Matter field wavefunctions in flux compactifications

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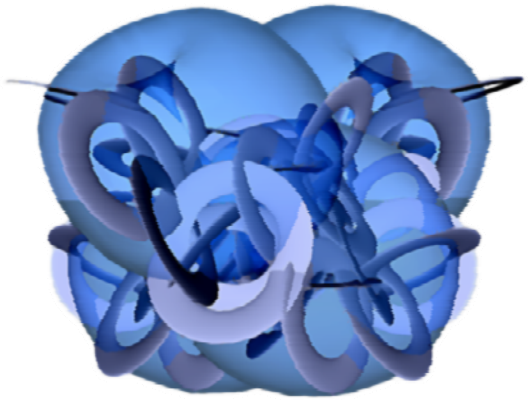
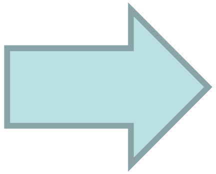
In collaboration with F. Marchesano (CERN). arXiv:**0906.xxxx** [hep-th]

[See also Marchesano's talk...]

String Phenomenology '09, Warsaw, June 15-19, 2009



String Theory



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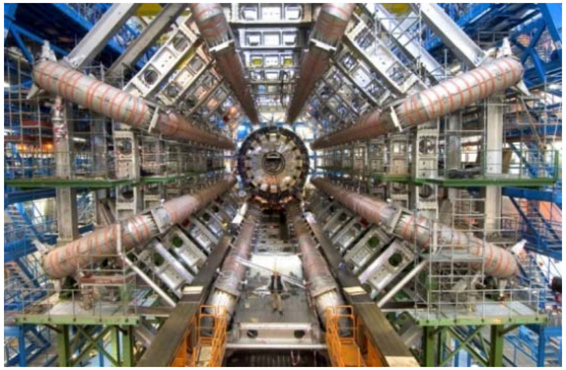


4D Physical observables

- Wavefunctions are useful in models where there is a higher dimensional sugra description valid for some regions of the moduli space
- Physical observables as overlap integrals of internal wavefunctions
- E.g. Yukawa couplings in toroidal orientifolds with magnetized branes:

$$Y_{ijk} = \int_{\mathcal{M}_6} \psi_i^{a\dagger} \Gamma^m \psi_j^b \phi_{k,m}^c f_{abc}$$

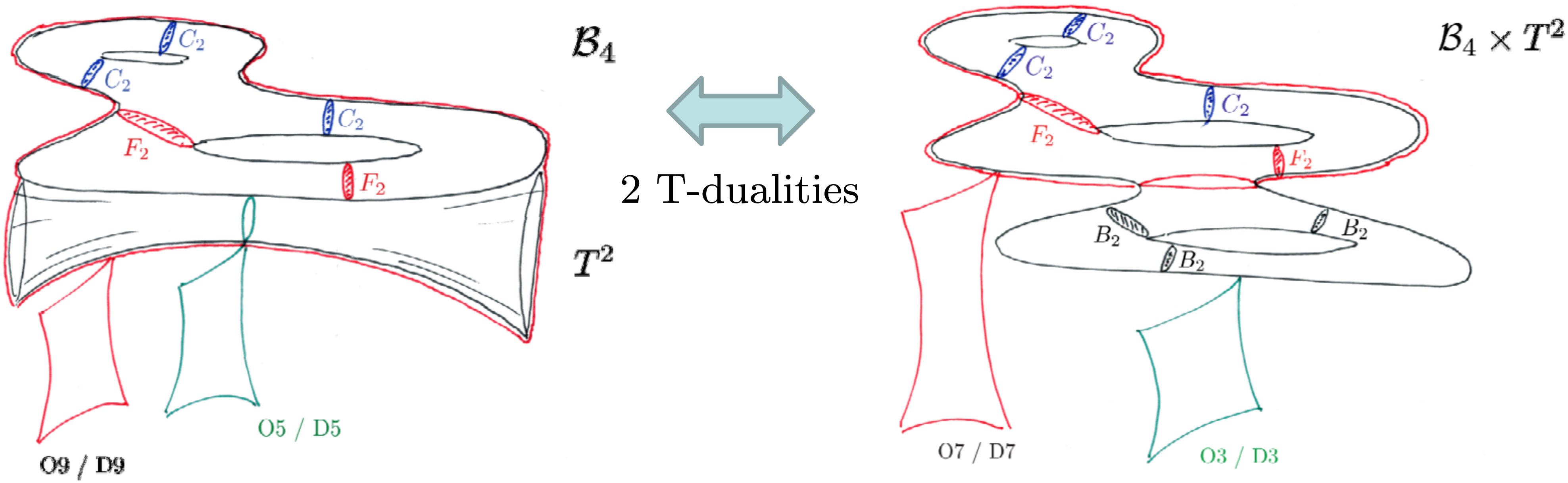
[Cremades,Ibanez,Marchesano]
[Conlon,Maharana,Quevedo]
[Di Vecchia et al.]
[Antoniadis,Kumar,Panda]

1.- 10D SYM describes the d.o.f. of a stack of magnetized D9-branes (to lower order)

2.- Wavefunctions are eigenmodes of the internal Dirac and Laplace operators

- We have extended this approach to toroidal compactifications with magnetized branes and closed string fluxes.

type I | type IIB [Dasgupta et al.; GKP]



torsion + RR 3-form flux

$$
\begin{aligned}
Ze^{\phi} &\equiv g_s = \text{const.} \\
g_s^{1/2} e^{\phi/2} F_3 &= *_{\mathcal{M}_6}\, e^{-3\phi/2} d(e^{3\phi/2} J) \\
d\left(e^{\phi} J \wedge J\right) &= 0
\end{aligned}
$$

NSNS & RR 3-form flux

$$
\begin{aligned}
e^{\phi} &\equiv g_s = \text{const.} \\
F_3 &= -e^{-\phi} *_{\mathcal{M}_6} H_3 \\
d(Z^{1/2} J) &= d(Z^{3/4}\Omega) = 0
\end{aligned}
$$

$$
ds^2 = Z^{-1/2} ds^2_{\mathbb{R}^{1,3}} + ds^2_{\mathcal{M}_6}
$$

- S-dual to heterotic strings with torsion [Hull; Strominger; Becker, Dasgupta]

- Magnetization in D9-branes Higgsses the gauge group: $G_{unbr} = \prod_\alpha G_\alpha$

$$F^\alpha_{mn} = 2\nabla^{\mathcal{M}_6}_{[m}\langle B^\alpha_{n]}\rangle \quad \Rightarrow \quad \tilde{D}_m \Phi^{\alpha\beta}_n \equiv \nabla^{\mathcal{M}_6}_m \Phi^{\alpha\beta}_n - i(\langle B^\alpha_m\rangle - \langle B^\beta_m\rangle)\Phi^{\alpha\beta}_n$$

⇨ W-bosons, scalars and fermions in bifundamental reps.

- If closed string fluctuations are frozen, there are modified internal Dirac and Laplace-Beltrami operators which account for the effect of the closed string background on the open string fluctuations.

W-bosons:

$$\tilde{D}^m \tilde{D}_m W^{\alpha\beta} - 2(\partial_m \log Z)\tilde{D}^m W^{\alpha\beta} = -Z^{1/2} m^2_W W^{\alpha\beta}$$

Charged scalars:

$$\tilde{D}^m \tilde{D}_m \Phi^{p,\alpha\beta} - [\nabla^{\mathcal{M}_6}_m, \nabla^{\mathcal{M}_6 p}]\Phi^{m,\alpha\beta} - 2(\partial_k \log\, Z)\tilde{D}^{[k}\Phi^{p],\alpha\beta} + \\ + 2i\Phi^{m,\alpha\beta}(F_m{}^{p,\alpha} - F_m{}^{p,\beta}) + e^{\phi/2}(\tilde{D}_m \Phi^{n,\alpha\beta})F_n{}^{mp} = -Z^{1/2} m^2_\Phi \Phi^{p,\alpha\beta}$$

Matter fields:

$$\Gamma_{(4)}\left(\not{D}^{\mathcal{M}_6} + \frac{1}{4}e^{\phi/2}\not{F}_3 - \frac{1}{2}\not{\partial}\ln Z\right)\chi_6 = Z^{1/4} m_\chi \mathcal{B}^*_6 \chi^*_6$$

- Simple vacua are $\mathcal{B}_4 = T^4 \quad \Rightarrow$ twisted tori

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c$$

[Schultz; PGC, Grana; Lust et al.]

- We consider $N = 2, 1$ and 0 vacua with RR 3-form fluxes

(diluted fluxes $\varepsilon \ll 1 \quad \Rightarrow$ warping negligible)

$$F_2 = 2\pi \sum_{k=1,2} \begin{pmatrix} m^k_\alpha \mathbb{I}_{n_\alpha} & 0 \\ 0 & m^k_\beta \mathbb{I}_{n_\beta} \end{pmatrix} dx^k \wedge dx^{k+3}$$

$$W \;\rightarrow\; \exp\left[i \oint_\gamma \langle B^\Lambda \rangle U_\Lambda\right] \cdot W \cdot \exp\left[-i \oint_\gamma \langle B^\Lambda \rangle U_\Lambda\right]$$

- FW anomalies forbid magnetization in the T^2 fiber $\xrightarrow{\text{T-duality}}$ D7-branes

$$\begin{aligned} x^k \rightarrow x^k + 1\,,\; x^a \rightarrow x^a - \frac{1}{2} f^a_{kq} x^q \;&:\quad W^{\alpha\beta} \rightarrow e^{i\pi I^k_{\alpha\beta} x^{k+3}} W^{\alpha\beta} \\ x^{k+3} \rightarrow x^{k+3} + 1\,,\; x^a \rightarrow x^a - \frac{1}{2} f^a_{k+3q} x^q \;&:\quad W^{\alpha\beta} \rightarrow e^{-i\pi I^k_{\alpha\beta} x^k} W^{\alpha\beta} \\ x^a \rightarrow x^a + 1 \;&:\quad W^{\alpha\beta} \rightarrow W^{\alpha\beta} \end{aligned}$$

$x^k,\; x^{k+3} \in \mathcal{B}_4$

$x^a \in T^2$

$I^k_{\alpha\beta} \equiv m^k_\alpha - m^k_\beta$

W bosons

$$\tilde{D}^m \tilde{D}_m W^{\alpha\beta} = -m_W^2 W^{\alpha\beta}$$

- A suitable ansatz is then: $W^{\alpha\beta}(\vec{x}) = V(x^k) e^{2\pi i (k_3 x^3 + k_6 x^6)}$

- The torsion acts on V as a standard gauge connection

$$W^{\alpha\beta,\ (j_1,j_2)}_{0,0,k_3,k_6} = \mathcal{N}\ e^{i\pi(\mathbf{N}\cdot\vec{z})\cdot(\mathrm{Im}\ \mathbf{\Omega_U})^{-1}\cdot\mathrm{Im}\ \vec{z}}\ \vartheta \begin{bmatrix} \vec{j} \\ 0 \end{bmatrix} (\mathbf{N}\cdot\vec{z};\ \mathbf{N}\cdot\mathbf{\Omega_U})\ e^{2\pi i(k_3 x^3 + k_6 x^6)}$$

- Closed string fluxes as non-diagonal (oblique) magnetization. For example :

$$\vec{z} = \begin{pmatrix} x^4 \\ x^2 \end{pmatrix} + \mathbf{\Omega_U}\cdot\begin{pmatrix} x^1 \\ x^5 \end{pmatrix} \qquad \mathbf{\Omega_U} = \bar{\mathbf{B}}^{-1}\cdot\bar{\mathbf{U}}\cdot\bar{\mathbf{B}}\cdot\mathbf{\Omega}$$

$$\mathbf{N} = \begin{pmatrix} -I^1_{\alpha\beta} & -k_6 M \\ k_6 M & I^2_{\alpha\beta} \end{pmatrix} \qquad \mathbf{U} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

$$\mathbf{B} = \sqrt{2}\pi\begin{pmatrix} R_4 & 0 \\ 0 & R_2 \end{pmatrix} \qquad \tan\phi = \frac{\rho_{\mathrm{cl}}}{\rho_{\mathrm{op}}} = \frac{k_6\varepsilon}{R_6\sigma_-}$$

- Higher levels given in terms of Hermite functions

- Landau degeneracies for KK modes:

$$m_W^2 = \left(\frac{k_3}{R_3}\right)^2 + \left(\frac{k_6}{R_6}\right)^2 + (n+1)\rho + (2k-n)\frac{\sigma_+\sigma_-}{\rho} \qquad k, n \in \mathbb{Z}$$

$$\rho = \sqrt{\rho_{\rm op}^2 + \rho_{\rm cl}^2} = \sqrt{\sigma_-^2 + \left(\frac{k_6 \varepsilon}{R_6}\right)^2}$$

- We can define a total effective magnetization: $(F_2)_{\rm eff} = F_2^{\rm op} + F_2^{\rm cl}$

open / closed string duality

closed string		open string
e^6	$\leftrightarrow$	A
x^6	$\leftrightarrow$	Λ
F_3	$\leftrightarrow$	ω_3

- In order the duality to hold at all levels, non-perturbative massive states transforming as bound states of bifundamentals are required

- Alternatively we can use the machinery of non-commutative harmonic analysis [see Marchesano's talk]

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c \qquad \Leftrightarrow \qquad [\hat{\partial}_b, \hat{\partial}_c] = -f^a_{bc} \hat{\partial}_a$$

- There is a correspondence between KK wavefunctions and irreducible unitary reps. of the isometry algebra:

$$B_{\vec{\omega}}(g) = \sum_{\gamma \in \Gamma} \pi_{\vec{\omega}}(\gamma g) \varphi(\vec{s}_0)$$

- For charged modes we include the D9-brane U(1) generators

$$[\tilde{D}_m, \tilde{D}_n] = -f^p_{mn} \tilde{D}_p + i F^\alpha_{mn} U_\alpha$$
$$[\tilde{D}_m, U_\alpha] = [U_\alpha, U_\beta] = 0$$

(Kaloper-Myers algebra)

- We study the irreps using the orbit method. The algebra contains also irreps. corresponding to the conjectured non-perturbative states

- Kaloper-Myers algebra is only part of the 4D $\mathcal{N} = 4$ gauged supergravity algebra (large amount of global symmetries)

$$
\begin{aligned}
[X^a, X^b] &= -\,\tilde{F}^{abp} Z_p + Q_p^{ab} X^p + \sqrt{2}\, p_k^I (c_I^k)^{ab} X_k^I \,, \\
[X^a, Z_b] &= -Q_b^{ap} Z_p \,, \\
[X^a, X_k^I] &= \sqrt{2}\, p_k^I (\mathcal{F}_k^I)^{aq} Z_q \,, \\
[X^a, \bar{X}^b] &= Q_p^{ab} \bar{X}^p - \tilde{F}^{abp} \bar{Z}_p + \sqrt{2}\, p_k^I (c_I^k)^{ab} \bar{X}_k^I \,, \\
[\bar{X}^a, Z_b] &= [X^a, \bar{Z}_b] = -Q_b^{ap} \bar{Z}_p \,, \\
[X^a, \bar{X}_k^I] &= [\bar{X}^a, X_k^I] = \sqrt{2}\, p_k^I (\mathcal{F}_k^I)^{aq} \bar{Z}_q \,, \\
[\bar{X}^a, \bar{X}^b] &= P_p^{ab} \bar{X}^p - \tilde{H}^{abp} \bar{Z}_p + \sqrt{2}\, q_k^I (c_I^k)^{ab} \bar{X}_k^I \,, \\
[\bar{X}^a, \bar{Z}_b] &= -P_b^{ap} \bar{Z}_p \,, \\
[\bar{X}^a, \bar{X}_k^I] &= \sqrt{2}\, q_k^I (\mathcal{F}_k^I)^{aq} \bar{Z}_q \,,
\end{aligned}
$$

[see also talk by G.Villadoro]

[Aldazabal, PGC, Rosabal]

Conjecture: irreducible unitary reps. of the full $\mathcal{N} = 4$ gauged supergravity algebra classify all massive excitations (KK, windings, non-perturbative states) of untwisted BPS sectors

- Using similar techniques we can also compute wavefunctions for scalars and fermions, in terms of bosonic wavefunctions.

- Let us illustrate all this machinery at work in some examples. We compute the full perturbative spectrum of light and massive modes for stacks of magnetized D9-branes in closed string flux vacua.

- For $N = 2$ supersymmetric vacua each irrep. leads to a tower of $N = 4$ vector multiplets, plus few extra $N = 2$ hypers.

- For $N = 1$ vacua, flux induced μ-terms break this structure further:

$$\begin{pmatrix} \mathcal{C}_1 & \mathcal{C}_2 \end{pmatrix} \begin{pmatrix} \varepsilon_\mu & m_{\mathcal{B}} \\ m_{\mathcal{B}} & 0 \end{pmatrix} \begin{pmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \end{pmatrix}$$

Each irrep. leads to a tower of N = 1 vector multiplets, plus two chiral multiplets with masses,

$$m_{\mathcal{C}_\pm}^2 - \varepsilon_\mu m_{\mathcal{C}_\pm} - m_{\mathcal{B}}^2 = 0 \quad \Longrightarrow \quad m_{\mathcal{C}_\pm}^2 = \frac{1}{4}\left(\varepsilon_\mu \pm \sqrt{\varepsilon_\mu^2 + 4m_{\mathcal{B}}^2}\right)^2$$

Example 1.

Multiplets	$(\text{Mass})^2$	Degeneracy
$(\mathcal{V}^\alpha)_{k_1,k_2,k_3,k_4,k_5}$	$\Delta^2_{k_1,k_2,k_3,k_4,k_5}$	1
$(\mathcal{V}^\alpha)^{(k,\delta_1,\delta_4)}_{n,k_3,k_6}$	$\|\varepsilon\|\Delta_{k_6}(n+1)+\Delta^2_{k_3,k_6}$	$(Mk_6)^2(n+1)$
$(\mathcal{H}^\alpha)^{(\delta_1,\delta_4)}_{k_3,k_6}$	$\Delta^2_{k_3,k_6}$	$(Mk_6)^2$

Table 1: Spectrum of neutral $\mathcal{N}=4$ and $\mathcal{N}=2$ supermultiplets for D9-brane fields in $\mathcal{N}=2$ flux compactifications with vanishing μ-terms.

Multiplets	$(\text{Mass})^2$	Degeneracy
$(\mathcal{V}^{\alpha\beta})^{(k,\delta_1,\delta_4)}_{n,k_3,k_6}$	$\rho(n+1)+\Delta^2_{k_3,k_6}$	$[(Mk_6)^2-I^1_{\alpha\beta}I^2_{\alpha\beta}](n+1)$
$(\mathcal{H}^{\alpha\beta})^{(j_1,j_2)}_{k_3,k_6}$	$\Delta^2_{k_3,k_6}$	$(Mk_6)^2-I^1_{\alpha\beta}I^2_{\alpha\beta}$
$(\mathcal{H}^{\alpha\beta})^{(j_1,j_2)}_0$	0	$\|I^1_{\alpha\beta}I^2_{\alpha\beta}\|$

Table 2: Spectrum of charged $\mathcal{N}=4$ and $\mathcal{N}=2$ multiplets for D9-brane fields in the same model.

Example 2.

Multiplets	$(\text{Mass})^2$	Degeneracy
$(\mathcal{A}^\alpha)_{k_1,k_2,k_4,k_5}$	$\Delta^2_{k_1,k_2,k_4,k_5}$	1
$(\mathcal{A}^\alpha)^{(\delta)}_{n,k_3,k_4,k_5}$	$\lvert\varepsilon_\mu\rvert\Delta_{k_3}(2n+1)+\Delta^2_{k_3,k_4,k_5}$	$\lvert k_3M_3\rvert$
$(\mathcal{A}^\alpha)^{(\delta)}_{n,k_2,k_4,k_6}$	$\lvert\varepsilon_\mu\rvert\Delta_{k_6}(2n+1)+\Delta^2_{k_2,k_4,k_6}$	$\lvert k_6M_6\rvert$
$(\mathcal{A}^\alpha)^{(\delta_2,\delta_5)}_{n,k_3,k_4,k_6}$	$\lvert\varepsilon_\mu\rvert\Delta_{k_3,k_6}(2n+1)+\Delta^2_{k_3,k_6}$	l.c.m.$(\lvert k_3M_3\rvert,\lvert k_6M_6\rvert)$
$(\mathcal{C}^\alpha_\pm)_{k_1,k_2,k_4,k_5}$	$\frac{1}{4}\left(\varepsilon_\mu\pm\sqrt{\varepsilon_\mu^2+4\Delta^2_{k_1,k_2,k_4,k_5}}\right)^2$	1
$(\mathcal{C}^\alpha_\pm)^{(\delta)}_{n,k_3,k_4,k_5}$	$\frac{1}{4}\left(\varepsilon_\mu\pm\sqrt{\varepsilon_\mu^2+4\lvert\varepsilon_\mu\rvert\Delta_{k_3}(2n+1)+4\Delta^2_{k_3,k_4,k_5}}\right)^2$	$\lvert k_3M_3\rvert$
$(\mathcal{C}^\alpha_\pm)^{(\delta)}_{n,k_2,k_4,k_6}$	$\frac{1}{4}\left(\varepsilon_\mu\pm\sqrt{\varepsilon_\mu^2+4\lvert\varepsilon_\mu\rvert\Delta_{k_6}(2n+1)+4\Delta^2_{k_2,k_4,k_6}}\right)^2$	$\lvert k_6M_6\rvert$
$(\mathcal{C}^\alpha_\pm)^{(\delta_2,\delta_5)}_{n,k_3,k_4,k_6}$	$\frac{1}{4}\left(\varepsilon_\mu\pm\sqrt{\varepsilon_\mu^2+4\varepsilon_\mu\Delta_{k_3,k_6}(2n+1)+4\Delta^2_{k_3,k_6}}\right)^2$	l.c.m.$(\lvert k_3M_3\rvert,\lvert k_6M_6\rvert)$
$(\mathcal{H}^\alpha)_0$	0	1
$(\mathcal{A}^\alpha)_0$	0	1
$\mathcal{C}^\alpha_3$	ε_μ^2	1

Table 1: Spectrum of neutral $\mathcal{N}=2$ and $\mathcal{N}=1$ supermultiplets for D9-brane fields in $\mathcal{N}=1$ compactifications with non-vanishing μ-term.

- There are modes that are insensitive to the closed string flux. In the regime of diluted fluxes, these are the lightest ones.

⇩

Explains why in this regime, the (truncated) 4D theory is insensitive to an expansion in harmonics of the fluxless manifold.

- We have compared the couplings obtained for the lightest modes, with the ones derived from 4D supergravity:

$$W = \int_{\mathcal{M}_6} \Omega \wedge (F_3 + i e^{-\phi/2} dJ)$$

- 2-point couplings match results previously derived in [PGC, Grana]

$$m_{3/2} = e^{\hat{K}/2} \langle \hat{W} \rangle = \frac{3}{4\sqrt{2s}} f_{ij}^{k} \qquad \mu_{kk} = \frac{e^{-\hat{K}/2} Z_{k\bar{k}}}{\sqrt{2s}} f_{ij}^{\bar{k}}$$

- 3-point couplings match the fluxless $N = 2$ result,

$$Y_{123} = \bar{Y}_{213} = \frac{1}{g_s^{1/4} \mathrm{Vol}_{\mathcal{M}_6}^{1/2}} \int_{\mathcal{M}_6} (\Psi_1^{\beta\alpha,(j_1,j_2)})^\dagger \tilde{\gamma}^3 \Psi_2^{\beta\alpha,(j_1',j_2')} = -i g_{YM} \delta_{j_1 j_1'} \delta_{j_2 j_2'}$$

Conclusions

- Suit of tools for computing wavefunctions in toroidal compactifications with magnetized D9-branes and closed string fluxes. These allow to compute the full spectrum of KK modes for D9-brane fields.

- Fluxes affect mainly the massive spectrum of KK modes, except for few flux induced μ-terms

⇓

gauge threshold corrections in flux compactifications (work in progress)

- There is a lot of information encoded in the irreps. of the gauged supergravity algebra ⇒ extra non-perurbative charged modes ?

- For more phenomenological models (chirality), one needs to consider also D5-branes.

- It would be interesting to understand how the warping fits into this picture. Required for applications to the AdS/CFT correspondence.

[see talk by G.Shiu]