	S_2	N_2
T_3	M_3	H_3^u	H_3^d	ν_3^c	S_3	N_3
T_X	M_X		ν_X^c			
T_X^c						



‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $E_6 \to SO_{10}$



	$E_6 \times$	$U_1^a \times$	U_1^b
T_1	$\mathbf{27}$	1	1
T_2	$\mathbf{27}$	1	-1
T_3	$\mathbf{27}$	-2	0
S_1	$\mathbf{1}$	3	-1
S_3	$\mathbf{1}$	3	1
S_2	$\mathbf{1}$	0	2

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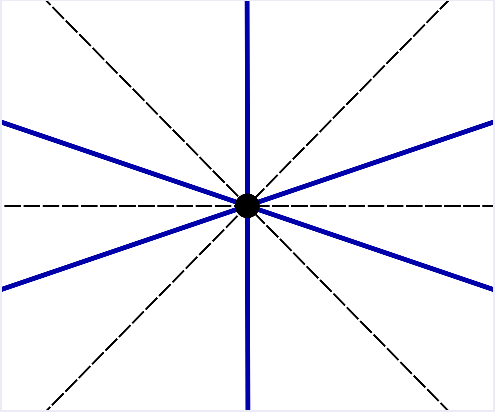

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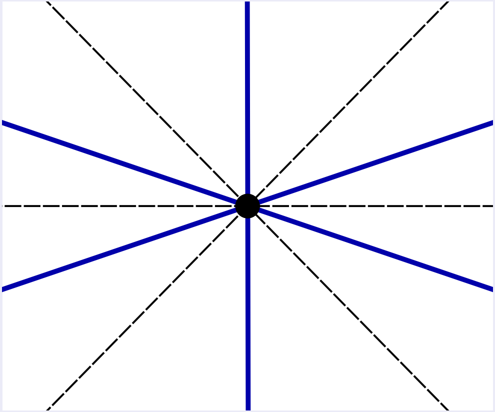

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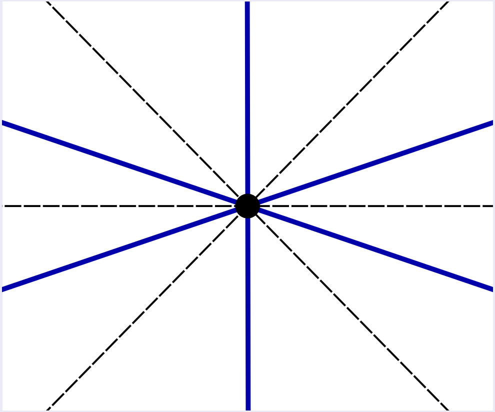

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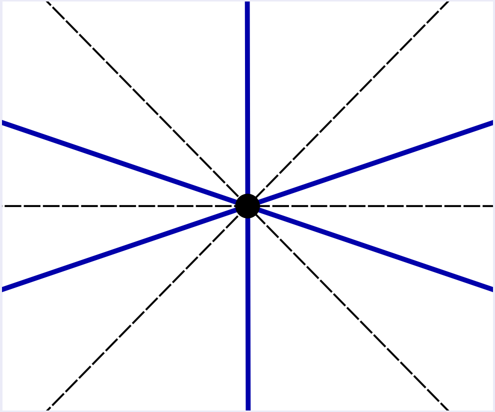

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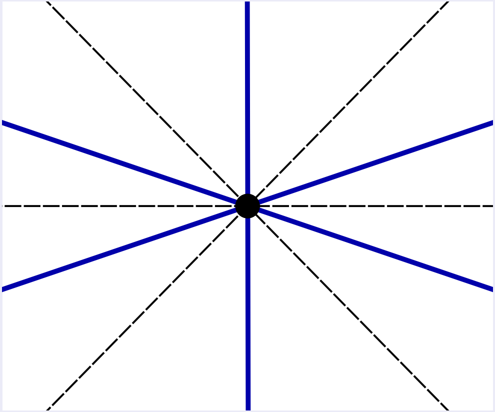

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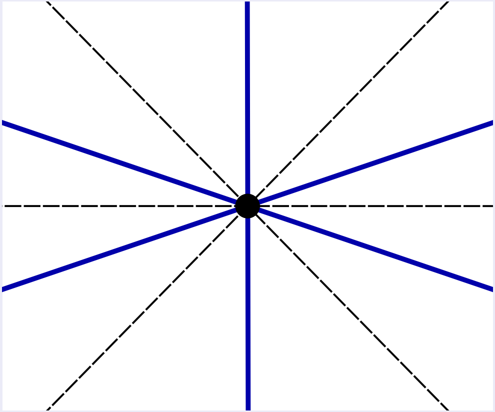

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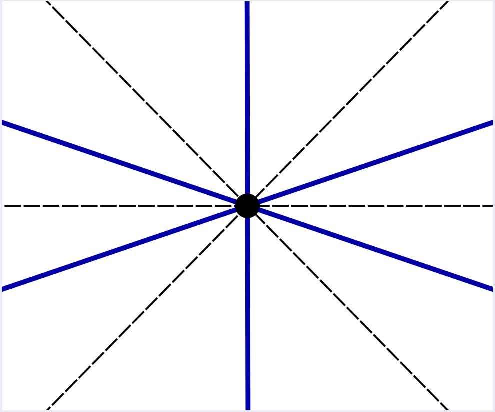

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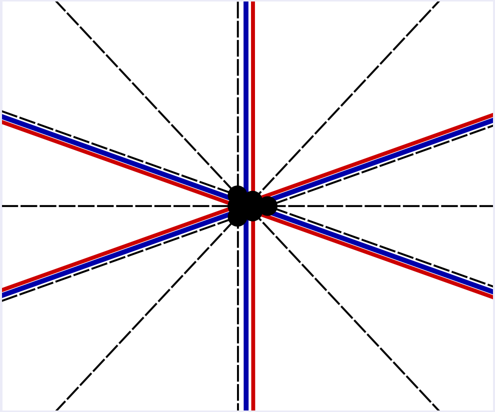

	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	−1
T_2	**16**	1	−1	−1
T_3	**16**	−2	0	−1
T_X^c	$\overline{\mathbf{16}}$	0	0	−3
H	**10**	1	1	2
Y_a	**10**	−1	1	−2
Y_b	**10**	2	0	−2
X_1	**1**	−1	−1	4
X_2	**1**	−1	1	4
N_2^c	**1**	−2	0	−4
N_3^c	**1**	3	−1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

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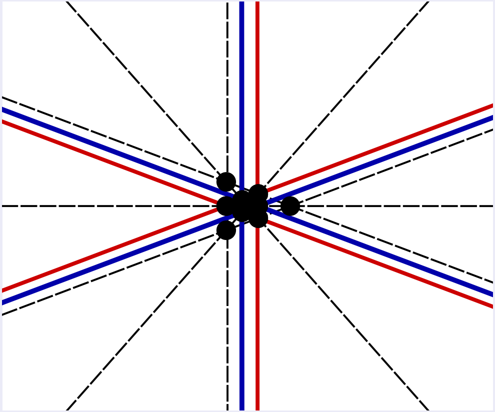

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T_1	**16**	1	1	−1
T_2	**16**	1	−1	−1
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Y_a	**10**	−1	1	−2
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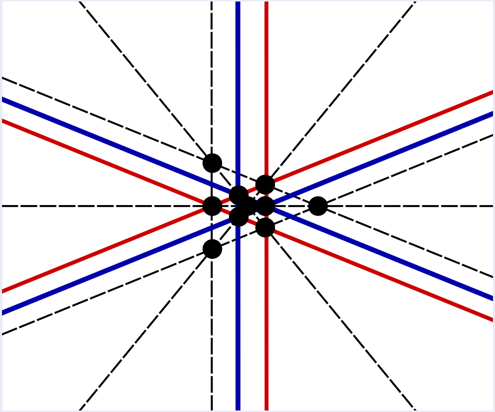

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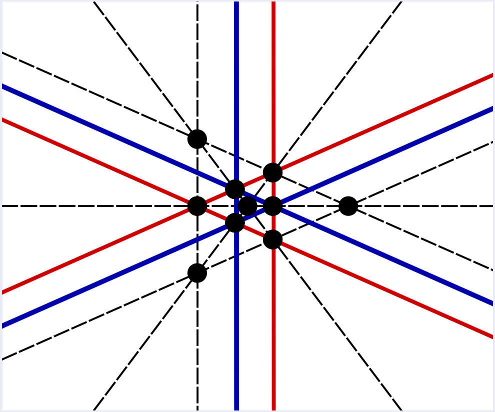

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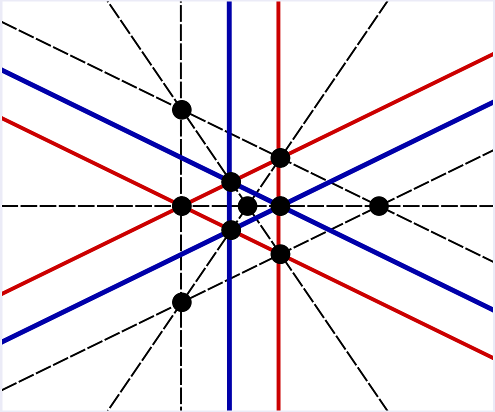

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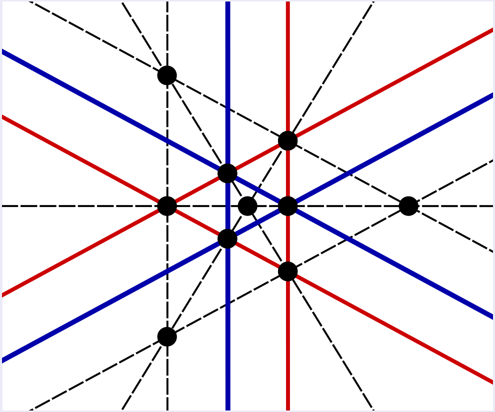

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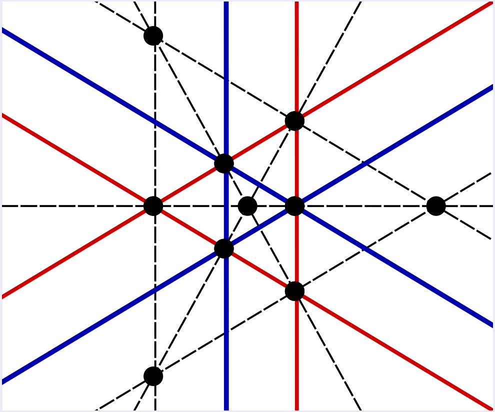

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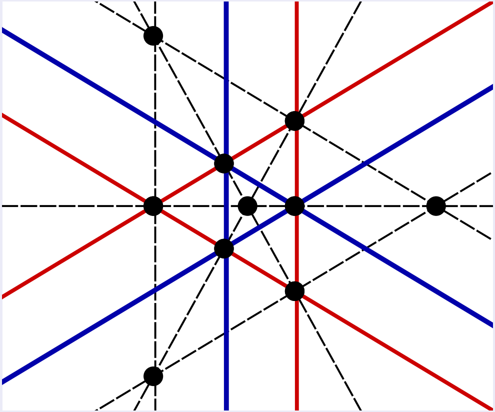

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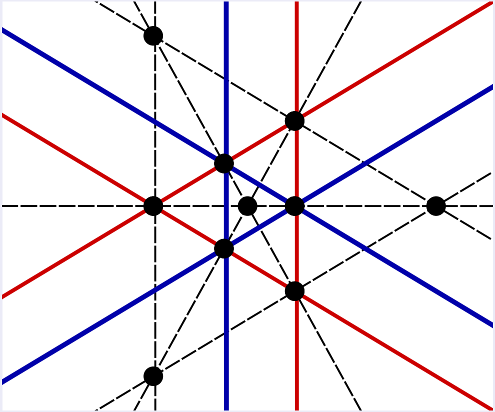

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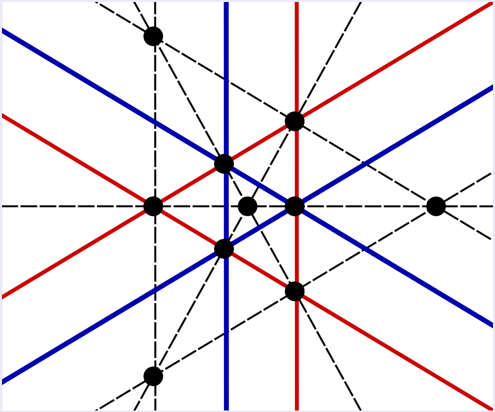

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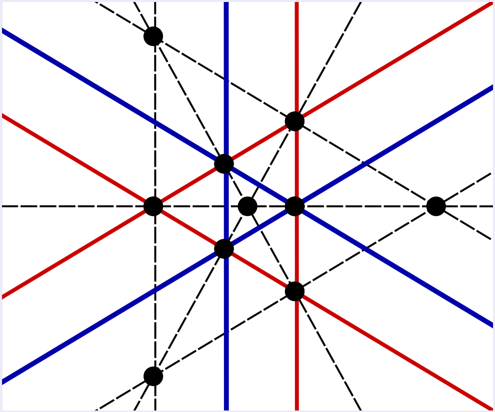

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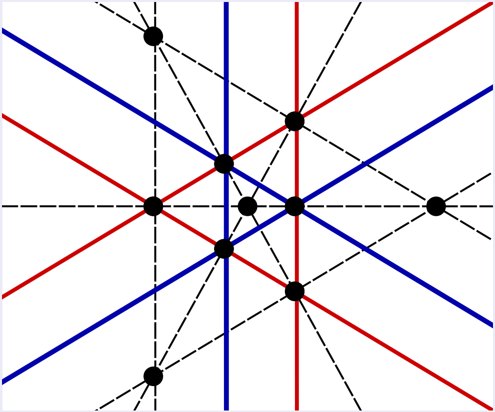
Unfolding $E_6 \to SO_{10}$

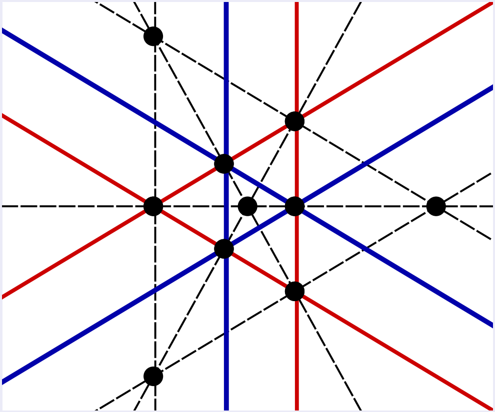


	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	$\mathbf{16}$	1	1	-1
T_2	$\mathbf{16}$	1	-1	-1
T_3	$\mathbf{16}$	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	$\mathbf{10}$	1	1	2
Y_a	$\mathbf{10}$	-1	1	-2
Y_b	$\mathbf{10}$	2	0	-2
X_1	$\mathbf{1}$	-1	-1	4
X_2	$\mathbf{1}$	-1	1	4
N_2^c	$\mathbf{1}$	-2	0	-4
N_3^c	$\mathbf{1}$	3	-1	0
S_1	$\mathbf{1}$	3	1	0
S_2	$\mathbf{1}$	0	2	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

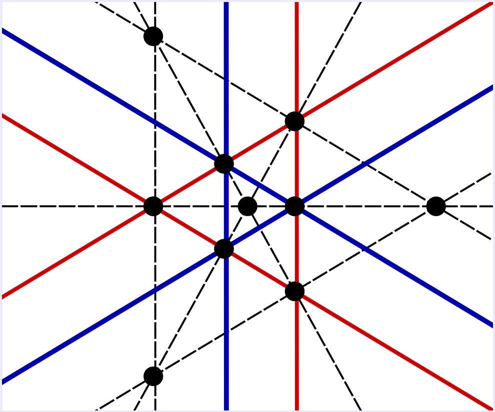


	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	$\mathbf{16}$	1	1	-1
T_2	$\mathbf{16}$	1	-1	-1
T_3	$\mathbf{16}$	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	$\mathbf{10}$	1	1	2
Y_a	$\mathbf{10}$	-1	1	-2
Y_b	$\mathbf{10}$	2	0	-2
X_1	$\mathbf{1}$	-1	-1	4
X_2	$\mathbf{1}$	-1	1	4
N_2^c	$\mathbf{1}$	-2	0	-4
N_3^c	$\mathbf{1}$	3	-1	0
S_1	$\mathbf{1}$	3	1	0
S_2	$\mathbf{1}$	0	2	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

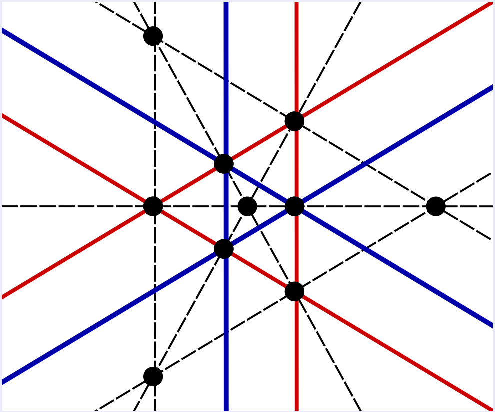


	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	-1
T_2	**16**	1	-1	-1
T_3	**16**	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	**10**	1	1	2
Y_a	**10**	-1	1	-2
Y_b	**10**	2	0	-2
X_1	**1**	-1	-1	4
X_2	**1**	-1	1	4
N_2^c	**1**	-2	0	-4
N_3^c	**1**	3	-1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

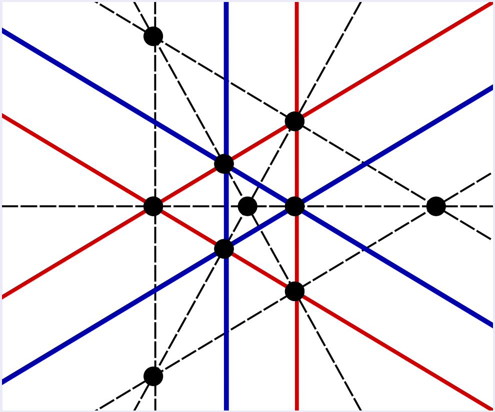


	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	−1
T_2	**16**	1	−1	−1
T_3	**16**	−2	0	−1
T_X^c	$\overline{\mathbf{16}}$	0	0	−3
H	**10**	1	1	2
Y_a	**10**	−1	1	−2
Y_b	**10**	2	0	−2
X_1	**1**	−1	−1	4
X_2	**1**	−1	1	4
N_2^c	**1**	−2	0	−4
N_3^c	**1**	3	−1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

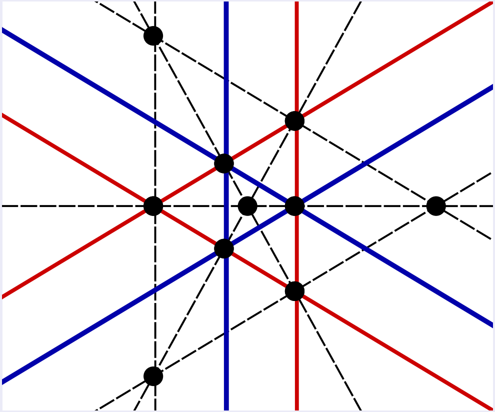


	SO_{10}	U_1^a	U_1^b	U_1^c
T_1	**16**	1	1	-1
T_2	**16**	1	-1	-1
T_3	**16**	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	**10**	1	1	2
Y_a	**10**	-1	1	-2
Y_b	**10**	2	0	-2
X_1	**1**	-1	-1	4
X_2	**1**	-1	1	4
N_2^c	**1**	-2	0	-4
N_3^c	**1**	3	-1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$



	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	-1
T_2	**16**	1	-1	-1
T_3	**16**	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	**10**	1	1	2
Y_a	**10**	-1	1	-2
Y_b	**10**	2	0	-2
X_1	**1**	-1	-1	4
X_2	**1**	-1	1	4
N_2^c	**1**	-2	0	-4
N_3^c	**1**	3	-1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

'Unfolding' Three Generations out of E_8

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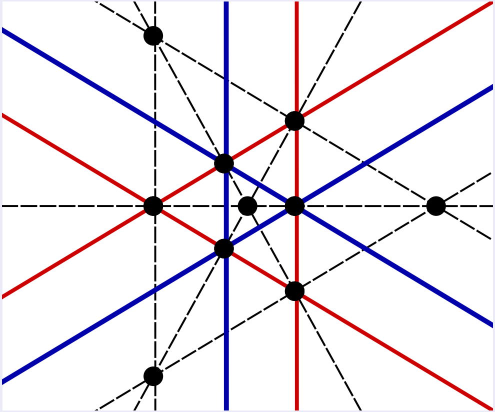
Unfolding $SO_{10} \to SU_5$



	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	−1
T_2	**16**	1	−1	−1
T_3	**16**	−2	0	−1
T_X^c	$\overline{\mathbf{16}}$	0	0	−3
H	**10**	1	1	2
Y_a	**10**	−1	1	−2
Y_b	**10**	2	0	−2
X_1	**1**	−1	−1	4
X_2	**1**	−1	1	4
N_2^c	**1**	−2	0	−4
N_3^c	**1**	3	−1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

'Unfolding' Three Generations out of E_8

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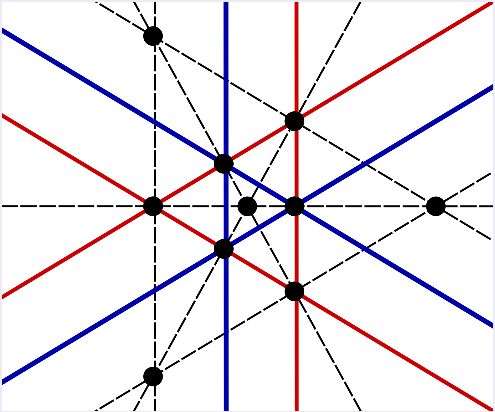
Unfolding $SO_{10} \to SU_5$

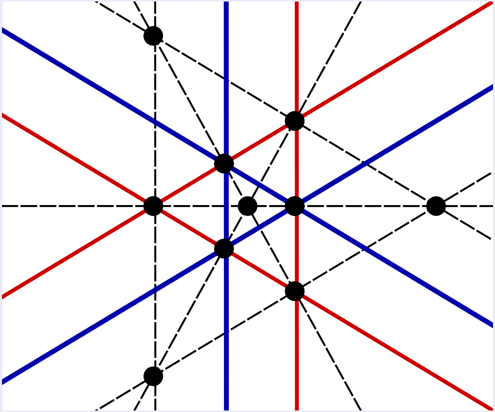


	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	-1
T_2	**16**	1	-1	-1
T_3	**16**	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	**10**	1	1	2
Y_a	**10**	-1	1	-2
Y_b	**10**	2	0	-2
X_1	**1**	-1	-1	4
X_2	**1**	-1	1	4
N_2^c	**1**	-2	0	-4
N_3^c	**1**	3	-1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

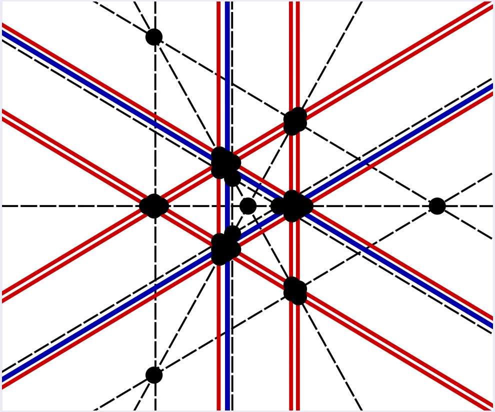


	SO_{10}	$\times U_1^a$	$\times U_1^b$	$\times U_1^c$
T_1	**16**	1	1	-1
T_2	**16**	1	-1	-1
T_3	**16**	-2	0	-1
T_X^c	$\overline{\mathbf{16}}$	0	0	-3
H	**10**	1	1	2
Y_a	**10**	-1	1	-2
Y_b	**10**	2	0	-2
X_1	**1**	-1	-1	4
X_2	**1**	-1	1	4
N_2^c	**1**	-2	0	-4
N_3^c	**1**	3	-1	0
S_1	**1**	3	1	0
S_2	**1**	0	2	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

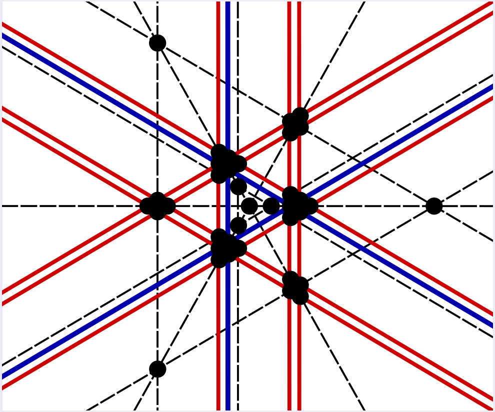


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$

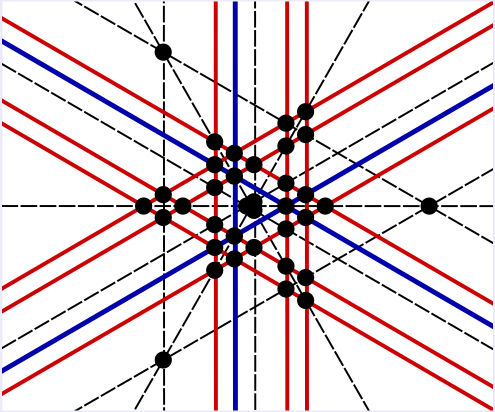


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$

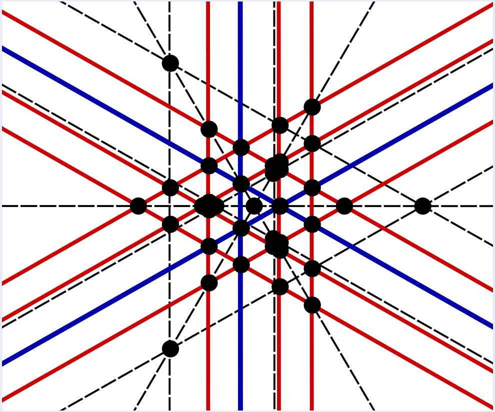


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

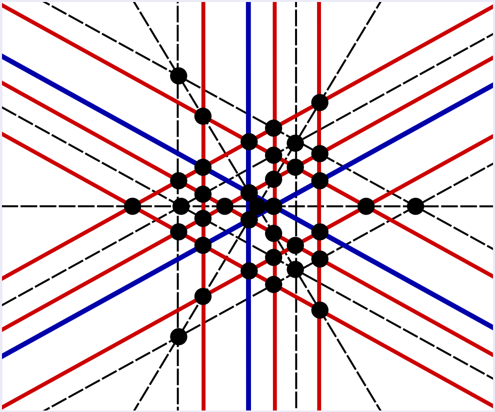


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

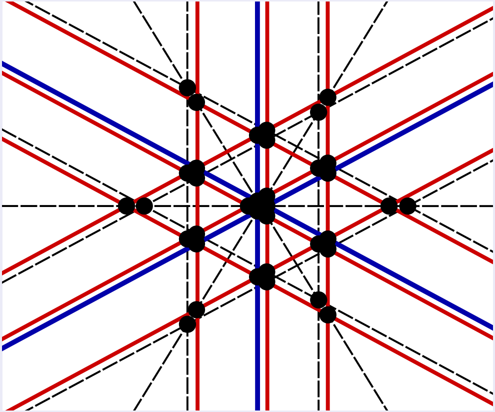


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$

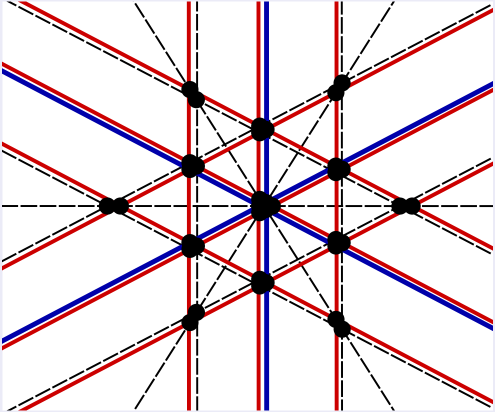


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

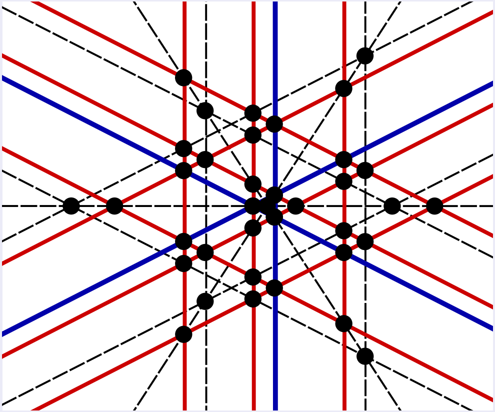


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

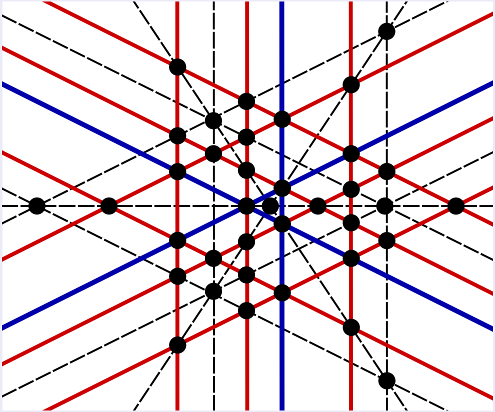


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$

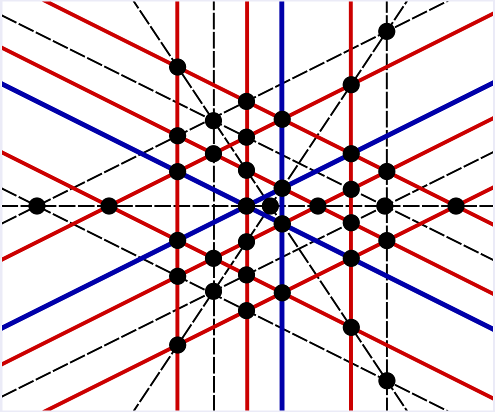


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$

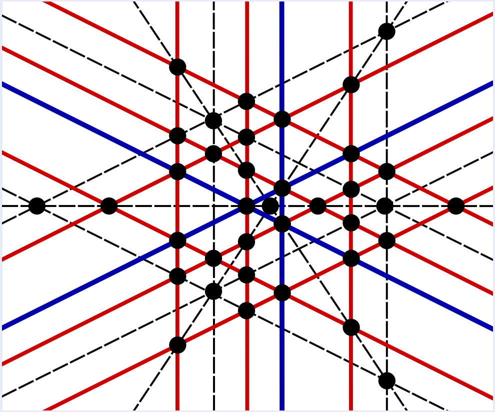


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

'Unfolding' Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential 'unfoldings':

Unfolding $SO_{10} \to SU_5$

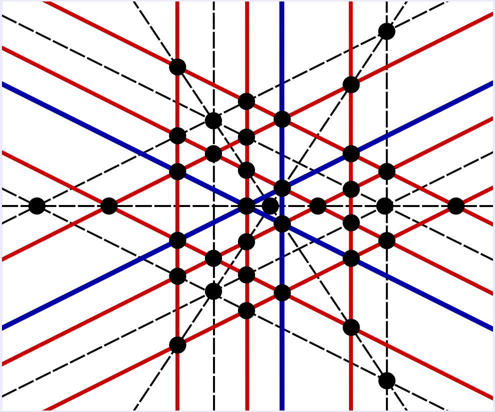


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \to SU_5$

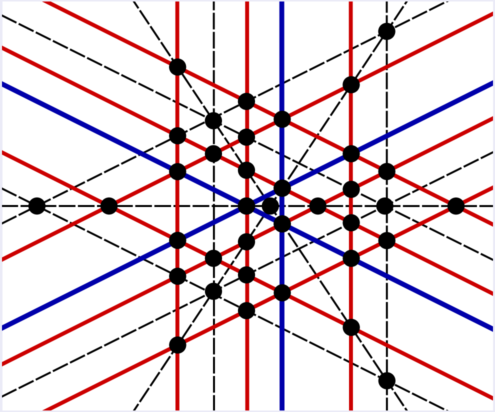


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

‘Unfolding’ Three Generations out of E_8

The matter content of a general $\widehat{E_8}$-fibred geometry can be derived by thinking in terms of sequential ‘unfoldings’:

Unfolding $SO_{10} \rightarrow SU_5$

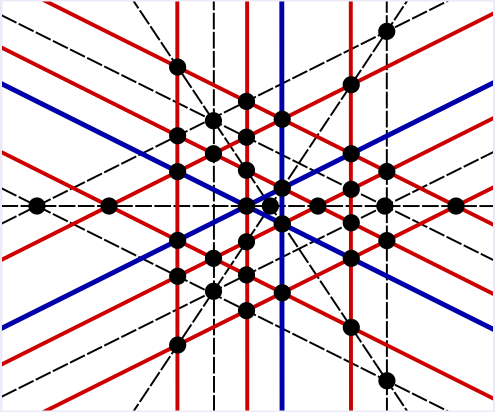


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

Realistic Examples of Phenomenology

This set of matter curves can lead to new, surprisingly realistic models that are nonetheless quite familiar to model builders

Unfolding $SO_{10} \rightarrow SU_5$



	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

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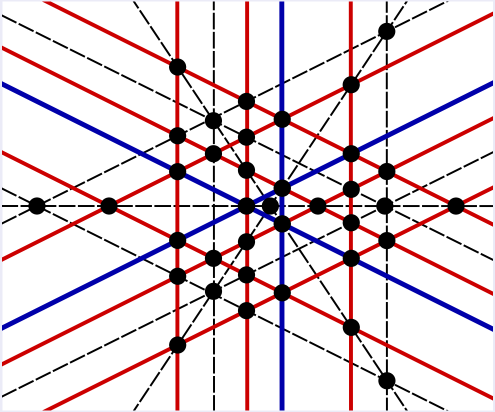
Unfolding $SO_{10} \to SU_5$



	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

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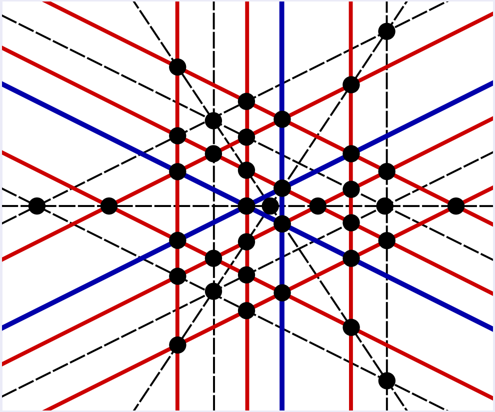
Unfolding $SO_{10} \to SU_5$



	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

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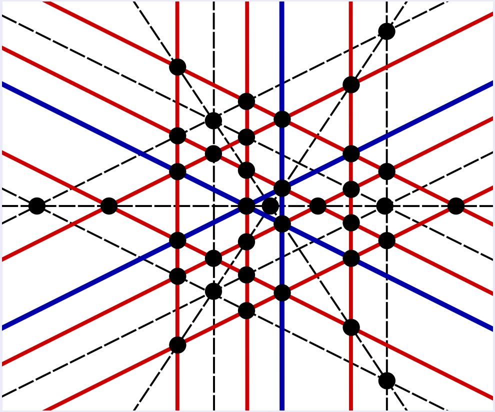
Unfolding $SO_{10} \to SU_5$

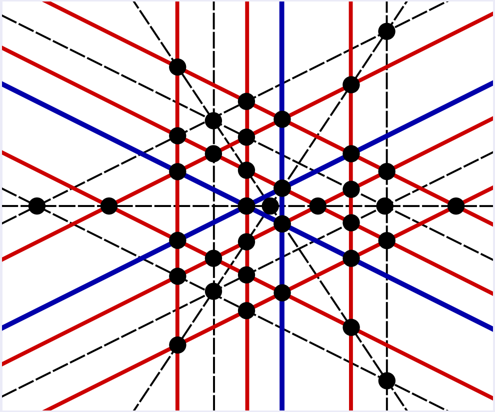


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

Realistic Examples of Phenomenology

This set of matter curves can lead to new, surprisingly realistic models that are nonetheless quite familiar to model builders

Unfolding $SO_{10} \to SU_5$



	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

Realistic Examples of Phenomenology

This set of matter curves can lead to new, surprisingly realistic models that are nonetheless quite familiar to model builders

- The freedom to 'conjugate' fields is quite unusual in traditional unified model building, but leads to powerful new phenomenological mechanisms.
- For any non-trivial choice of fluxes, there are always anomalous U_1-symmetries, which become Higgsed by the Green-Schwarz mechanism, which also generates mass-terms for some fields: 'vacuum realignment.'
- By choosing fluxes appropriately, one can find models that are surprisingly realistic in both M-Theory and F-Theory.

	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

Realistic Examples of Phenomenology

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	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

Realistic Examples of Phenomenology

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- The freedom to 'conjugate' fields is quite unusual in traditional unified model building, but leads to powerful new phenomenological mechanisms.
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- By choosing fluxes appropriately, one can find models that are surprisingly realistic in both M-Theory and F-Theory.

	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
$(T_X^c$	$\overline{\mathbf{10}}$	0	0	−3	1)
$(T_X$	$\mathbf{10}$	0	0	0	4)
$(M_X^c$	$\mathbf{5}$	0	0	−3	−3)
$(S_1$	$\mathbf{1}$	3	1	0	0)
$(S_2$	$\mathbf{1}$	0	2	0	0)

Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

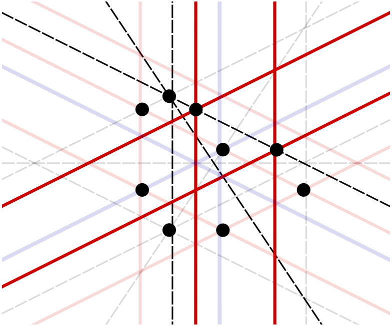
$$W = T_2 T_3 H^u + T_2 M_3 H^d + T_3 M_2 H^d + H^u M_2 \nu_3^c + H^u M_3 \nu_2^c + X_1 Y_1 Y_1^c + X_1 Y_2 Y_2^c + X_1 N_1^c \nu_1^c + X_2 N_1^c \nu_2^c + X_2 N_2^c N_3^c.$$

	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
$(T_X^c$	$\overline{\mathbf{10}}$	0	0	−3	1)
$(T_X$	$\mathbf{10}$	0	0	0	4)
$(M_X^c$	$\mathbf{5}$	0	0	−3	−3)
$(S_1$	$\mathbf{1}$	3	1	0	0)
$(S_2$	$\mathbf{1}$	0	2	0	0)

Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

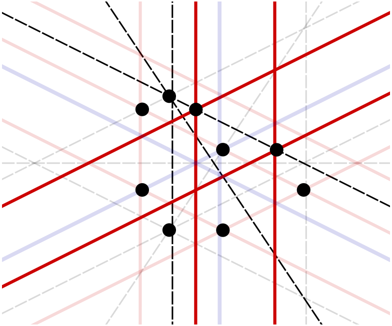
$$W = T_2 T_3 H^u + T_2 M_3 H^d + T_3 M_2 H^d + H^u M_2 \nu_3^c + H^u M_3 \nu_2^c + X_1 Y_1 Y_1^c + X_1 Y_2 Y_2^c + X_1 N_1^c \nu_1^c + X_2 N_1^c \nu_2^c + X_2 N_2^c N_3^c .$$



Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

$$W = T_2 T_3 H^u + T_2 M_3 H^d + T_3 M_2 H^d + H^u M_2 \nu_3^c + H^u M_3 \nu_2^c + X_1 Y_1 Y_1^c + X_1 Y_2 Y_2^c + X_1 N_1^c \nu_1^c + X_2 N_1^c \nu_2^c + X_2 N_2^c N_3^c.$$



Notice that T_1 and M_1 do not appear in the superpotential at all!

Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

$$W = T_2\,T_3\,H^u + T_2\,M_3\,H^d + T_3\,M_2\,H^d + H^u\,M_2\,\nu_3^c + H^u\,M_3\,\nu_2^c + X_1\,Y_1\,Y_1^c + X_1\,Y_2\,Y_2^c + X_1\,N_1^c\,\nu_1^c + X_2\,N_1^c\,\nu_2^c + X_2\,N_2^c\,N_3^c.$$

Notice that T_1 and M_1 do not appear in the superpotential at all!

	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
$(T_X^c$	$\overline{\mathbf{10}}$	0	0	−3	1)
$(T_X$	$\mathbf{10}$	0	0	0	4)
$(M_X^c$	$\mathbf{5}$	0	0	−3	−3)
$(S_1$	$\mathbf{1}$	3	1	0	0)
$(S_2$	$\mathbf{1}$	0	2	0	0)

Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

$$\begin{aligned} W = \; & T_2\,T_3\,H^u \;+\; T_2\,M_3\,H^d \;+\; T_3\,M_2\,H^d \\ & + H^u\,M_2\,\nu_3^c \;+\; H^u\,M_3\,\nu_2^c \;+\; X_1\,Y_1\,Y_1^c \\ & + X_1\,Y_2\,Y_2^c \;+\; X_1\,N_1^c\,\nu_1^c \;+\; X_2\,N_1^c\,\nu_2^c \\ & + X_2\,N_2^c\,N_3^c . \end{aligned}$$

Consider the operator $T_1\,T_1\,H^u$, it has charges $(3,3,0,0)$ under $U_1^a\times U_1^b\times U_1^c\times U_1^d$.

	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\overline{\mathbf{5}}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\overline{\mathbf{5}}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
$(T_X^c$	$\overline{\mathbf{10}}$	0	0	-3	$1)$
$(T_X$	$\mathbf{10}$	0	0	0	$4)$
$(M_X^c$	$\mathbf{5}$	0	0	-3	$-3)$
$(S_1$	$\mathbf{1}$	3	1	0	$0)$
$(S_2$	$\mathbf{1}$	0	2	0	$0)$

Re-Folding the Geometry to Enforce Couplings

- The choice of fluxes listed in the table on the right would generate a superpotential of the form:

$$\begin{aligned} W = \ & T_2 T_3 H^u + T_2 M_3 H^d + T_3 M_2 H^d \\ & + H^u M_2 \nu_3^c + H^u M_3 \nu_2^c + X_1 Y_1 Y_1^c \\ & + X_1 Y_2 Y_2^c + X_1 N_1^c \nu_1^c + X_2 N_1^c \nu_2^c \\ & + X_2 N_2^c N_3^c . \end{aligned}$$

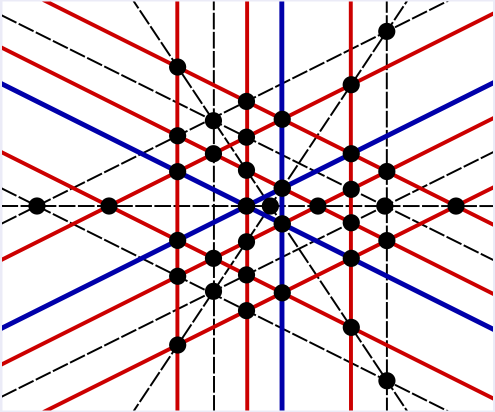
Consider the operator $T_1 T_1 H^u$, it has charges $(3,3,0,0)$ under $U_1^a \times U_1^b \times U_1^c \times U_1^d$.

	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
$(T_X^c$	$\overline{\mathbf{10}}$	0	0	-3	$1)$
$(T_X$	$\mathbf{10}$	0	0	0	$4)$
$(M_X^c$	$\mathbf{5}$	0	0	-3	$-3)$
$(S_1$	$\mathbf{1}$	3	1	0	$0)$
$(S_2$	$\mathbf{1}$	0	2	0	$0)$

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

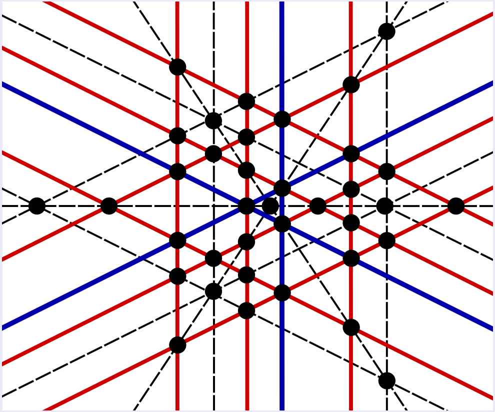


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\overline{\mathbf{5}}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

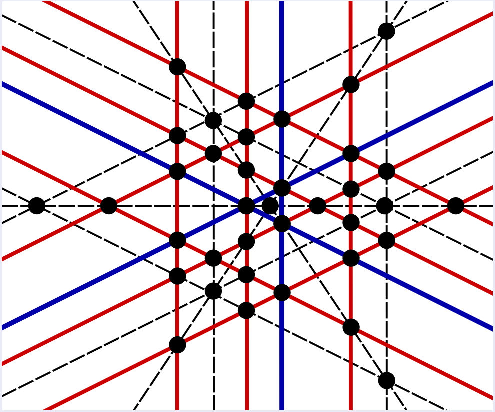


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

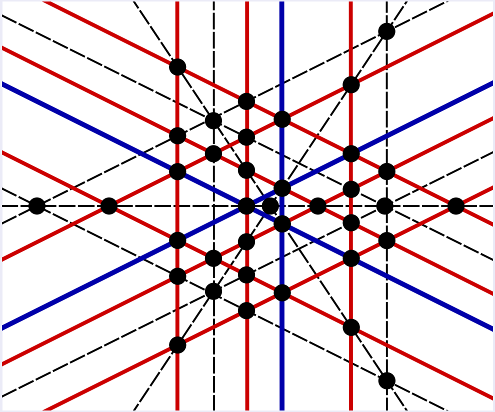


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

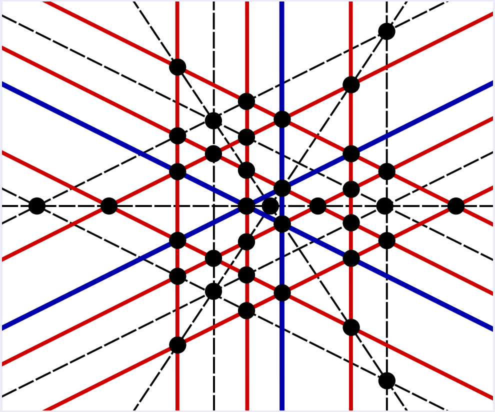


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

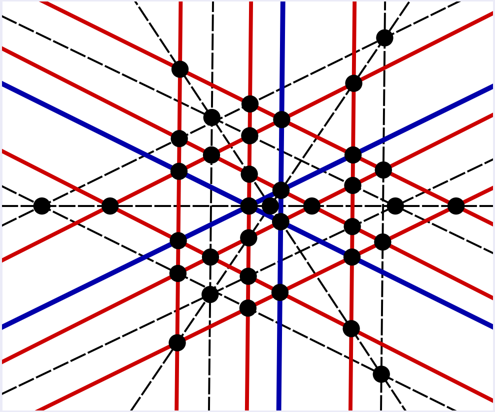


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Constraining the Moduli Space

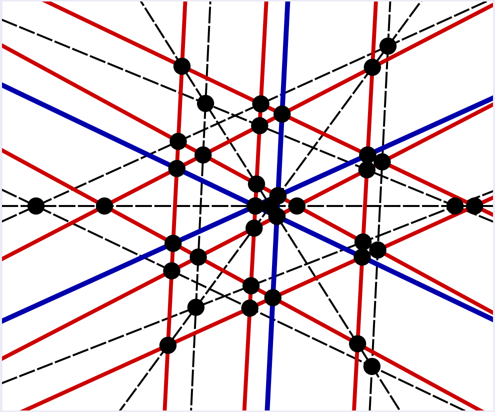


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	**10**	1	1	−1	−1
T_2	**10**	1	−1	−1	−1
T_3	**10**	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	**5**	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\overline{\mathbf{5}}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\overline{\mathbf{5}}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	**1**	−1	−1	4	0
X_2	**1**	−1	1	4	0
ν_1^c	**1**	1	1	−1	−5
ν_2^c	**1**	1	−1	−1	−5
ν_3^c	**1**	−2	0	−1	−5
N_1^c	**1**	0	0	−3	5
N_2^c	**1**	−2	0	−4	0
N_3^c	**1**	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	**10**	0	0	0	4
M_X^c	**5**	0	0	−3	−3
S_1	**1**	3	1	0	0
S_2	**1**	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Constraining the Moduli Space

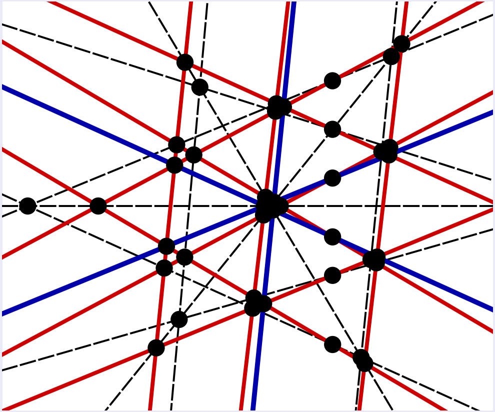


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

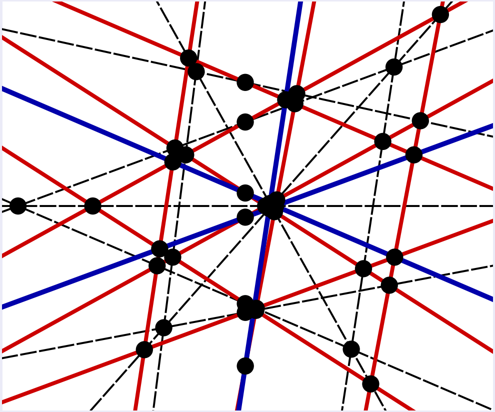


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

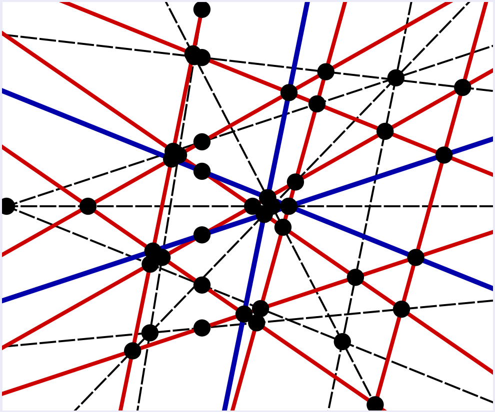


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\, T_1\, H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

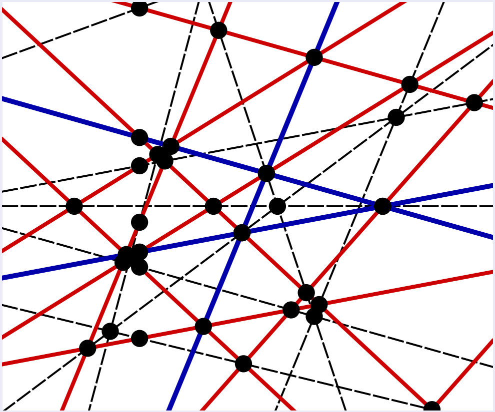


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Constraining the Moduli Space

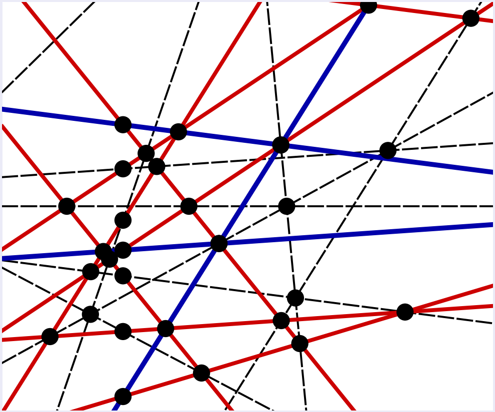


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\overline{\mathbf{5}}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\overline{\mathbf{5}}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\, T_1\, H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Constraining the Moduli Space

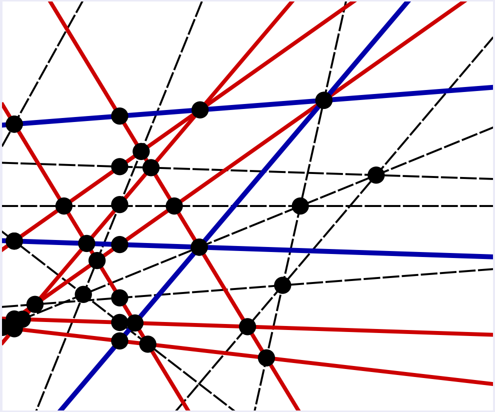


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\overline{\mathbf{5}}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\overline{\mathbf{5}}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\, T_1\, H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

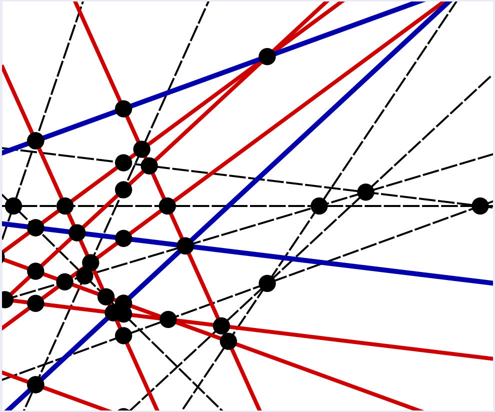


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

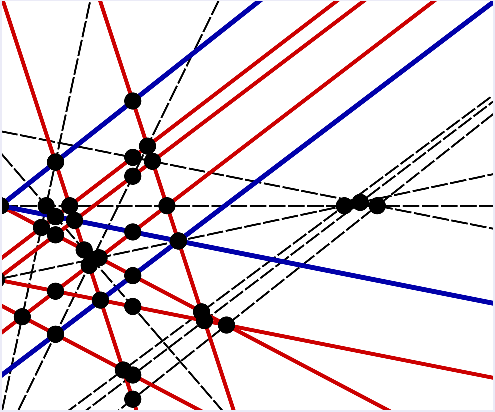


	$SU_5 \times$	$U_1^a \times$	$U_1^b \times$	$U_1^c \times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Constraining the Moduli Space

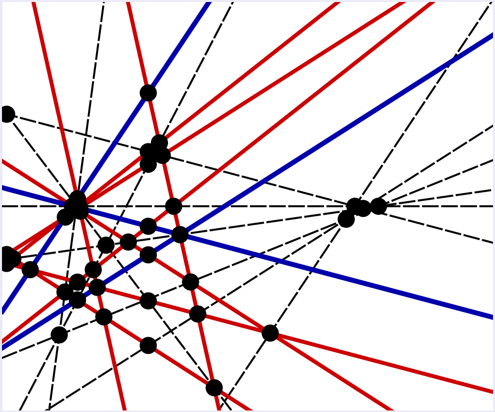


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

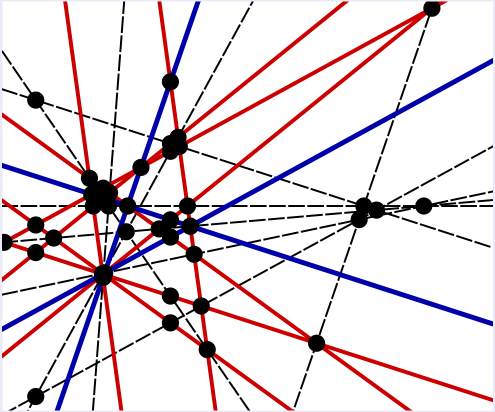


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

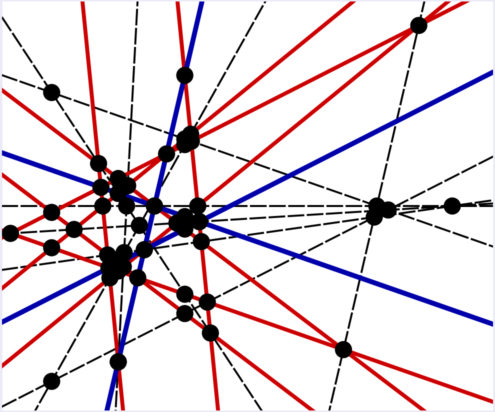


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

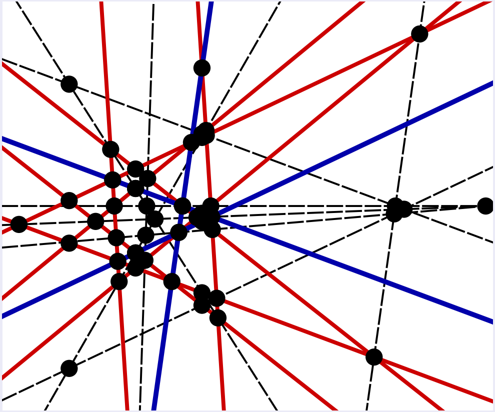


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

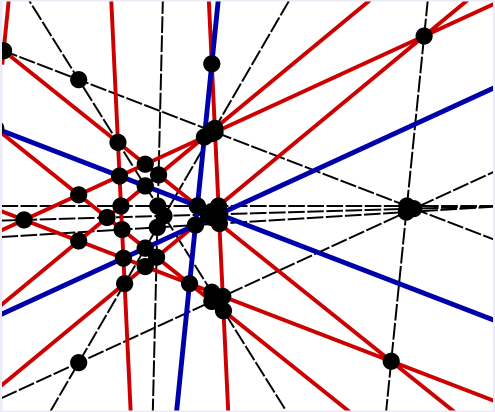


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

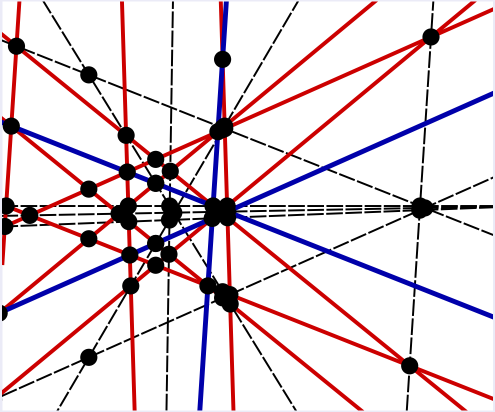


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

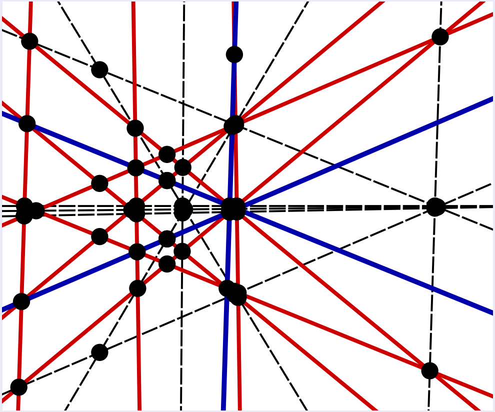


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	−1	−1
T_2	$\mathbf{10}$	1	−1	−1	−1
T_3	$\mathbf{10}$	−2	0	−1	−1
M_1	$\overline{\mathbf{5}}$	1	1	−1	3
M_2	$\overline{\mathbf{5}}$	1	−1	−1	3
M_3	$\overline{\mathbf{5}}$	−2	0	−1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	−2
Y_1	$\mathbf{5}$	−1	1	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	−2	−2
Y_2	$\mathbf{5}$	2	0	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−1	1	−2	−2
X_1	$\mathbf{1}$	−1	−1	4	0
X_2	$\mathbf{1}$	−1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	−1	−5
ν_2^c	$\mathbf{1}$	1	−1	−1	−5
ν_3^c	$\mathbf{1}$	−2	0	−1	−5
N_1^c	$\mathbf{1}$	0	0	−3	5
N_2^c	$\mathbf{1}$	−2	0	−4	0
N_3^c	$\mathbf{1}$	3	−1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	−3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	−3	−3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, -3c{+}d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

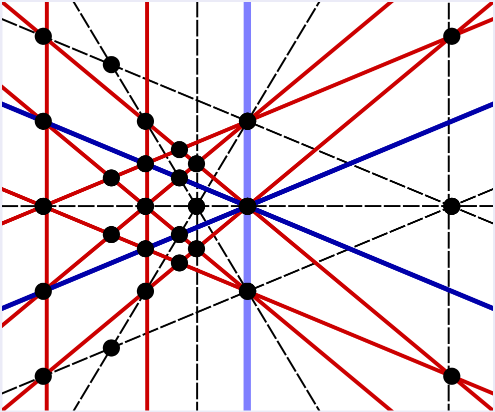


	$SU_5\times$	$U_1^a\times$	$U_1^b\times$	$U_1^c\times$	U_1^d
T_1	$\mathbf{10}$	1	1	-1	-1
T_2	$\mathbf{10}$	1	-1	-1	-1
T_3	$\mathbf{10}$	-2	0	-1	-1
M_1	$\overline{\mathbf{5}}$	1	1	-1	3
M_2	$\overline{\mathbf{5}}$	1	-1	-1	3
M_3	$\overline{\mathbf{5}}$	-2	0	-1	3
H^u	$\mathbf{5}$	1	1	2	2
H^d	$\overline{\mathbf{5}}$	1	1	2	-2
Y_1	$\mathbf{5}$	-1	1	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	0	-2	-2
Y_2	$\mathbf{5}$	2	0	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-1	1	-2	-2
X_1	$\mathbf{1}$	-1	-1	4	0
X_2	$\mathbf{1}$	-1	1	4	0
ν_1^c	$\mathbf{1}$	1	1	-1	-5
ν_2^c	$\mathbf{1}$	1	-1	-1	-5
ν_3^c	$\mathbf{1}$	-2	0	-1	-5
N_1^c	$\mathbf{1}$	0	0	-3	5
N_2^c	$\mathbf{1}$	-2	0	-4	0
N_3^c	$\mathbf{1}$	3	-1	0	0
T_X^c	$\overline{\mathbf{10}}$	0	0	-3	1
T_X	$\mathbf{10}$	0	0	0	4
M_X^c	$\mathbf{5}$	0	0	-3	-3
S_1	$\mathbf{1}$	3	1	0	0
S_2	$\mathbf{1}$	0	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\, T_1\, H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

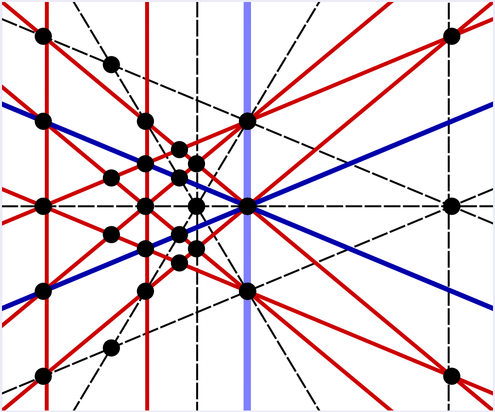


	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\mathbf{5}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\mathbf{5}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c+d,2a+c+d,-3c+d,-4d,0,0,0,0)$

Constraining the Moduli Space

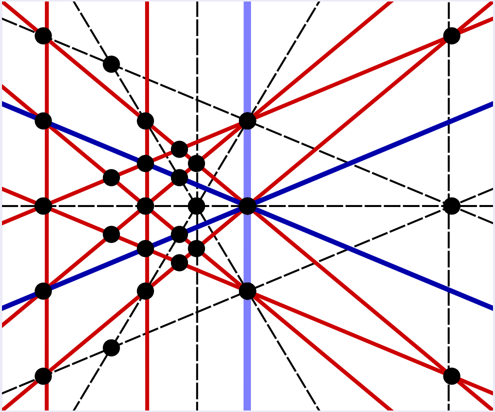


	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\mathbf{5}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\mathbf{5}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

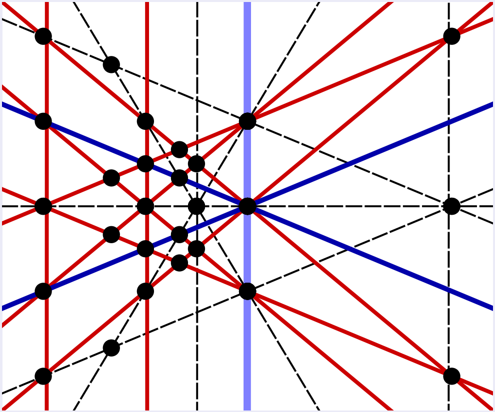


	$SU_5 \times U_1^a \times U_1^c \times U_1^d$			
T_1	$\mathbf{10}$	0	−1	−1
T_2	$\mathbf{10}$	2	−1	−1
T_3	$\mathbf{10}$	−2	−1	−1
M_1	$\overline{\mathbf{5}}$	0	−1	3
M_2	$\overline{\mathbf{5}}$	2	−1	3
M_3	$\overline{\mathbf{5}}$	−2	−1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	−2
Y_1	$\mathbf{5}$	−2	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	−2	−2
Y_2	$\mathbf{5}$	2	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−2	−2	−2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	−2	4	0
ν_1^c	$\mathbf{1}$	0	−1	−5
ν_2^c	$\mathbf{1}$	2	−1	−5
ν_3^c	$\mathbf{1}$	−2	−1	−5
N_1^c	$\mathbf{1}$	0	−3	5
N_2^c	$\mathbf{1}$	−2	−4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	−3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	−3	−3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, -3c{+}d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

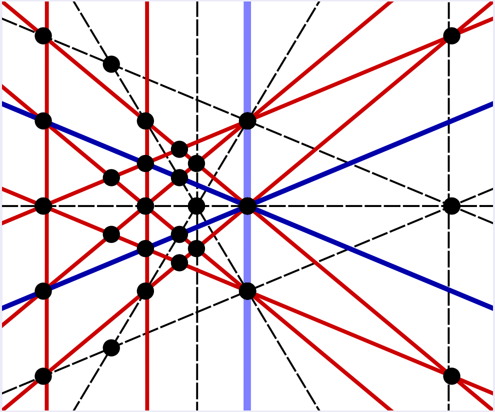


	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\mathbf{5}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\mathbf{5}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\, T_1\, H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Constraining the Moduli Space

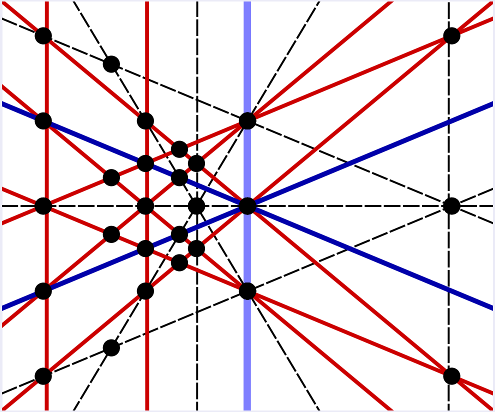


	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	−1	−1
T_2	$\mathbf{10}$	2	−1	−1
T_3	$\mathbf{10}$	−2	−1	−1
M_1	$\overline{\mathbf{5}}$	0	−1	3
M_2	$\overline{\mathbf{5}}$	2	−1	3
M_3	$\overline{\mathbf{5}}$	−2	−1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	−2
Y_1	$\mathbf{5}$	−2	−2	2
Y_1^c	$\overline{\mathbf{5}}$	2	−2	−2
Y_2	$\mathbf{5}$	2	−2	2
Y_2^c	$\overline{\mathbf{5}}$	−2	−2	−2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	−2	4	0
ν_1^c	$\mathbf{1}$	0	−1	−5
ν_2^c	$\mathbf{1}$	2	−1	−5
ν_3^c	$\mathbf{1}$	−2	−1	−5
N_1^c	$\mathbf{1}$	0	−3	5
N_2^c	$\mathbf{1}$	−2	−4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	−3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	−3	−3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Constraining the Moduli Space



	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\overline{\mathbf{5}}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\overline{\mathbf{5}}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Once this condition is imposed, we obtain the following superpotential,

$$W = T_1\,T_1\,H^u + T_2\,T_3\,H^u + T_1\,M_1\,H^d + T_2\,M_3\,H^d + T_3\,M_2\,H^d + H^u\,M_1\,\nu_1^c + H^u\,M_2\,\nu_3^c + H^u\,M_3\,\nu_2^c + X_1\,Y_1\,Y_1^c + X_1\,Y_2\,Y_2^c + X_1\,N_1^c\,\nu_1^c + X_2\,N_1^c\,\nu_2^c + X_2\,N_2^c\,N_3^c.$$

	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\mathbf{5}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\mathbf{5}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Once this condition is imposed, we obtain the following superpotential,

$$
\begin{aligned}
W = \;& T_1\,T_1\,H^u \;+\; T_2\,T_3\,H^u \;+\; T_1\,M_1\,H^d \\
&+ T_2\,M_3\,H^d \;+\; T_3\,M_2\,H^d \;+\; H^u\,M_1\,\nu_1^c \\
&+ H^u\,M_2\,\nu_3^c \;+\; H^u\,M_3\,\nu_2^c \;+\; X_1\,Y_1\,Y_1^c \\
&+ X_1\,Y_2\,Y_2^c \;+\; X_1\,N_1^c\,\nu_1^c + \; X_2\,N_1^c\,\nu_2^c \\
&+ X_2\,N_2^c\,N_3^c.
\end{aligned}
$$

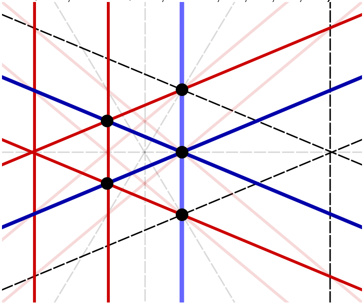
	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\mathbf{5}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\mathbf{5}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Once this condition is imposed, we obtain the following superpotential,

$$W = T_1\,T_1\,H^u + T_2\,T_3\,H^u + T_1\,M_1\,H^d + T_2\,M_3\,H^d + T_3\,M_2\,H^d + H^u\,M_1\,\nu_1^c + H^u\,M_2\,\nu_3^c + H^u\,M_3\,\nu_2^c + X_1\,Y_1\,Y_1^c + X_1\,Y_2\,Y_2^c + X_1\,N_1^c\,\nu_1^c + X_2\,N_1^c\,\nu_2^c + X_2\,N_2^c\,N_3^c.$$

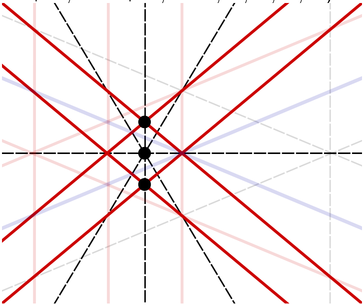


A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b=-a$: $E_8(c{+}d, 2a{+}c{+}d, {-}3c{+}d, {-}4d, 0, 0, 0, 0)$

Once this condition is imposed, we obtain the following superpotential,

$$\begin{aligned} W = \;& T_1\,T_1\,H^u + T_2\,T_3\,H^u + T_1\,M_1\,H^d \\ &+ T_2\,M_3\,H^d + T_3\,M_2\,H^d + H^u\,M_1\,\nu_1^c \\ &+ H^u\,M_2\,\nu_3^c + H^u\,M_3\,\nu_2^c + X_1\,Y_1\,Y_1^c \\ &+ X_1\,Y_2\,Y_2^c + X_1\,N_1^c\,\nu_1^c + X_2\,N_1^c\,\nu_2^c \\ &+ X_2\,N_2^c\,N_3^c. \end{aligned}$$



A Local Diamond Ring in F-Theory

To enforce the gauge-invariance of $T_1\,T_1\,H^u$, we should impose the **geometric** condition $b = -a$: $E_8(c+d, 2a+c+d, -3c+d, -4d, 0, 0, 0, 0)$

Once this condition is imposed, we obtain the following superpotential,

$$W = T_1\,T_1\,H^u + T_2\,T_3\,H^u + T_1\,M_1\,H^d + T_2\,M_3\,H^d + T_3\,M_2\,H^d + H^u\,M_1\,\nu_1^c + H^u\,M_2\,\nu_3^c + H^u\,M_3\,\nu_2^c + X_1\,Y_1\,Y_1^c + X_1\,Y_2\,Y_2^c + X_1\,N_1^c\,\nu_1^c + X_2\,N_1^c\,\nu_2^c + X_2\,N_2^c\,N_3^c.$$

Notice that the operator $X_1^\dagger\,H^u\,H^d$ is gauge invariant.

	SU_5	$\times\, U_1^a$	$\times\, U_1^c$	$\times\, U_1^d$
T_1	$\mathbf{10}$	0	-1	-1
T_2	$\mathbf{10}$	2	-1	-1
T_3	$\mathbf{10}$	-2	-1	-1
M_1	$\overline{\mathbf{5}}$	0	-1	3
M_2	$\overline{\mathbf{5}}$	2	-1	3
M_3	$\overline{\mathbf{5}}$	-2	-1	3
H^u	$\mathbf{5}$	0	2	2
H^d	$\overline{\mathbf{5}}$	0	2	-2
Y_1	$\mathbf{5}$	-2	-2	2
Y_1^c	$\overline{\mathbf{5}}$	2	-2	-2
Y_2	$\mathbf{5}$	2	-2	2
Y_2^c	$\overline{\mathbf{5}}$	-2	-2	-2
X_1	$\mathbf{1}$	0	4	0
X_2	$\mathbf{1}$	-2	4	0
ν_1^c	$\mathbf{1}$	0	-1	-5
ν_2^c	$\mathbf{1}$	2	-1	-5
ν_3^c	$\mathbf{1}$	-2	-1	-5
N_1^c	$\mathbf{1}$	0	-3	5
N_2^c	$\mathbf{1}$	-2	-4	0
N_3^c	$\mathbf{1}$	4	0	0
T_X^c	$\overline{\mathbf{10}}$	0	-3	1
T_X	$\mathbf{10}$	0	0	4
M_X^c	$\mathbf{5}$	0	-3	-3
S_1	$\mathbf{1}$	2	0	0

Conclusions and Future Directions

- We have seen that explicit, purely local phenomenological models with three generations exist in both F-theory and M-theory, but only just-so.
- These models are 'in principle' explicit enough to be exhaustively studied, and even make falsifiable predictions for low-energy physics.
  - To what extent is this claim true in both F-theory and M-theory?
  - How many assumptions are required in each case?
- What about moduli stabilization? Are there compact manifolds with these local patches? How is the locally continuous landscape of fibrations quantized by compactification?
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