Global Aspects of F-theory Compactification

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with Tae-Won Ha (Bonn)

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F-theory

IIB string has $SL(2, \mathbb{Z})$ symmetry

axion-dilaton $\tau \equiv$ complex structure of a torus

- 1 more complex dimension:
  Elliptic equation, with one section

$$y^2 = x^3 + fx + g$$

$\dim 2 - 1$, genus 1: torus.

- In total 12 real dimensions:
  Calabi–Yau manifold, with more compact base space $B'$

$$f \sim K_{B'}^{-4}, \quad g \sim K_{B'}^{-6}. $$

$f, g$ are holomorphic polynomials of degrees 8, 12 on $B'$, resp.
F-theory

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$$f \sim K_{B'}^{-4}, \quad g \sim K_{B'}^{-6}.$$  

$f, g$ are holomorphic polynomials of degrees 8, 12 on $B'$, resp.

► Fibered: $\tau$ vary on $B'$

$$j(\tau) = \frac{f^3}{\Delta}, \quad \Delta = 4f^3 + 27g^2 \sim K_{B'}^{-12}$$

► Going close to $\Delta = 0$ surface, fiber singular.
IIB string has $SL(2, \mathbb{Z})$ symmetry

axion-dilaton $\tau \equiv$ complex structure of a torus

- Elliptic equation, with one section
  \[ y^2 = x^3 + fx + g \]
  
  \text{dim} \ 2 - 1, \text{ genus} \ 1: \text{torus.}

- To be Calabi–Yau manifold
  \[ f \in H^0_\partial (B, -4K_B), \quad g \in H^0_\partial (B, -6K_B) \]
  
  $f, g$ are resp. holomorphic polynomials of orders 8, 12 on $B$.

- Fibered: $\tau$ vary on $B$
  \[ j(\tau) = \frac{f^3}{\Delta}, \quad \Delta = 4f^3 + 27g^2 \]

- Going close to $\Delta = 0$ surface, fiber singular.

- Gauge symmetry: how singular the fiber is $\text{ord} (f, g, \Delta)$.
  \[ \text{Identification: Kodaira Table.} \]
  \[ \text{Equation: Tate’s algorithm. [Bershadsky et al].} \]

- In general $\Delta$ is reducible. How to reduce?
Gauge symmetry

Singularity of the fiber

- gauge symmetry of the same name.

Matter fields

- off-diagonal component of the adjoint. \[\text{[Katz Vafa]}\]
  cf. Bifundamentals at the intersections of branes.

Ex. \(U(m + n) \rightarrow U(m) \times U(n)\)

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<th>ord (f)</th>
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<tbody>
<tr>
<td>0</td>
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<td>(n)</td>
<td>(A_{n-1})</td>
</tr>
<tr>
<td>2</td>
<td>(\geq 3)</td>
<td>(n+6)</td>
<td>(D_{n+4})</td>
</tr>
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Gauge symmetry

Singularity of the fiber
  ▶ gauge symmetry of the same name.

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  ▶ off-diagonal component of the adjoint. [Katz Vafa]
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Ex. $U(m+n) \rightarrow U(m) \times U(n)$

$$y^2 = x^2 + (z - u(z'))^m(z - t(z'))^n$$

▶ If $u = t$ the symmetry is enhanced to $U(m+n)$.
▶ Even $u \neq t$ at $\{z = u\} \cap \{z = t\}$, local symmetry enhancement.
▶ Branching

$$(m + n)^2 \rightarrow (m^2, 1) + (1, n^2) + (1, 1) + (m, n) + (\bar{m}, \bar{n})$$

Chiral fields are localized

$$(\bar{m}, \bar{n}) : CPT \ conjugate.$$
Intersection and divisors

Divisor

- Codimension one subspace specified by an equation
- Ex. \((x - a_0)^2(x - a_1)(x - a_2)^{-3} = 0\).

\[ D = 2P_0 + P_1 - 3P_2 \]

- Extended to higher dimension
Intersection and divisors

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Intersection number

- A natural product between homological cycles
- Ex. On \(T^2\), two one-cycles \(C_1\) and \(C_2\),

\[ C_1 \cdot C_2 = +1. \]

- Curves: the net number of intersections (topological quantity).
- Surfaces: the intersection divisors (higher codimension object).
Matter curves

Ex. $U(m + n) \rightarrow U(m) \times U(n)$

$$y^2 = x^2 + (z - u)^m (z - t)^n$$

$C_1 = \{ z = u(z') \}, \quad C_2 = \{ z = t(z') \}.$
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$$(m + n)^2 \rightarrow (m^2, 1) \oplus (1, n^2) \oplus (1, 1) + (m, n) + (\overline{m}, \overline{n}).$$

Under the reduction

- $u = t$: $D = (m + n)C$.
- $u \neq t$: $D = mC_1 + nC_2$.

$(m, n)$ is localized at

$$C_1 \cdot C_2 = \{ z = u(z') \} \cap \{ z = t(z') \} = \sum m_a P_a.$$ 

Matter curves [Katz, Vafa] [Beasley, Heckman, Vafa]
Calabi–Yau manifold

12D with 32 SUSY: On Calabi–Yau 4-fold, we have $\mathcal{N} = 1$ SUSY in 4D.

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General structure: $B'_3$ is a $\mathbb{P}^1$ fibration over $B_2$. 
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General structure: $B'_3$ is a $\mathbb{P}^1$ fibration over $B_2$.

$\mathbb{P}^1$ described by two line bundles $r$ (base) and $t$ ($\mathcal{O}_{B_2}(1)$ fiber) satisfying

$r \cdot (r + t) = 0$. [Friedan, Morgan, Witten]
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- **definition of F-theory**
  - $"$ B'$_3$ $T$
- **F-theory on K3 = heterotic on T**
  - $"$ $B_2$ $K3$
- **K3 = T fiber over $\mathbb{P}^1$**
  - $"$ $B_2$ $\mathbb{P}^1 = S^2$ $T$

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$$r \cdot (r + t) = 0.$$  [Friedan, Morgan, Witten]

Putting the dual gauge group $E_8 \times E_8$ on $r$, $(r + t)$, resp.

$$F = -4K_{B'_3} = 4r + 4(r + t) + 8t$$
$$G = -6K_{B'_3} = 5r + 5(r + t) + 2r + 6c_1(B_2) + t,$$
$$D = -12K_{B'_3} = 10r + 10(r + t) + 4r + 12c_1(B_2) + 2t.$$

Two ends of the interval of heterotic-M-theory  [Horava, Witten] [Morrison, Vafa I]

Information on $B_2$ is its divisors $\{s_i\}$. $t, c_1(B_2)$ are also expressed in terms of them.

Maximal gauge symmetry at $r$ is $E_8 \times E_8 +$ zero size instantons (blowing-ups on the base).

**cf.** two global sections: Spin(32)/$\mathbb{Z}_2$ [Aspinwall, Gross]
Global consistency condition

Ex. Case $B_1 = \mathbb{P}^1$. A $\mathbb{P}^1$ fibration over this gives the Hirzebruch surface $\mathbb{F}_n$.

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$\mathbb{F}_n$ is generated by two divisors $C_0, f$ such that $C_0 \cdot (C_0 + nf) = 0$, $C_0^2 = -n, f^2 = 0$. 
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\[ F = -4K_{B_2} = 4C_0 + 4(C_0 + nf) + 8f, \]
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\[ D = -12K_{B_2} = 10C_0 + 10(C_0 + nf) + 4C_0 + (24 + 2nf). \]

Induced 6-dimensional objects

\[ C_0 \cdot D' = 2(12 - n), \quad (C_0 + nf) \cdot D' = 2(12 + n) \quad \text{cf. } \mathbb{Z}_2 \text{ monodromy.} \]

Bianchi identity on the heterotic side with background bundles $\mathcal{V}_1, \mathcal{V}_2$.

\[ c_2(\mathcal{V}_1) + c_2(\mathcal{V}_2) + \delta n_3 = c_2(K3) = 24 \]
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\[
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\]

Some of 24 points are blown-up. 4D compactification: missing part

\[
\frac{\chi(X_4)}{24} = n_3 + \frac{1}{2} \int_{X_4} G_4 \wedge G_4.
\]

Sufficiently smooth Calabi–Yau condition = ‘charge conservation’ of ‘branes’

Symmetry breaking preserving this form.
Symmetry breaking

Along $\Delta = 0$, gauge theory on the 8D worldvolume.
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Two ways of gauge symmetry breaking

1. $\phi_{89} \sim K_B \otimes \text{adj}G$
   - adjoint Higgs
   - parameterizes the normal direction to the base ‘brane’
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   - tuning the parameters of $\Delta = \text{re-decomposing } D$
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   - nonconstant profile: intersecting branes
   - tuning the parameters of \( \Delta = \) re-decomposing \( D \)

2. \( A_m \sim \Omega_B \otimes \text{adj}G \)
   - HYM equation with DUY condition: instanton solution
   - background gauge field on the brane
   - analogous to magnetized brane
   - blowing up some intersection of \( \Delta = \) replacing the divisors

Reduction of the discriminant locus \( \Delta \)
Reduction of discriminant locus

A nontrivial scalar profile \( \langle \varphi \rangle \) gives rise the reduction. \( \varphi \sim K_B \otimes \text{adj}G_S \)

We re-decompose \( D \) within \( E_8 \times E_8 \).

Ex. \( E_8 \to E_6 \times U(2) \)

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\begin{align*}
F &= 4r + 4(r + t) + 8t \\
G &= 5r + 5(r + t) + 2r + 6c_1(B_2) + t, \\
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$\downarrow$

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4r &= 3S_1 + 0S_2 + 1S_3 \\
5r &= 4S_1 + 0S_2 + 1S_3 \\
10r &= 8S_1 + 2S_2 + 0S_3
\end{align*}
$$

cf. $S_3$ plays no role in gauge theory.

$\triangleright$ Instanton number untouched

\[ 248 \rightarrow (3, 1) + \langle (1, 1) \rangle + (1, 78) + (2, 1)_3 + (1, 27)_2 + (2, 27)_1 + CPT \text{ conj}, \]

‘Off-diagonal’ matters are localized along the matter curves

$$
S_1 \cdot S_2 = \sum m_a \Sigma^a_{12}
$$
Matter curves

Line bundle background:
- ‘off-diagonal’ components with different $U(1)$ charges.

\[ S_i \cdot S_j = \sum m^a \Sigma_{ij}^a \]

ex. $E_8 \rightarrow SU(2) \times E_6$ in 6D, we had $10r \rightarrow 2C_1 + 6C_2$. 

\[ z = u(z')^{(m, n)} \]
\[ z = t(z')^{(\bar{m}, \bar{n})} \]
\[ (m, n) \]
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$\ ▶ \ C \sim C_i \sim C_j$

$$10r \rightarrow 2r + 6r$$

$$C_1 \cdot C_2 = r^2 = -n.$$  

Not allowed unless the base is blown-up.
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\[C \sim C_i \sim C_j\]

\[10r \to 2r + 6r\]

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Not allowed unless the base is blown-up.

\[C_i \not\sim C_j\]

\[10r \to 2(r + 6t) + 6(r - 2t)\]

\[C_1 \cdot C_2 = (r + 6t) \cdot (r - 2t) = 4 - n\]

if $n \leq 4$, we have $(4 - n)(2, 27)$s.

$n = 4$ ‘parallel separation’

cf. If $n > 4$, the minimal gauge group should be bigger than $E_7$. [Morrison, Vafa]
We have obtained

1. **Gauge surfaces** $D = \sum \text{ord} \Delta_i S_i + D'$
   by the decomposition preserving the $E_8 \times E_8$ structure

2. **Matter curves** $S_i \cdot S_j = \sum m_a \Sigma_{ij}^a$
   from the intersections

We can also turn on the background gauge bundle $\langle A_m \rangle \to \mathcal{V}$

**Multiplicity: index theorem**

$$
\chi(S_i, \mathcal{V}_i) = \int_{S_i} \text{ch} (\mathcal{V}_i) \text{Td}(S_i)
$$
We studied global issues of F-theory compactification. The important problem is decomposition of the discriminant locus.

- Intersection theory is useful for enumerative operation among geometric objects.
- The adjoint scalar $\varphi$ normal to the base $B$ parameterizes the geometry of discriminant locus. $\langle \varphi \rangle \neq 0$ corresponding to reducing the discriminant locus.
- Preserving the charges of discriminant locus: susy conditions, ‘brane charges’, instanton no are preserved. We also need 3-branes.
- We have analogous phenomena of parallel separation and recombination in the D-brane picture.
- Chiral fermions emerge as ‘off-diagonal’ component of the adjoint during the reduction. We can calculate their matter curve and localization.
- With background gauge field, we obtain the spectrum using the index theorem.