Global Aspects of F-theory Compactification

Kang Sin Choi
Kyoto University

with Tae-Won Ha (Bonn)

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F-theory

IIB string has $SL(2,\mathbb{Z})$ symmetry

axion-dilaton $\tau \equiv$ complex structure of a torus

- 1 more complex dimension:
  Elliptic equation, with one section

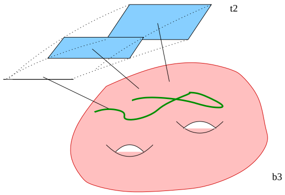
$$y^2 = x^3 + fx + g$$

  dim 2 − 1, genus 1: torus.

- In total 12 real dimensions:
  Calabi–Yau manifold, with more compact base space B'

$$f \sim K_{B'}^{-4}, \quad g \sim K_{B'}^{-6}.$$

  f, g are holomorphic polynomials of degrees $8, 12$ on B', resp.



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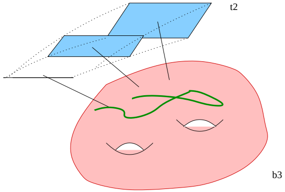

- Fibered: τ vary on B'

$$j(\tau) = f^3/\Delta, \quad \Delta = 4f^3 + 27g^2 \sim K_{B'}^{-12}$$

- Going close to $\Delta = 0$ surface, fiber singular.

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- To be Calabi–Yau manifold

$$f \in H^0_{\bar\partial}(B, -4K_B), \quad g \in H^0_{\bar\partial}(B, -6K_B)$$

f, g are resp. holomorphic polynomials of orders 8, 12 on B.

- Fibered: τ vary on B

$$j(\tau) = f^3/\Delta, \quad \Delta = 4f^3 + 27g^2$$

- Going close to $\Delta = 0$ surface, fiber singular.
- Gauge symmetry: how singular the fiber is ord (f, g, Δ).
  - Identification: Kodaira Table.
  - Equation: Tate's algorithm. [Bershadsky et al].

ord f	ord g	ord Δ	name
0	0	n	A_{n-1}
2	≥ 3	n+6	D_{n+4}
≥ 2	3	n+6	D_{n+4}
≥ 3	4	8	E_6
3	≥ 5	9	E_7
≥ 4	5	10	E_8

[Kodaira]

In general Δ is reducible. How to reduce?

Gauge symmetry

Singularity of the fiber

- gauge symmetry of the same name.

Matter fields

- off-diagonal component of the adjoint. [Katz Vafa]
  cf. Bifundamentals at the intersections of branes.

Ex. $U(m+n) \to U(m) \times U(n)$

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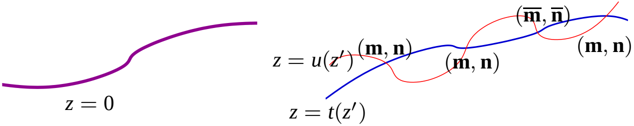
Ex. $U(m+n) \to U(m) \times U(n)$

$$y^2 = x^2 + (z - u(z'))^m (z - t(z'))^n$$

- If $u = t$ the symmetry is enhanced to $U(m+n)$.
- Even $u \neq t$ at $\{z = u\} \cap \{z = t\}$, local symmetry enhancement.
- Branching

$$(\mathbf{m}+\mathbf{n})^2 \to (\mathbf{m}^2, \mathbf{1}) + (\mathbf{1}, \mathbf{n}^2) + (\mathbf{1}, \mathbf{1}) + (\mathbf{m}, \mathbf{n}) + (\overline{\mathbf{m}}, \overline{\mathbf{n}}).$$

Chiral fields are localized



$(\overline{\mathbf{m}}, \overline{\mathbf{n}})$: *CPT* conjugate.

Intersection and divisors

Divisor

- Codimension one subspace specified by an equation
- Ex. $(x-a_0)^2(x-a_1)(x-a_2)^{-3}=0.$

$$D = 2P_0 + P_1 - 3P_2$$

- Extended to higher dimension

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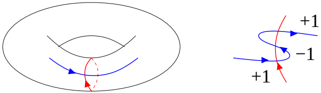
Intersection number

- A natural product between homological cycles
- Ex. On T^2, two one-cycles C_1 and C_2,



$$C_1 \cdot C_2 = +1.$$

- Curves: the **net** number of intersections (topological quantity).
- Surfaces: the intersection divisors (higher codimension object).

Matter curves

Ex. $U(m+n) \to U(m) \times U(n)$

$$y^2 = x^2 + (z-u)^m (z-t)^n$$

$$C_1 = \{z = u(z')\}, \quad C_2 = \{z = t(z')\}.$$

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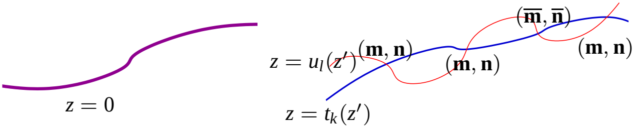
$$(\mathbf{m}+\mathbf{n})^2 \to (\mathbf{m}^2, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{n}^2) \oplus (\mathbf{1}, \mathbf{1}) + (\mathbf{m}, \mathbf{n}) + (\overline{\mathbf{m}}, \overline{\mathbf{n}}).$$



Under the reduction

- $u = t$: $D = (m+n)C$.
- $u \neq t$: $D = mC_1 + nC_2$.
  $(\mathbf{m}, \mathbf{n})$ is localized at

$$C_1 \cdot C_2 = \{z = u(z')\} \cap \{z = t(z')\} = \sum m_a P_a.$$

Matter curves [Katz, Vafa] [Beasley, Heckman, Vafa]

Calabi–Yau manifold

12D with 32 SUSY: On Calabi–Yau 4-fold, we have $\mathcal{N} = 1$ SUSY in 4D.

direction	0 1 2 3	4 5 6 7	8 9	10 11
	$M^{1,3}$	Calabi–Yau 4-fold		
definition of F-theory	″	B_3'		T
F-theory on K3 = heterotic on T	″	B_2	K3	
K3 = T fiber over $\mathbb{P}^1$	″	B_2	$\mathbb{P}^1 = S^2$	T

General structure: B_3' is a $\mathbb{P}^1$ fibration over B_2.

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$r \cdot (r + t) = 0$. [Friedan, Morgan, Witten]
Putting the dual gauge group $E_8 \times E_8$ on r, $(r + t)$, resp.

$$
\begin{aligned}
F &= -4K_{B_3'} = 4r + 4(r+t) + 8t \\
G &= -6K_{B_3'} = 5r + 5(r+t) + 2r + 6c_1(B_2) + t, \\
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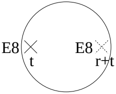

Two ends of the interval of heterotic-M-theory [Horava, Witten] [Morrison, Vafa I]
Information on B_2 is its divisors $\{s_i\}$. t, $c_1(B_2)$ are also expressed in terms of them.
Maximal gauge symmetry at r is $E_8 \times E_8$ + zero size instantons (blowing-ups on the base).
cf. two global sections: $\mathrm{Spin}(32)/\mathbb{Z}_2$ [Aspinwall, Gross]

Global consistency condition

Ex. Case $B_1 = \mathbb{P}^1$. A $\mathbb{P}^1$ fibration over this gives the Hirzebruch surface $\mathbb{F}_n$.

0 1 2 3 4 5	6 7	8 9	10 11
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G &= -6\,K_{B_2} = 5C_0 + 5(C_0+nf) + 2C_0 + (12+n)f\,, \\
D &= -12\,K_{B_2} = 10C_0 + 10(C_0+nf) + \underbrace{4C_0 + (24+2n)f}_{D'}\,.
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Induced 6-dimensional objects

$$C_0 \cdot D' = 2(12-n), \quad (C_0+nf)\cdot D' = 2(12+n) \quad \text{cf. } \mathbb{Z}_2 \text{ monodromy.}$$

Bianchi identity on the heterotic side with backgroud bundles $\mathcal{V}_1, \mathcal{V}_2$.

$$c_2(\mathcal{V}_1) + c_2(\mathcal{V}_2) + \delta n_3 = c_2(\mathrm{K3}) = 24$$

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Some of 24 points are blown-up. 4D compactification: missing part

$$\frac{\chi(X_4)}{24} = n_3 + \frac{1}{2}\int_{X_4} G_4 \wedge G_4.$$

Sufficiently smooth Calabi–Yau condition = ‘charge conservation’ of ‘branes’

Symmetry breaking preserving this form.

Symmetry breaking

Along $\Delta = 0$, gauge theory on the 8D worldvolume.
Field contents

direction	0 1 2 3	4 5 6 7	8 9	10 11
geometry	$M^{1,3}$	B	$\mathbb{P}^1$	T^2
fields	A_μ	A_m	φ_{89}	(τ)

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1. $\phi_{89} \sim K_B \otimes \mathrm{adj}G$
   - adjoint Higgs
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2. $A_m \sim \Omega_B \otimes \text{adj}G$
   - HYM equation with DUY condition: instanton solution
   - background gauge field on the brane
   - analogous to magnetized brane
   - blowing up some intersection of Δ = replacing the divisors

Reduction of the discriminant locus Δ

Reduction of discriminant locus

A nontrivial scalar profile $\langle\varphi\rangle$ gives rise the reduction. $\varphi \sim K_B \otimes adjG_S$
We re-decompose D **within** $E_8 \times E_8$.
Ex. $E_8 \longrightarrow E_6 \times U(2)$

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  cf. S_3 plays no role in gauge theory.

- Instanton number untouched

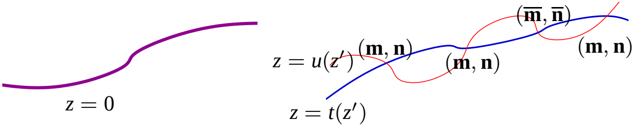
$$\mathbf{248} \longrightarrow (\mathbf{3},\mathbf{1}) + \langle(\mathbf{1},\mathbf{1})\rangle + (\mathbf{1},\mathbf{78}) + (\mathbf{2},\mathbf{1})_3 + (\mathbf{1},\mathbf{27})_2 + (\mathbf{2},\mathbf{27})_1 + CPT \text{ conj},$$

'Off-diagonal' matters are localized along the matter curves

$$S_1 \cdot S_2 = \sum m_a \Sigma_{12}^a$$

Matter curves

Line bundle background
: ‘off-diagonal’ components with different $U(1)$ charges.



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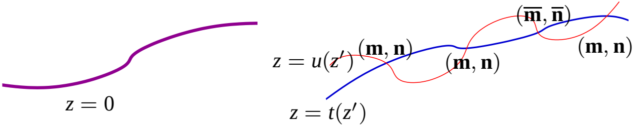

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- $C \sim C_i \sim C_j$

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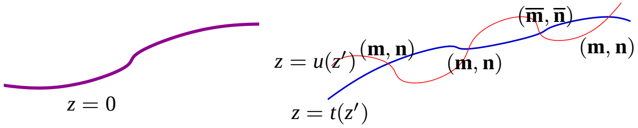

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- $C_i \not\sim C_j$

$$10r \longrightarrow 2(r + 6t) + 6(r - 2t)$$

$$C_1 \cdot C_2 = (r + 6t) \cdot (r - 2t) = 4 - n$$

  if $n \leq 4$, we have $(4 - n)(\mathbf{2}, \mathbf{27})$s.
  $n = 4$ 'parallel separtaion'
  cf. If $n > 4$, the minimal gauge group should be bigger than E_7. [Morrison, Vafa]

Spectrum

We have obtained

1. Gauge surfaces $D = \sum \operatorname{ord} \Delta_i S_i + D'$
   by the decomposition preserving the $E_8 \times E_8$ structure
2. Matter curves $S_i \cdot S_j = \sum m_a \Sigma_{ij}^a$
   from the intersections

We can also turn on the background gauge bundle $\langle A_m \rangle \longrightarrow \mathcal{V}$
Multiplicity: index theorem

$$\chi(S_i, \mathcal{V}_i) = \int_{S_i} \mathrm{ch}(\mathcal{V}_i) \mathrm{Td}(S_i)$$

Conclusion

We studied global issues of F-theory compactification. The important problem is

decomposition of the discriminant locus

- Intersection theory is useful for enumerative operation among geometric objects.
- The adjoint scalar φ normal to the base B parameterizes the geometry of discriminant locus.
  $\langle \varphi \rangle \neq 0$ corresponding to reducing the discriminant locus.
- Preserving the charges of discriminant locus: susy conditions, 'brane charges', instanton no are preserved.
  We also need 3-branes.
- We have analogous phenomena of parallel separtaion and recombination in the D-brane picture.
- Chiral fermions emerge as 'off-diagonal' component of the adjoint during the reduction.
  We can calculate their matter curve and localization
- With background gauge field, we obtain the spectrum using the index theorem.