An Effective Description of the Landscape

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Integrating Out v.s. Freezing Fields

Effective Theory

- Effective theories are an important tool for particle physics, leading to reliable simplifications.
- However, even at the classical level, the integration of H^i heavy fields:

$$\mathcal{L}_{eff}(L^\alpha) = \mathcal{L}(H^i(L^\alpha), L^\alpha) \quad , \quad \left.\frac{\partial \mathcal{L}}{\partial H^i}\right|_{H^i(L^\alpha)} = 0 \,. \tag{1}$$

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is a very hard task for many interesting systems, e.g., $4D$ fields theories arising from String theory compactifications.

A common approach is to study a simplified version

$$\mathcal{L}_{sim}(L^\alpha) = \mathcal{L}(H_0^i, L^\alpha) \,, \tag{2}$$

H_0^i the leading solution for H^i independent of L^α.

$\mathcal{N} = 1$ Supersymmetric theories

Freezing complete susy multiplets

The simplified version is described by

$$\begin{aligned} K_{sim}(L^\alpha, \bar{L}^{\bar{\alpha}}) &= K(H_0^i, \bar{H}_0^{\bar{i}}, L^\alpha, \bar{L}^{\bar{\alpha}})\ ,\ W_{sim}(L^\alpha) = W(H_0^i, L^\alpha)\ , \\ f_{sim,ab}(L^\alpha) &= f_{ab}(H_0^i, L^\alpha)\ . \end{aligned} \tag{3}$$

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Moduli stabilization (Two Steps Stabilization)

Flux compactifications, (e.g. KKLT)

$$W(T) = W_{flux}(U_0, S_0) + W_{np}(U_0, S_0, T)\,. \tag{4}$$

Some works addressing this:

- Extensions to KKLT. K. Choi, *et. al.* '04
- Conditions on the mass matrix. H. Abe, T. Higaki & T. Kobayashi '06
- Comments on the proper integration. S. P. de Alwis '05

UNDER WHAT CONDITIONS THIS IS A GOOD APPROXIMATION?

That is the Question

We focus on a *particular* class of $\mathcal{N} = 1$ SUSY theories inspired by flux compactifications.

Outline

1. Basic Set-up: No Matter Mutiplets
   - Component Approach
   - Supersymmetric Approach
2. Matter Multiplets and Gauge Interactions
   - $\mathcal{O}(1)$ Yukawa Couplings.
   - Vector Multiplets
3. Conclusions

$4D, \mathcal{N}=1$ SUGRA theory described by

$$W(H^i, L^\alpha) = W_0(H^i) + \epsilon\, W_1(H^i, L^\alpha)\,, \quad (5)$$

with $\epsilon \sim m_L/m_H$.

We allow arbitrary, but regular, Kähler potential.

- The eigenvalues of $g_{M\bar{N}} = \partial_M \partial_{\bar{N}} K$ are $\mathcal{O}(\epsilon^0)$.

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The following discussion excludes, then, the LARGE volume compactification scenario. This belongs to the so called *factorizable models*.

Canonically Normalized Fluctuations

- The eigenvalues of $g_{M\bar{N}} = \partial_M \partial_{\bar{N}} K$ are $\mathcal{O}(1)$.
- Then physical heavy and light modes are uniquely identified by its appearance in the scalar potential:

$$\begin{aligned} \langle g_{M\bar{N}} \rangle &= [(T^{-1})^\dagger (T^{-1})]_{M\bar{N}} \quad \text{(Cholesky decomposition)} \\ T &= \begin{pmatrix} (T_H)^i_j & 0 \\ (T_{HL})^\alpha_j & (T_L)^\alpha_\beta \end{pmatrix}, \\ \begin{pmatrix} \hat{H} \\ \hat{L} \end{pmatrix} &= T \cdot \begin{pmatrix} \hat{H}_c \\ \hat{L}_c \end{pmatrix}. \end{aligned} \tag{6}$$

The $\hat{H}^i_c$ are linear combinations of only the $\hat{H}^i$ with $\mathcal{O}(1)$ coefficients:

$$V(\langle H \rangle + \hat{H}, \langle L \rangle + \hat{L}) = V(\langle H \rangle + T_H \cdot \hat{H}_c, \langle L \rangle + T_{HL} \cdot \hat{H}_c + T_L \cdot \hat{L}_c)\,. \tag{7}$$

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The kinetic part and canonical normalization are irrelevant for our discussion, so we can focus in the potential part of the Lagrangian.

Scalar potential and vacuum structure

Scalar potential, $G = K + \ln|W|^2$,

$$V = e^G \left(g^{\bar{M}N} \overline{G}_{\bar{M}} G_N - 3 \right) . \tag{8}$$

Expanding in ϵ the leading terms are

$$V_0 = e^K \left(g^{\bar{M}N} \overline{F}_{0,\bar{M}} F_{0,N} - 3|W_0|^2 \right) , \tag{9}$$

where $F_{0,i} = \partial_i W_0 + (\partial_i K) W_0$, $F_{0,\alpha} = (\partial_\alpha K) W_0$.

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- Solutions for $F_{0,i} = 0$ are not decoupled for generic K.
- Decoupling requires $\langle W_0\rangle \sim \mathcal{O}(\epsilon)$. (This also ensures an $\mathcal{O}(\epsilon)$ hierarchy).

With this in mind solve $\partial_M V = 0$, $\phi^M = \phi^M_0 + \epsilon\phi^M_1$,

- $\mathcal{O}(1)$: decoupling at the SUSY solution $\partial_i W_0 = 0$, fixing all H^i_0 if all eigenvalues for $\partial_i\partial_j W_0$ are $\mathcal{O}(1)$.
- $\mathcal{O}(\epsilon)$: shift in H^i is determined

$$H^i_1 = -(\hat{K}^{-1})^i_j g^{\bar{j}M}\left(\partial_M W_1 + W\partial_M K\right), \text{ where } \hat{K}^{\bar{i}}_j = g^{\bar{i}k}\partial_k\partial_j W_0 . \tag{10}$$

- At $H^i = H^i_0 + \epsilon H^i_1$: $G_M = \mathcal{O}(1)$, $G^i = \mathcal{O}(\epsilon)$, $G^\alpha = \mathcal{O}(1)$.

Effective action

$$V_{full} = V(\langle H \rangle, L) + V_{int}(\langle H \rangle, L)\,.$$

At the Gaussian level,

$$V_{int} = -\frac{1}{2} V_I V^{IJ} V_J \big|_{H=\langle H \rangle}\ , \quad I = i, \bar{\imath}\,. \tag{11}$$

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Up to $\mathcal{O}(\epsilon^2)$ we have, $\tilde{g}_{i\bar{j}} = (g^{\bar{j}i})^{-1}$

$$\left.\begin{array}{r} \partial_i\partial_{\bar{j}} V|_0 = e^K \partial_{\bar{j}}\partial_{\bar{k}}\overline{W}_0\ g^{\bar{k}j}\ \partial_i\partial_j W_0 \\ \partial_i V|_1 = e^G \partial_i\partial_j W_0 G^j/\overline{W} \end{array}\right\} \Rightarrow V_{int} = -e^G G^i \tilde{g}_{i\bar{j}} \overline{G}^{\bar{j}}\,. \tag{12}$$

Thus with $\tilde{g}^{\bar{\alpha}\alpha} = (g_{\alpha\bar{\alpha}})^{-1}$, satisfying $\tilde{g}^{\bar{\alpha}\alpha} = g^{\bar{\alpha}\alpha} - g^{\bar{\alpha}i}\tilde{g}_{i\bar{j}} g^{\bar{j}\alpha}$,

$$\begin{aligned} V_{full} &= e^G\left[\overline{G}_{\bar{M}}\left(g^{\bar{M}N} - g^{\bar{M}i}\tilde{g}_{i\bar{j}}g^{\bar{j}N}\right)G_N - 3\right] + \mathcal{O}(\epsilon^3) \\ &= e^G\left(\tilde{g}^{\bar{\alpha}\alpha}G_\alpha\overline{G}_{\bar{\alpha}} - 3\right) + \mathcal{O}(\epsilon^3) = (1+\mathcal{O}(\epsilon))\,V_{sim}\,. \end{aligned} \tag{13}$$

E.o.m. and two derivative truncation

L. Brizi, M. Gómez-Reino & C. Scrucca '09

Exploit the fact we are working with a SUSY theory.

$$\mathcal{L} = \int d\theta^4 (-3e^{-K/3}\bar{\Phi}\Phi) + \left(\int d\theta^2 W + h.c. \right) . \tag{14}$$

The e.o.m. for a H^i chiral multiplet is

$$\partial_i W - \frac{1}{4}\overline{D}^2 \left(e^{-K/3}\bar{\Phi}\partial_i K \right) \Phi^{-2} = 0 \overset{\text{two derivatives}}{\Longrightarrow} \partial_i W = 0 . \tag{15}$$

The resulting effective theory is exact up to leading order in ∂^μ / m_H, $\psi^\alpha / m_H^{3/2}$, F^α / m_H^2 and F^Φ / m_H.

The two descriptions then differ at $(F^\alpha)^3$, $(F^\alpha)^2 F^\Phi$, $F^\alpha (F^\Phi)^2$ and $(F^\Phi)^3$, where

$$F^\Phi = \frac{1}{3} K_M F^M - e^{K/2}\overline{W} . \tag{16}$$

With $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$ all these extra terms are $\mathcal{O}(\epsilon^3)$.

Solving $\partial_i W = 0$

Solvin expanding in ϵ, $H^i = H_0^i + \epsilon H_1^i$ (now as a chiral multiplet!)

$$\partial_i W_0(H_0^i) = 0 \,, \quad H_1^i = -W_0^{ij} \partial_j W_1|_{H_0^i} \,. \tag{17}$$

So with $W_{sim} = W(H_0^i)$ and $K_{sim} = K(\bar{H}_0^{\bar{i}}, H_0^i)$,

$$\begin{aligned} W_{full} &= W_{sim} + \epsilon^2 \left(\frac{1}{2} \partial_i \partial_j W_0 H_1^i H_1^j + \partial_i W_1 H_1^i \right) + \mathcal{O}(\epsilon^3) \,, \\ K_{full} &= K_{sim} + \epsilon \left(\partial_i K_{sim} H_1^i + \partial_{\bar{i}} K_{sim} \bar{H}_1^{\bar{i}} \right) + \mathcal{O}(\epsilon^2) \,. \end{aligned} \tag{18}$$

Solving $\partial_i W = 0$

Solvin expanding in ϵ, $H^i = H^i_0 + \epsilon H^i_1$ (now as a chiral multiplet!)

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These corrections are clearly negligible,

$$V_{full} = \left(1 + \mathcal{O}(\epsilon)\right) V_{sim} \,. \tag{19}$$

Generalized setup: $\mathcal{O}(1)$ Yukawa couplings

Introducing C^α-multiplets with almost vanishing VEV's,

$$
\begin{aligned}
W &= W_0(H^i) + \eta\,\widetilde{W}_0(H^i, M^\mu, C^\alpha) + \epsilon\,W_1(H^i, M^\mu, C^\alpha)\,, \\
K &= K_0 + K_{1,\alpha\bar\beta}C^\alpha\bar C^{\bar\beta} + (K_{2,\alpha\beta}C^\alpha C^\beta + c.c.) + \mathcal{O}(C^3)\,,
\end{aligned} \tag{20}
$$

M^μ denoting any kind of multiplet with $\mathcal{O}(1)$, VEV.

$$
\begin{aligned}
\widetilde{W}_0 &= Y_{3,\alpha\beta\gamma}(H^i, M^\mu)C^\alpha C^\beta C^\gamma + \mathcal{O}(C^4)\,, \\
W_1 &= \widetilde{W}_1(H^i, M^\mu) + \mu_{2,\alpha\beta}(H^i, M^\mu)C^\alpha C^\beta + \mathcal{O}(C^3)\,.
\end{aligned} \tag{21}
$$

The following analysis can be generalized allowing $\mathcal{O}(1)$ mass terms.

Effective Theory

Solving $\partial_i W = 0$ around $\partial_i W_0(H_0^i) = 0$

$$
\begin{aligned}
W_{full} &= W_{sim} - \frac{1}{2}\left(\eta\partial_i\widetilde{W}_0 + \partial_i W_1\right) W_0^{ij}\left(\eta\partial_j\widetilde{W}_0 + \partial_j W_1\right) + \mathcal{O}(\eta^3, \eta^2\epsilon, \eta\epsilon^2, \epsilon^3)\,, \\
K_{full} &= K_{sim} - \eta\left[\partial_i K_{sim} W_0^{ij}\partial_j\widetilde{W}_0 + \partial_{\bar i}K_{sim}\overline{W}_0^{\bar i\bar j}\partial_{\bar j}\overline{\widetilde{W}}_0\right] + \mathcal{O}(\epsilon, \eta^2)\,.
\end{aligned} \tag{22}
$$

Corrections

In the field-space region $|C| \lesssim \mathcal{O}(\epsilon)$

C-dependent parts

- W_{sim} and K_{sim} are of $\mathcal{O}(\epsilon^3)$ and $\mathcal{O}(\epsilon^2)$ respectively.
- With $W_0 = \mathcal{O}(\epsilon)$ these induce $\mathcal{O}(\epsilon^4)$ terms in V.
- The induced couplings in W_{full} and K_{full} are at most of $\mathcal{O}(\epsilon^4)$ and $\mathcal{O}(\epsilon^3)$ respectively., then

$$V(C)_{full} = (1 + \mathcal{O}(\epsilon))\, V(C)_{sim}\,. \tag{23}$$

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Non-trustable operators

Schematically if $W \supset Y_N C^N$:

$$\delta W \supset \frac{1}{m_H} Y_{N_i} Y_{N_j} C^{N_i+N_j} \quad , \quad \delta K \supset \frac{1}{m_H} \partial_i K_{sim} Y_{N_i} C^{N_i} \; + h.c. \tag{24}$$

Gauge Dynamics (Valid also in flat space)

- Gauging an isometry group $\mathcal{G}$ generated by holomorphic Killing vectors X_A:
$$\delta_\lambda \phi^M = \lambda^A X_A^M \,, \quad \delta_\lambda \bar{\phi}^{\bar{M}} = \bar{\lambda}^A \bar{X}_A^{\bar{M}}. \tag{25}$$
With holomorphic gauge kinetic functions
$$f_{AB} = \delta_{AB}\, f_A(H^i, M^\mu, C^\alpha) \,, \quad \mathcal{R}e(f_A) = g_A^{-2} \,. \tag{26}$$
- D-term potential
$$V_D = \frac{1}{2} \sum_A g_A^2 D_A^2 \,, \quad \text{with} \quad D_A = i X_A^M G_M \,. \tag{27}$$

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Comments on freezing

- Gauge invariance of W_0, relates the e.o.m.'s $X_A^i \partial_i W_0 = 0$.
- Is not a meaningful gauge invariant statement for charged fields.

We impose
$$X_A^i = 0 \,. \tag{28}$$

Unbroken Symmetry (No charged M^μ)

The solution to $\partial_i W = 0$, H^i, now further induces

$$f_{AB,full} = f_{AB,sim} - \partial_i f_{AB} W_0^{ij} \partial_j \widetilde{W}_0 + \mathcal{O}(\epsilon, \eta^2)\,. \tag{29}$$

New terms in the scalar potential

$$\delta V_D \supset \frac{g_A^2 Y_{N_i}}{m_H} C^{N_i+4} + \frac{\epsilon g_A^2 \mu_{M_i}}{m_H} C^{M_i+4}\,. \tag{30}$$

Taking $C \sim \epsilon$ these are again negligible.

Comments

- Even at two derivative level neglecting the covariant derivatives misses FD and D^2 terms.
- In particular this approach cannot lead to g_A^4 terms.

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- In particular this approach cannot lead to g_A^4 terms.
- These are suppressed by powers of m_H.

Broken symmetry (Charged M^μ)

$\mathcal{G}$ spontaneously broken to $\mathcal{H}$: $\hat{a} \in \mathcal{G}/\mathcal{H}$, and $a \in \mathcal{H}$.

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SUSY integration of the Vector multiplet

Arkani-Hamed, Dine, Martin & Martin '98

- E.o.m. neglecting covariant derivatives, $\langle D \rangle / m_V^2 \ll 1$,

$$\partial_{V_{\hat{a}}} K = 0 \,. \tag{31}$$

- Gauge fixing: $M^{\hat{\mu}} = \langle M^{\hat{\mu}} \rangle = M_0^{\hat{\mu}}$, $M^{\hat{\mu}}$ such that $\langle \chi_{\hat{a},\hat{\mu}} \rangle \neq 0$, i.e., non-vanishin component in the would-be Goldstone direction.
- Denoting $L^{\mathcal{A}'}$ the remaining chiral fields and $V_{\hat{a}}^0(L^{\mathcal{A}'})$ the solutions, the effective theory is described by

$$K' = K(M_0^{\hat{\mu}}, L^{\mathcal{A}'}, V_{\hat{a}}^0(L^{\mathcal{A}'}), V_a) \tag{32}$$

Back to the unbroken case

The new theory is described by

$$K' \ , \quad W' = W_0(H^i) + \epsilon W_1(H^i, M_0^{\hat{\mu}}, L^{\alpha'}) \ , \quad f'_a = f(M_0^{\hat{\mu}}, L^{\mathcal{A}'}) \ , \tag{33}$$

and

- Gauge symmetry is un-broken.
- Is possible to define a simplified theory

$$K'_{sim} = K'(H_0^i, \bar{H}_0^{\bar{i}}) \ , \quad W'_{sim} = W'(H_0^i) \ , \quad f'_{a,sim} = f'_a(H_0^i) \ , \tag{34}$$

  and re-do our previous analysis for the matching.

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  and re-do our previous analysis for the matching.

- This simplified theory coincide with the one obtained from,

$$K_{sim} = K(H_0^i, \bar{H}_0^{\bar{i}}) \; , \quad W_{sim} = W(H_0^i) \; , \quad f_{A,sim} = f_A(H_0^i) \; , \tag{35}$$

  after the integration of the heavy vector multiplet using the very same gauge fixing.

Summary

- In systems where the superpotential for the "moduli" is of the form

$$W = W_0(H^i) + \epsilon W_1(H^i, M^\mu)\,, \tag{36}$$

with *arbitrary* sufficiently regular Kähler potential, freezing of the H chiral multiplets is a reliable approach provided that these are neutral and at H_0^i

$$\langle W_0 \rangle \sim \mathcal{O}(\epsilon)\,,\ \ \partial_i W_0 \sim \mathcal{O}(\epsilon)\,,\ \ \partial_i \partial_j W_0 \sim \mathcal{O}(1)\,. \tag{37}$$

- Higher order couplings not described by the simple description are due to the presence of $\mathcal{O}(1)$ couplings in the matter sector. $W \supset Y_M C^M$:

$$\delta W \supset Y_{M_1} Y_{M_2} C^{M_1+M_2}\quad,\quad \delta K \supset Y_M C^M\,.$$

The End

Thanks!