# Fixing D7 Brane Positions by F-Theory Fluxes

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A. Braun, A. Hebecker, CL, R. Valandro,
Nucl.Phys.B815:256-287,2009 [arXiv:0811.2416 [hep-th]]

# Motivation

- F-Theory: Nonperturbative version of type IIB string theory

  [Vafa;Sen]

- Add two auxiliary dimensions, singularities of compactification manifold encode brane positions
- Recently, lots of interest in F-theory for model building interest

  [Beasley,Heckman,Vafa;Saulina,Schäfer-Nameki;Bourjaily;Tatar,Watari. . . ]

- Local models do not address global constraints like tadpole cancellation
- Four-form flux can stabilise moduli, including brane positions
- Simple example: F-Theory on $K3 \times \widetilde{K3}$, where $\widetilde{K3}$ is an elliptic fibration over $\mathbb{P}^1$ [Görlich et al.;Lust et al.; Aspinwall,Kallosh;Dasgupta et al.]
- Includes as special case the type IIB orientifold $K3 \times T^2/\mathbb{Z}_2$

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# F-Theory/M-Theory Duality

M-Theory on $X_6 \times T^2$ → (Compactify one $S^1$) → Type IIA on $X_6 \times S^1_A$ → ($T$-dualise along $S^1_A$) → Type IIB on $X_6 \times S^1_B$ $(R_B = 1/R_A)$

Fibrewise duality: $X_6 \times T^2 \rightsquigarrow$ elliptically fibred $CY_4$
dual to type IIB on base of fibration

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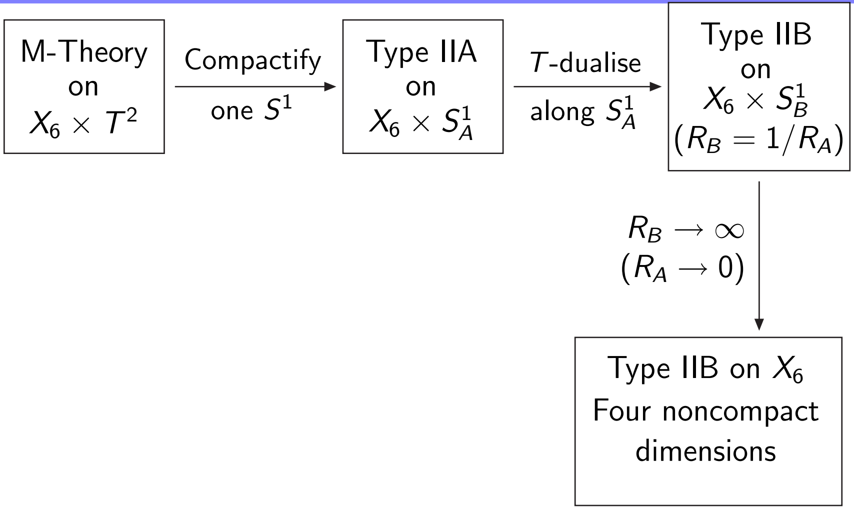


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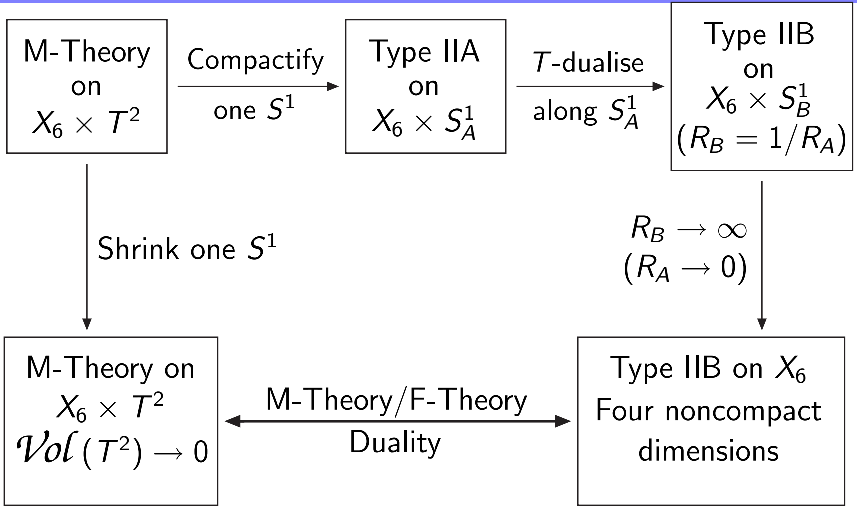


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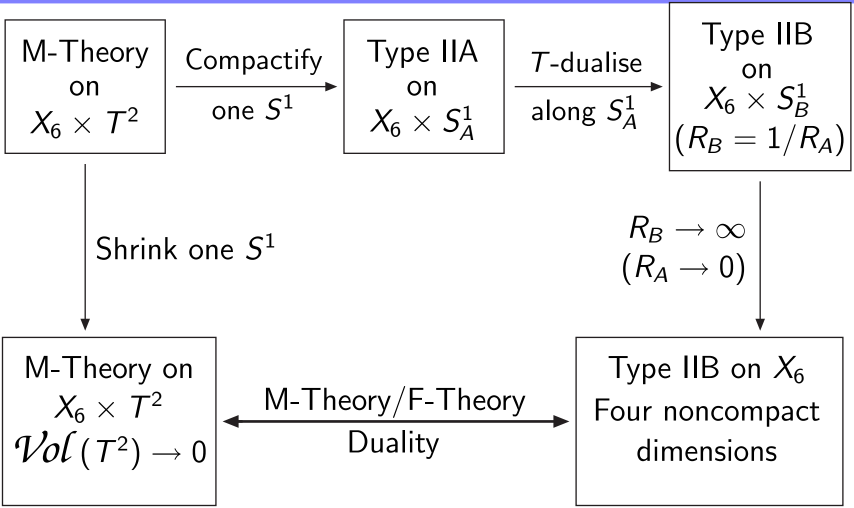


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# K3: Calabi–Yau Two-Fold

- $H^2(K3, \mathbb{R})$ has signature $(3, 19)$
- Holomorphic two-form and Kähler form spanned by three real forms $\omega_i$ with $\omega_i \cdot \omega_j = \delta_{ij}$ and overall volume $\nu$:

$$\omega = \omega_1 + \mathrm{i}\omega_2 \qquad\qquad j = \sqrt{2\nu}\,\omega_3$$

- K3 is hyperkähler, i.e. $SO(3)$ rotating the $\omega_i$ $\rightsquigarrow$ geometry fixed by positive-norm three-plane $\Sigma \subset H^2(K3, \mathbb{R})$ and $\nu$
- Moduli space has $3 \times 19 + 1 = 58$ dimensions
- Integral basis for $H^2(K3)$ with intersection matrix $U \oplus U \oplus U \oplus (-E_8) \oplus (-E_8)$, where $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $E_8$ is Cartan matrix of $E_8$

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# K3: Elliptic Fibration and F-Theory Limit

- For an elliptically fibred $K3$, require integral cycles $B$ and $F$ (base and fibre) with
  - intersection matrix $\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$
  - $B \cdot \omega = F \cdot \omega = 0$

$\Rightarrow$ $(B, F)$ spans a $U$ block, and we can parametrise the Kähler form as

$$j = b\, B + f\, F + c^a u_a \qquad \text{(where } u_a \cdot \omega = 0\text{)}$$

- F-theory limit: Fibre volume shrinks to zero $\Rightarrow$ $b \to 0$. $K3$ volume is $\nu \sim bf - c^a c^a$, so we have to take $c^a \to 0$ as fast as $\sqrt{b}$ (as intuitively expected)
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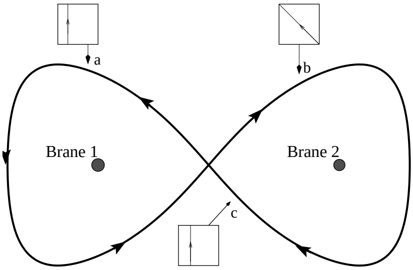
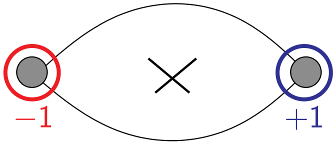
# Cycles Between Branes

[Braun,Hebecker,Triendl]

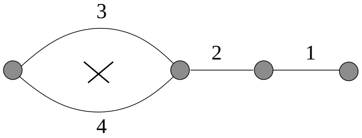



- One leg in the base, one in the fibre torus
- Shrink to zero when the branes are moved on top of each other.
- They are topologically a sphere ↔ self-intersection −2.
- Cycles meeting at a brane intersect once, cycles encircling O planes ( $\times$ ) do not intersect

# Shrinking Cycles and Gauge Enhancement

Intersection matrix of shrinking cycles determines gauge group:
Consider e.g. $T^2/\mathbb{Z}_2$ orientifold: One O7, four D7s $\rightsquigarrow SO(8)$




$$\rightsquigarrow \begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

In appropriate basis, complex structure of $\widetilde{K3}$ is [Braun,Hebecker,Triendl]

$$\omega = \frac{\alpha}{2} + u\, e_2 + s\, \frac{\beta}{2} - \left( u\, s - \frac{z^2}{2} \right) e_1 + z_I \widehat{E}_I$$

Explicit mapping between complex structure and brane positions!

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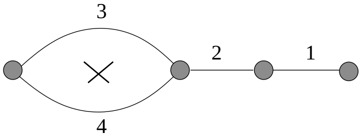


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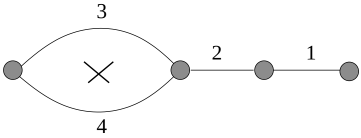


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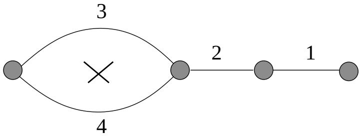


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- $z_I$: brane positions, $z_I = 0$ is $SO(8)^4$ point

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# Flux Potential

- Type IIB: Three-form flux $G_3$ on the bulk, two-form gauge flux $F_2$ on the branes can stabilise geometric and brane moduli
- In M-theory, these are combined into four-form flux $G_4$ (brane moduli become four-form geometric moduli)
- Consistency conditions:
  - Flux quantisation: flux needs to be integral
  - Tadpole cancellation (without spacetime-filling M2 branes)

$$\frac{1}{2}\int_{K3\times\widetilde{K3}} G_4 \wedge G_4 = \frac{\chi}{24} = 24$$

- $G_4$ needs to have exactly one leg on each on base and fibre for Lorentz invariance, hence two on each $K3$: $G = G^{I\Lambda}\eta_I \wedge \tilde{\eta}_\Lambda$, but no flux along $B$ or $F$

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# Potential

- Flux potential ($\mathcal{V}$ is the volume) : [Haack,Louis]

$$V = \frac{1}{4\mathcal{V}^3}\left(\int_{K3\times\widetilde{K3}} G\wedge *G - \frac{\chi}{12}\right)$$

- $K3\times\widetilde{K3}$ is not a proper $CY_4$: Holonomy is $SU(2)\times SU(2)$
- $G_4$ induces map $G: H^2(\widetilde{K3})\to H^2(K3)$ and its adjoint $G^a$ by

$$G\tilde{\eta} = \int_{\widetilde{K3}} G\wedge\tilde{\eta} \qquad G^a\eta = \int_{K3} G\wedge\eta$$

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# $K3 \times \widetilde{K3}$ Flux Potential

$$V = -\frac{1}{2(\nu \cdot \widetilde{\nu})^3} \left( \sum_j \| G\, \tilde{\omega}_j \|_\perp^2 + \sum_i \| G^a \omega_i \|_{\widetilde{\perp}}^2 \right)$$

Here $\|\cdot\|_\perp^2$ is the norm orthogonal to the three-plane

- Positive definite potential
- Manifestly symmetric under $SO(3)$
- Minima at $V = 0$:

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# Minima: Existence, Flat Directions

- Minkowski minima do not necessarily exist: $G^a G$ must be diagonalisable and positive semi-definite (not guaranteed although $G^a G$ is self-adjoint, since metric is indefinite!)
- Flat directions generally exist and are desired: M-theory moduli become part of 4D vector fields in F-theory limit $\rightsquigarrow$ fixing these moduli breaks the gauge group (rank-reducing)
- Flux also induces explicit mass term for three-dimensional vectors
- Vacua can preserve $\mathcal{N} = 4$, $\mathcal{N} = 2$ or $\mathcal{N} = 0$ supersymmetry in four dimensions, depending on the action of $G$ on the three-plane

# Stabilisation Strategy

- F-theory limit fixes Kähler form (up to base volume), $j = f\, F$
- Holomorphic two-form determines shrinking cycles, i.e. gauge enhancement
- To stabilise a desired brane configuration:
  - Identify set of shrinking cycles to obtain desired brane stacks
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We give explicit examples of

- The $T^2/\mathbb{Z}_2$ orientifold with $SO(8)^4$: Four stacks of four D7 branes and one O7 plane each
- Moving one brane off a stack. $\rightsquigarrow SO(8)^3 \times SO(6) \times U(1)$ or $SO(8)^3 \times SO(6)$
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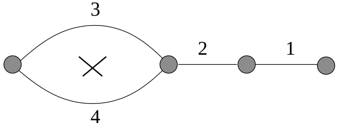

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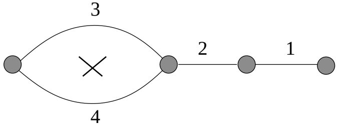


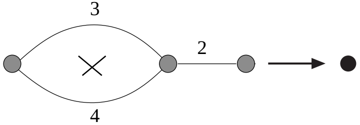

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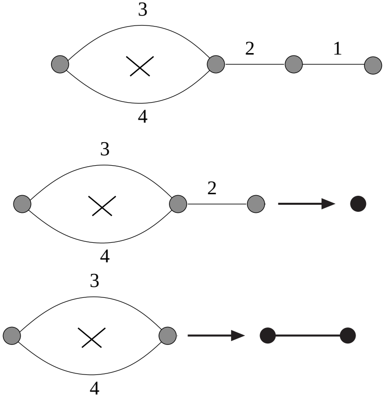

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- Translation to F-theory $\Rightarrow$ recipe to find fluxes that stabilise a desired situation
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