Integrability methods in AdS/CFT

Romuald A. Janik

Jagiellonian University
Krakow

Z. Bajnok, RJ: 0807.0499
Z. Bajnok, RJ, T. Łukowski: 0811.4448
Z. Bajnok, A. Hegedus, RJ, T. Łukowski: 0906.????

Outline

1. **Motivation**
2. **The worldsheet QFT of the superstring in $AdS_5 \times S^5$**
3. **Spectrum on a cylinder**
4. **The gauge theory side**
5. **Computation of Konishi anomalous dimension from strings in $AdS_5 \times S^5$**
6. **Generalizations: twist two operators**
7. **Generalizations: 5-loop Konishi**
8. **Conclusions**

Motivation

$\mathcal{N} = 4$ Super Yang-Mills theory $\equiv$ Superstrings on $AdS_5 \times S^5$

Parameters

't Hooft coupling

$\lambda = g_{YM}^2 N_c$

number of colours

N_c

Parameters

string scale

$\alpha'_{eff} \propto \frac{1}{\sqrt{\lambda}}$ (keeping $R_{AdS} = 1$)

string coupling

$g_s \propto \frac{1}{N_c}$ (keeping λ fixed)

- The gravity/string side is 'easy' at strong coupling
  – conventional supergravity description
  – strings are classical/semiclassical
- α' corrections difficult for the string worldsheet theory $\longrightarrow$ Difficult to extend to smaller λ.
- **Integrability:** Solve the worldsheet theory exactly for *any* α' (with $g_s = 0$)

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Goal: Solve exactly $\mathcal{N} = 4$ SYM or superstrings in $AdS_5 \times S^5$ in the large N_c limit for any value of the coupling $\lambda = g_{YM}^2 N_c$

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Gauge theory side:

- $\mathcal{N} = 4$ is an exact CFT
- Find the spectrum of the dilatation operator
- Equivalently, find the anomalous dimensions of all operators in $\mathcal{N} = 4$ SYM as a function of the coupling constant $g^2 = \lambda/16\pi^2$

$$\langle O(x)O(y)\rangle = \frac{const}{|x-y|^{2\Delta}}$$

String theory side:

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AdS/CFT dictionary

Operators in $\mathcal{N} = 4$ SYM	$\longleftrightarrow$	(quantized) string states in $AdS_5 \times S^5$
Single trace operators	$\longleftrightarrow$	single string states
Multitrace operators	$\longleftrightarrow$	multistring states
Large N_c limit	$\longleftrightarrow$	suffices to consider single string states
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The worldsheet QFT of the superstring in $AdS_5 \times S^5$

- Consider the worldsheet theory of the string in $AdS_5 \times S^5$ in a (generalized) light cone gauge
- Worldsheet hamiltonian corresponds to translations in global AdS time
- One $U(1)_R$ charge is uniformly spread on the string worldsheet
  – this defines the σ coordinate
  – identifies the *size* of the worldsheet cylinder with the charge J of the corresponding state
- One obtains a highly interacting theory
- The theory is *not* conformal (c.f. BMN limit/pp-wave)
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- Assuming quantum integrability one may proceed to solve the theory exactly on an infinite plane (historically this was considered in the spin-chain language [Beisert,Staudacher])
- Identify explicit global symmetry — $su_c(2|2) \times su_c(2|2)$ [Beisert]
- Guess the set of asymptotic states (using information from pp-wave limit/gauge theory)
- Find the S-matrix between these states which satisfies the Yang-Baxter Equation and has the appropriate global symmetry

$$S_{12}S_{23}S_{13} = S_{13}S_{23}S_{12}$$

- This fixes the S-matrix up to an overall scalar factor ($\equiv$ 'dressing phase')

$$S(p_1, p_2) = S_0(p_1, p_2) \cdot \left[\hat{S}_{su_c(2|2)}(p_1, p_2) \otimes \hat{S}_{su_c(2|2)}(p_1, p_2)\right]$$

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- [Beisert,Hernandez,Lopez], [Beisert,Eden,Staudacher] found an exact solution of the crossing equation — the S-matrix is currently known exactly
- Poles of the S-matrix lead to an infinite set of bound states labelled by Q. These have to be added to the set of asymptotic states
- The theory is solved in the infinite volume limit!
- The S-matrix is a highly nontrivial function of λ (equivalently α') which incorporates α' corrections to all orders!

Caveat: We still have to use this information to find the spectrum on a cylinder of finite size

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Spectrum on a cylinder

- In general this is a very complicated problem even for integrable QFT's
- Something may be said about large volume behaviour
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$$e^{ip_i L} = \prod_{k \neq i} S(p_i, p_k)$$

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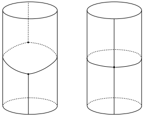

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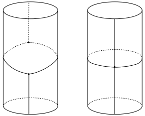

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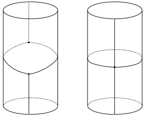

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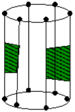
Gauge theory counterpart

In $\mathcal{N} = 4$ SYM compute anomalous dimensions from two-point correlation functions

$$\langle O(x) O(y) \rangle = \frac{const}{|x-y|^{2\Delta}}$$



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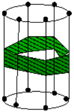

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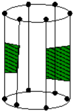

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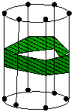

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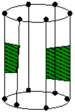

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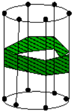

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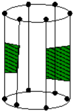

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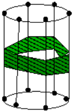

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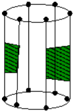

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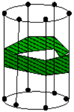

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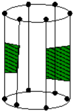

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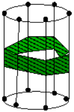

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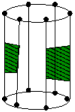

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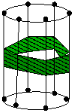

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The Konishi operator

- Simplest operator which is not protected by supersymmetry — the Konishi operator

$$\mathrm{tr}\,\Phi_i^2 \quad \longleftrightarrow \quad \mathrm{tr}\,Z^2X^2 + \ldots \quad \longleftrightarrow \quad \mathrm{tr}\,ZD^2Z + \ldots$$

- Its anomalous dimension should be given by the ABA exactly up to 3 loops:

$$E_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The true result is

$$E = E_{Bethe} + \Delta_{wrapping} E$$

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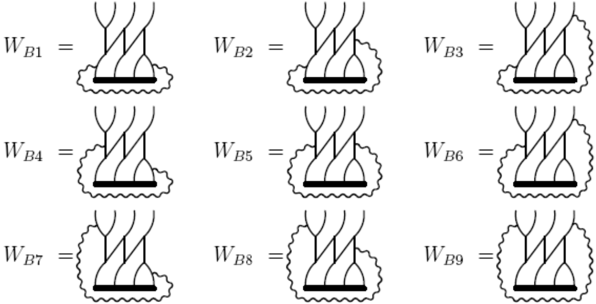


Figure C.1: Wrapping diagrams with chiral structure $\chi(1,2,3)$

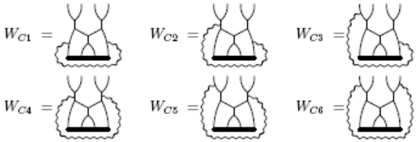


Figure C.2: Wrapping diagrams with chiral structure $\chi(1,3,2)$

$W_{C1} \rightarrow *$	1	$W_{C4} \rightarrow$ finite	
$W_{C2} \rightarrow *$	2	$W_{C5} \rightarrow -W_{C3}$	
$W_{C3} \rightarrow -W_{C5}$		$W_{C6} \rightarrow$ finite	

Table C.2: Results of D-algebra for diagrams with structure $\chi(1,3,2)$

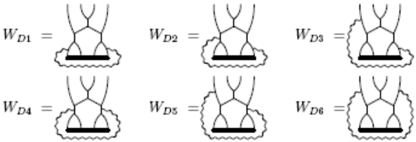
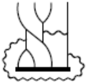
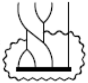
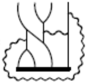


Figure C.3: Wrapping diagrams with chiral structure $\chi(2,1,3)$

$W_{E1} =$ 
$W_{E2} =$ 
$W_{E3} =$ 
$W_{E4} =$ 

$W_{E5} =$ 
$W_{E6} =$ 
$W_{E7} =$ 
$W_{E8} =$ 

$W_{E9} =$ 
$W_{E10} =$ 
$W_{E11} =$ 
$W_{E12} =$ 

$W_{E13} =$ 
$W_{E14} =$ 
$W_{E15} =$ 
$W_{E16} =$ 

$W_{E17} =$ 
$W_{E18} =$ 
$W_{E19} =$ 
$W_{E20} =$ 

$W_{E21} =$ 
$W_{E22} =$ 
$W_{E23} =$ 
$W_{E24} =$ 

$W_{E25} =$ 
$W_{E26} =$ 
$W_{E27} =$ 

$W_{F1} =$  $W_{F2} =$  $W_{F3} =$  $W_{F4} =$ 

$W_{F5} =$  $W_{F6} =$  $W_{F7} =$  $W_{F8} =$ 

$W_{F9} =$  $W_{F10} =$  $W_{F11} =$  $W_{F12} =$ 

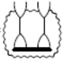
$W_{F13} =$  $W_{F14} =$  $W_{F15} =$  $W_{F16} =$ 

$W_{F17} =$  $W_{F18} =$  $W_{F19} =$  $W_{F20} =$ 

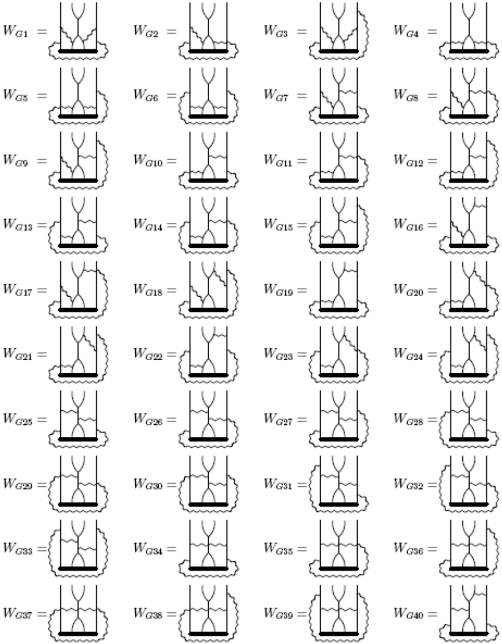
$W_{F21} =$  $W_{F22} =$  $W_{F23} =$  $W_{F24} =$ 

$W_{F25} =$  $W_{F26} =$  $W_{F27} =$  $W_{F28} =$ 

$W_{F29} =$  $W_{F30} =$  $W_{F31} =$  $W_{F32} =$ 

$W_{F33} =$  $W_{F34} =$  $W_{F35} =$  $W_{F36} =$ 

$W_{F37} =$ 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$W_{G13} =$ $W_{G14} =$ $W_{G15} =$ $W_{G16} =$
$W_{G17} =$ $W_{G18} =$ $W_{G19} =$ $W_{G20} =$
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$W_{G37} =$ $W_{G38} =$ $W_{G39} =$ $W_{G40} =$

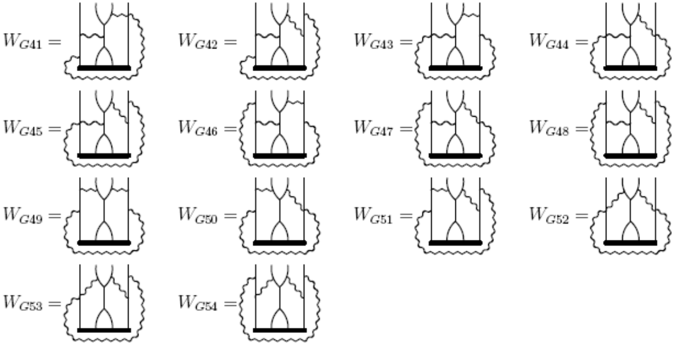
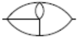


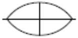
Figure C.6: Wrapping diagrams with chiral structure $\chi(1)$ (continued)

$I_1 = J_1 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{5}{4\varepsilon}\Big)$

$I_2 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{1}{\varepsilon}\Big(\frac{5}{4} - \zeta(3)\Big)\Big)$

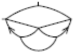
$I_3 = J_5 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{12\varepsilon^4} + \frac{1}{3\varepsilon^3} - \frac{5}{12\varepsilon^2} - \frac{1}{\varepsilon}\Big(\frac{1}{2} - \zeta(3)\Big)\Big)$

$I_4 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{6\varepsilon^4} + \frac{1}{3\varepsilon^3} + \frac{1}{3\varepsilon^2} - \frac{1}{\varepsilon}(1 - \zeta(3))\Big)$

$I_5 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}5\zeta(5)$

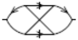
$I_6 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{12\varepsilon^2} - \frac{7}{12\varepsilon}\Big)$

$I_7 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}(-\zeta(3))$

$I_8 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{4\varepsilon^2} - \frac{11}{12\varepsilon}\Big)$

$I_9 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(\frac{1}{2}\zeta(3) - \frac{5}{2}\zeta(5)\Big)$

$I_{10} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{2} - \frac{1}{2}\zeta(3) + \frac{5}{2}\zeta(5)\Big)$

$I_{11} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{4} - \frac{3}{2}\zeta(3) + \frac{5}{2}\zeta(5)\Big)$

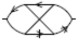
$I_{12} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{8} - \frac{1}{4}\zeta(3) + \frac{5}{4}\zeta(5)\Big)$

Table C.8: Loop integrals for 4-loop wrapping diagrams. The arrows of the same type indicate contracted spacetime derivatives

Perturbative 4-loop result for the Konishi

- The final result for the anomalous dimension of the Konishi operator is

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

($-2584 \longrightarrow -2496$ after the appearance of our paper)

- The wrapping part is thus

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

Compute the same 4-loop anomalous dimension from string theory

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The Konishi computation

- For the Konishi at 4 loops only the F-term like expression contributes

$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left(\frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons ($Q = 1$)
  - Q-magnon bound states (in the antisymmetric representation)
  - Moreover we should sum over all internal states of these particles $\equiv \sum_b$
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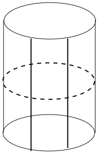

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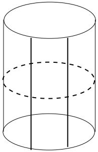

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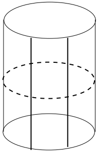

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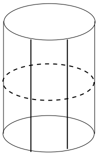

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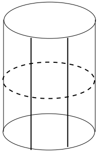

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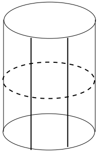

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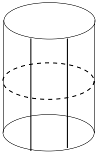

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$$\left(\frac{z^-}{z^+}\right)^2 = \frac{16g^4}{(Q^2+q^2)^2} + \ldots$$

- The scalar part gives

$$S_{Q-1}^{scalar,sl(2)} = \frac{3q^2 - 6iQq + 6iq - 3Q^2 + 6Q - 4}{3q^2 + 6iQq - 6iq - 3Q^2 + 6Q - 4} \cdot \frac{16}{9q^4 + 6(3Q(Q+2)+2)q^2 + (3Q(Q+2)+4)^2}$$

- The matrix part (summed over b) evaluates to

$$S_{Q-1}^{matrix,sl(2)} = \frac{5184 Q^2 (3q^2 + 3Q^2 - 4)^2 g^4}{(q^2+Q^2)^2((3q - 3iQ + 3i)^2 - 3)^2}$$

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- The result is

$$\sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4 - 3Q^2 + 1)^4 (27Q^6 - 27Q^4 + 36Q^2 + 16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$num(Q) = 7776\,Q(19683\,Q^{18} - 78732\,Q^{16} + 150903\,Q^{14} - 134865\,Q^{12} + \\ + 1458\,Q^{10} + 48357\,Q^8 - 13311\,Q^6 - 1053\,Q^4 + 369\,Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

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$$\begin{aligned}num(Q) =& 7776Q(19683Q^{18}-78732Q^{16}+150903Q^{14}-134865Q^{12}+\\ &+1458Q^{10}+48357Q^8-13311Q^6-1053Q^4+369Q^2-10)\end{aligned}$$

- Two last terms give at once $864\,\zeta(3)-1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping}E=(324+864\zeta(3)-1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

- The integral over q can be carried out analytically by residues
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Generalizations: twist two operators

- We computed the 4-loop wrapping corrections for arbitrary twist two operators

$$O_M = \mathrm{tr}\, Z D^M Z + \ldots$$

  where D is a light-cone derivative and M is any even integer [Bajnok,RJ,Łukowski]
- The Konishi operator is just $O_{M=2}$.
- So far there is no gauge theory perturbative computation for arbitrary M
- The 4-loop correction obeys the maximal transcendentality principle of [Kotikov,Lipatov]
- There is a prediction of the pole structure of the anomalous dimensions analytically continued to $M = -1 + \omega$ from BFKL and NLO BFKL equations.
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Generalizations: 5-loop Konishi (to appear)

- The computation of the Konishi anomalous dimension from multiparticle Lüscher corrections can be extended to 5 loops.
- Several new features appear...
- Due to the special form of kinematics already an *infinite* set of coefficients of the BES dressing phase starts to contribute
- One has to take into account the modification of the Bethe ansatz quantization due to the virtual particles
- How to check the result?
  - Not much hope for a direct perturbative computation but there are other cross-checks...
  - The final answer should have a nice transcendentality structure...
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Conclusions

- The agreement of the Konishi computation with the 4-loop weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The result came from a single diagram – in contrast to direct perturbative computations in gauge theory which are much more complex
- This suggests that one can use string theory methods of AdS/CFT as an efficient calculational tool also at *weak coupling*
- May be possible to access information on strings in highly curved $AdS_5 \times S^5$
- Various groups proposed formulations (functional relations/systems of nonlinear integral equations) which aim to give an exact solution for any λ
- Use 5-loop Konishi as a testing ground for these formulations...

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