Phenomenological Aspects of Magnetized Brane Models

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based on

Abe, T.K., Ohki, arXiv: 0806.4748 [hep-th]

Abe, Choi, T.K., Ohki, 0812.3534, 0903.3800, 0904.2631

1 Introduction

Extra dimensional field theories,

in particular

string-derived extra dimensional field theories,

play important roles in particle physics

as well as cosmology .

Chiral theory

When we start with extra dimensional field theories,

how to realize chiral theories is one of important issues from the viewpoint of particle physics.

$$i\gamma^{m} D_{m}\psi = 0$$

Zero-modes between chiral and anti-chiral fields are different from each other on certain backgrounds, e.g. CY.

Torus with magnetic flux

$$i\gamma^m D_m \psi = 0$$

The limited number of solutions with non-trivial backgrounds are known.

Torus background with magnetic flux is one of interesting backgrounds, where one can solve zero-mode Dirac equation.

Magnetic flux

Indeed, several studies have been done in both extra dimensional field theories and string theories with magnetic flux background.

In particular, magnetized D-brane models are T-duals of intersecting D-brane models.

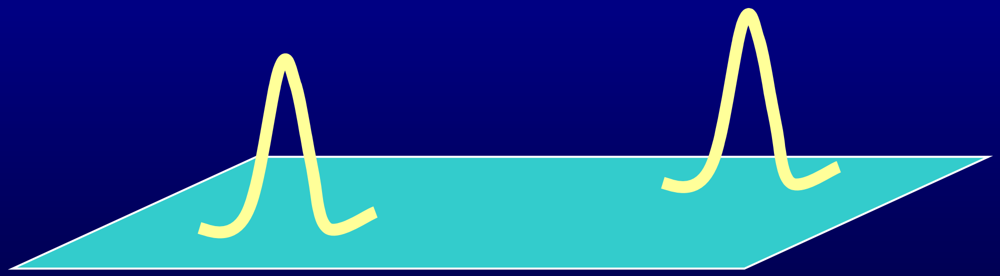
Several interesting models have been constructed in intersecting D-brane models.

Magnetized D-brane models

The (generation) number of zero-modes
is determined by the size of magnetic flux.

Zero-mode profiles are quasi-localized.

=> several interesting phenomenology



Phenomenology of magnetized brane models

It is important to study phenomenological aspects of magnetized brane models such as Yukawa couplings and higher order n-point couplings in 4D effective theory, their symmetries like flavor symmetries, Kahler metric, etc.

It is also important to extend such studies on torus background to other backgrounds with magnetic fluxes, e.g. orbifold backgrounds.

2. Magnetized torus model

We start with N=1 super Yang-Mills theory in D = 4+2n dimensions.

We consider 2n-dimensional torus compactification with magnetic flux background.

Higher Dimensional SYM theory with flux

4D Effective theory <= dimensional reduction

$$\mathcal{L}_{SYM} = -\frac{1}{4g^2} Tr\{F^{MN}F_{MN}\} + \frac{i}{2g^2} Tr\{\bar{\lambda}\Gamma^M D_M \lambda\}$$

$$\begin{aligned} \lambda(x^\mu, y^m) &= \sum_n \chi_n(x^\mu) \times \psi_n(y_m), \\ A_M(x^\mu, y^m) &= \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m) \end{aligned}$$

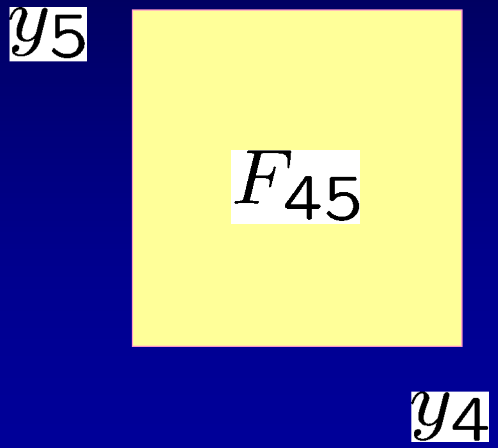
$$\Rightarrow$$

$$\begin{aligned} i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\ \Delta_6 \phi_{n,M}(y) &= M^2_{n,M} \phi_{n,M} \end{aligned}$$

The wave functions $\longrightarrow$ eigenstates of corresponding internal Dirac/Laplace operator.

U(N) theory on T6

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & \ddots & \\ 0 & & m_k \mathrm{I}_{N_k} \end{pmatrix}$$



$$y_4 \sim y_4 + 1,$$
$$y_5 \sim y_5 + 1$$

$\mathrm{I}_N : (N \times N)$ identity matrix

$$z = y_4 + i y_5$$

gauge group

$$U(N) \Rightarrow \prod_{i=1}^{k} U(N_i)$$

Bi-fundamental

Gaugino fields in off-diagonal entries correspond to bi-fundamental matter fields and the difference M= m-m' of magnetic fluxes appears in their Dirac equation.

Zero-modes

Cremades, Ibanez, Marchesano, '04

$$\psi^{j}_{M}(z) = N_M \exp[i\pi M z \,\mathrm{Im}(z)] \cdot \vartheta\begin{bmatrix} j/M \\ 0 \end{bmatrix}(Mz, iM)$$

N_M : normalization factor, $\quad j = 1, \cdots, M$

Zero-mode w.f. = gaussian x theta-function

$$\psi^{i}_{M}(z) \cdot \psi^{j}_{N}(z) = \sum_{m=1}^{M+N} y_{ijm} \psi^{i+j+Mm}_{M+N}(z),$$

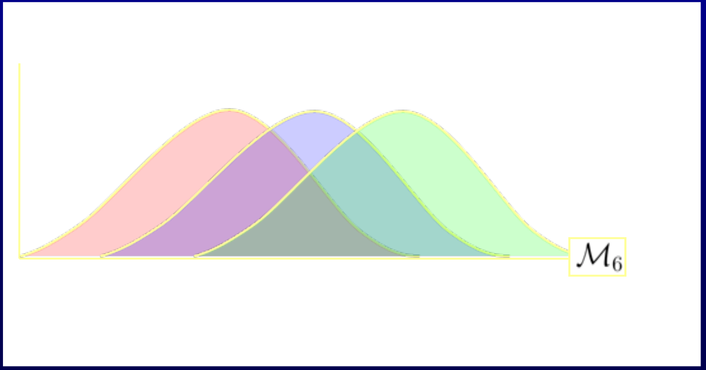
up to normalization factor

$$y_{ijm} = \vartheta\begin{bmatrix} (Ni - Mj + MNm)/(MN(M+N)) \\ 0 \end{bmatrix}(0, iMN(M+N))$$

3. N-point couplings

The N-point couplings are obtained by overlap integral of their zero-mode w.f.'s.

$$Y = g \int d^2 z \, \psi_M^i(z) \psi_N^j(z) \ldots \psi_P^k(z)$$



3-point couplings

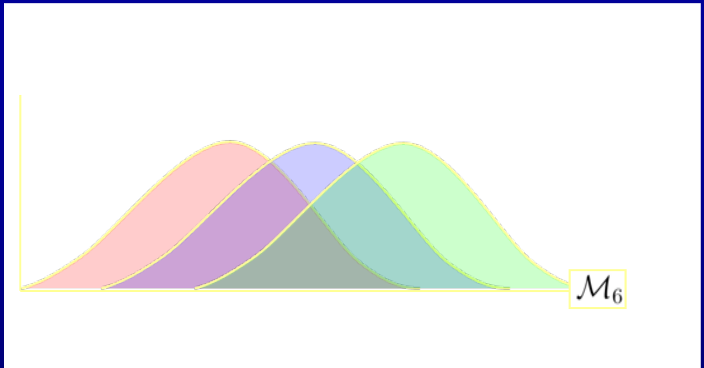
Cremades, Ibanez, Marchesano, ‘04

The 3-point couplings are obtained by overlap integral of three zero-mode w.f.’s.

$$Y_{ijk} = \int d^2 z\, \psi_M^i(z) \psi_N^j(z) \left(\psi_{M+N}^k(z)\right)^*$$

$$\int d^2 z\, \psi_M^i(z) \left(\psi_M^k(z)\right)^* = \delta^{ik}$$

$$Y_{ijk} = \sum_{m=1}^{M+N} \delta_{i+j+mM,k}\, y_{ijm}$$



up to normalization factor

Selection rule

$$\delta_{i+j+mM,k} \Rightarrow i + j + mM = k(M+N)$$

$$i + j = k \pmod{g} \quad \text{when} \quad g = \gcd(M,N)$$

Each zero-mode has a Zg charge,
which is conserved in 3-point couplings.

$$y_{ijm} = \vartheta\begin{bmatrix} (Ni - Mj + MNm)/(MN(M+N)) \\ 0 \end{bmatrix}(0, iMN(M+N))$$

up to normalization factor

4-point couplings

Abe, Choi, T.K., Ohki, '09

The 4-point couplings are obtained by overlap integral of four zero-mode w.f.'s.

$$Y_{ijkl} = \int d^2 z \, \psi^i_M(z) \psi^j_N(z) \psi^k_P(z) \left(\psi^l_{M+N+P}(z) \right)^*$$

split

$$\int d^2 z d^2 z' \psi^i_M(z) \psi^j_N(z) \delta(z - z') \psi^k_P(z') \left(\psi^l_{M+N+P}(z') \right)^*$$

insert a complete set

$$\delta(z - z') = \sum_{\text{all modes}} \left(\psi^n_K(z) \right)^* \psi^n_K(z')$$

up to normalization factor

$$Y_{ijk\bar{l}} = \sum_s y_{ij\bar{s}} y_{sk\bar{l}}$$

for K=M+N

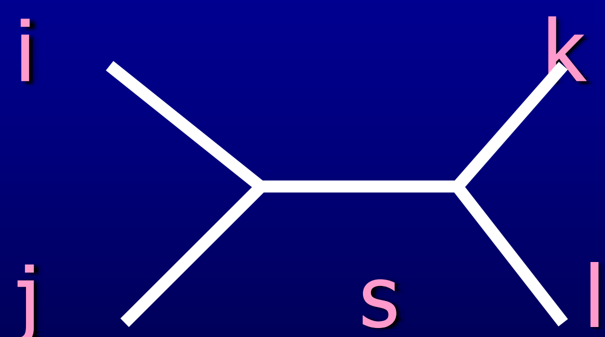
4-point couplings: another splitting

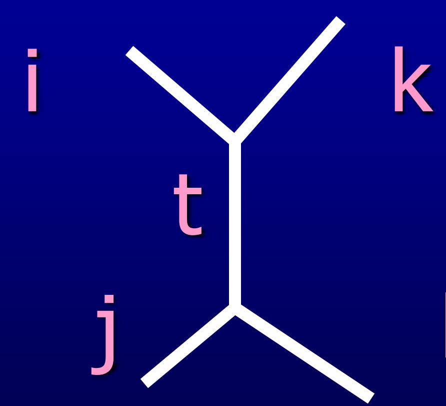
$$\int d^2z\,d^2z'\,\psi^i_M(z)\,\psi^k_P(z)\,\delta(z-z')\,\psi^j_N(z')\left(\psi^l_{M+N+P}(z')\right)^*$$

$$Y_{ijk\bar{l}} = \sum_t y_{ik\bar{t}}\,y_{tj\bar{l}}$$

$$Y_{ijk\bar{l}} = \sum_s y_{ij\bar{s}}\,y_{sk\bar{l}}$$

$$Y_{ijk\bar{l}} = \sum_t y_{ik\bar{t}}\,y_{tj\bar{l}}$$





N-point couplings

Abe, Choi, T.K., Ohki, '09

We can extend this analysis to generic n-point couplings.

N-point couplings = products of 3-point couplings
= products of theta-functions

This behavior is non-trivial. (It's like CFT.)
Such a behavior would be satisfied
not for generic w.f.'s, but for specific w.f.'s.

However, this behavior could be expected
from T-duality between magnetized
and intersecting D-brane models.

T-duality

The 3-point couplings coincide between magnetized and intersecting D-brane models.

explicit calculation

Cremades, Ibanez, Marchesano, '04

Such correspondence can be extended to 4-point and higher order couplings because of CFT-like behaviors, e.g.,

$$Y_{ijk\bar{l}} = \sum_{s} y_{ij\bar{s}} y_{sk\bar{l}}$$

Abe, Choi, T.K., Ohki, '09

Heterotic orbifold models

$$\left(\text{open string amplitude}\right)^2 = \left(\text{closed string amplitude}\right)$$

$$\begin{pmatrix} \text{couplings in} \\ \text{intersecting brane} \end{pmatrix}^2 = \begin{pmatrix} \text{coupling in} \\ \text{heterotic orbifold} \end{pmatrix}$$

Our results would be useful to n-point couplings of twsited sectors in heterotic orbifold models.

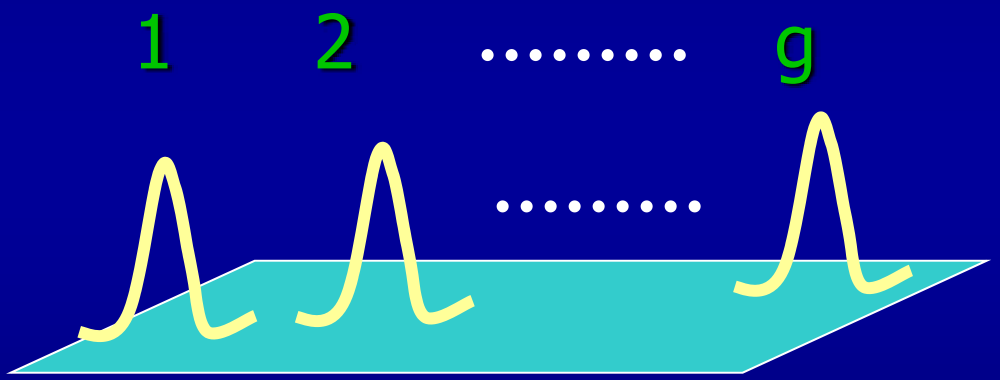
Twisted strings on fixed points might correspond to quasi-localized modes with magnetic flux,

zero modes profile = gaussian x theta-function

Non-Abelian discrete flavor symmetry

The coupling selection rule is controlled by
Z_g charges.

For $M=g$,



Effective field theory also has a cyclic permutation symmetry of g zero-modes.

Non-Abelian discrete flavor symmetry

The total flavor symmetry corresponds to
the closed algebra of

$$\begin{pmatrix} 1 & & & \\ & \rho & & \\ & & \ddots & \\ & & & \rho^{g-1} \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho = \exp[2\pi i / g]$$

That is the semidirect product of Zg x Zg and Zg.

For example,

g=2 D4

g=3 Δ(27) details => Ohki's talk

Cf. heterotic orbifolds, T.K. Raby, Zhang, '04
T.K. Nilles, Ploger, Raby, Ratz, '06

3. Magnetized orbifold models

We consider orbifold compactification
with magnetic flux.

Orbifolding is another way to obtain chiral theory.

Magnetic flux is invariant under the Z2 twist.

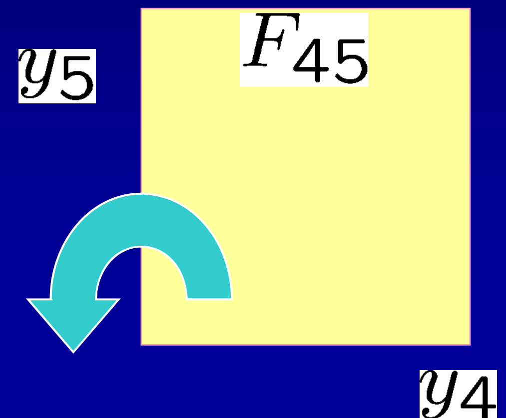
We consider the Z2 and Z2xZ2' orbifolds.

Orbifold with magnetic flux

Abe, T.K., Ohki, '08

$$Z_2 : z(= y_4 + iy_5) \to -z$$

$$Z_2 : \psi_M^j(z) \to \psi_M^{M-j}(z)$$



$$Z_2 \text{ even mode} : \psi_M^j + \psi_M^{M-j}$$

$$Z_2 \text{ odd mode} : \psi_M^j - \psi_M^{M-j}$$

Note that there is no odd massless modes on the orbifold without magnetic flux.

Zero-modes

Even and/or odd modes are allowed as zero-modes on the orbifold with magnetic flux.

On the usual orbifold without magnetic flux, odd zero-modes correspond only to massive modes.

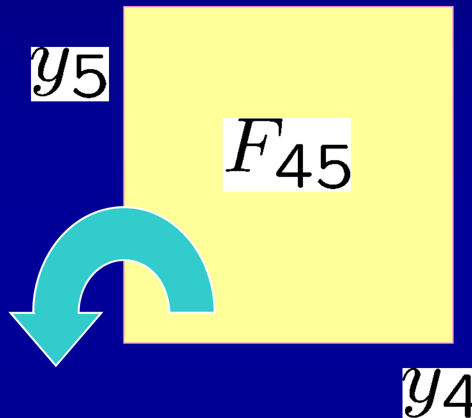
Adjoint matter fields are projected by orbifold projection.

Orbifold with magnetic flux

Abe, T.K., Ohki, '08

The number of even and odd zero-modes

$M = I^{ab}$	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4



We can also embed Z2 into the gauge space.

$$\mathsf{Z}_2 : \psi(\mathsf{y}_4, \mathsf{y}_5) \to \psi(-\mathsf{y}_4, -\mathsf{y}_5) = (-\mathsf{i})\Gamma^4\Gamma^5 \mathsf{P}\psi(-\mathsf{y}_4, -\mathsf{y}_5)$$

$$(P^2 = 1)$$

=> various models, various flavor structures

Localized modes on fixed points

We have degree of freedom to
introduce localized modes on fixed points
like quarks/leptons and higgs fields.

That would lead to richer flavor structure.

Summary

We have studied phenomenological aspects of magnetized brane models.

N-point couplings are comupted.
4D effective field theory has non-Abelian flavor symmetries, e.g. D4, Δ(27).

Orbifold background with magnetic flux is also important.