Open String Wavefunctions in Flux Compactifications

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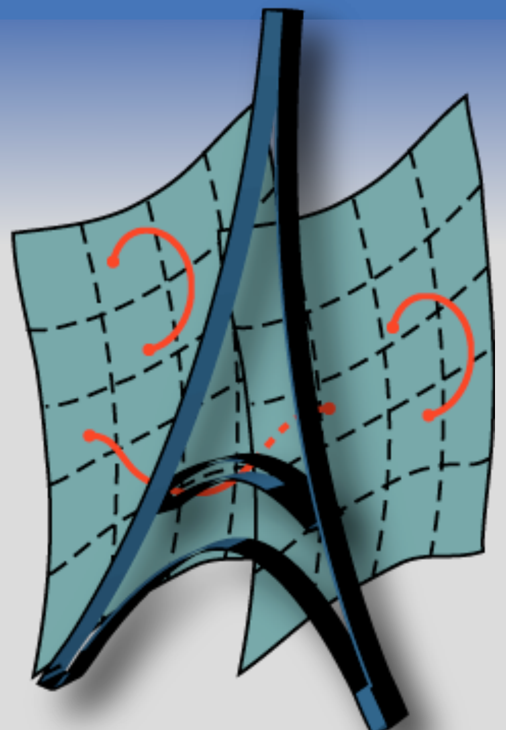
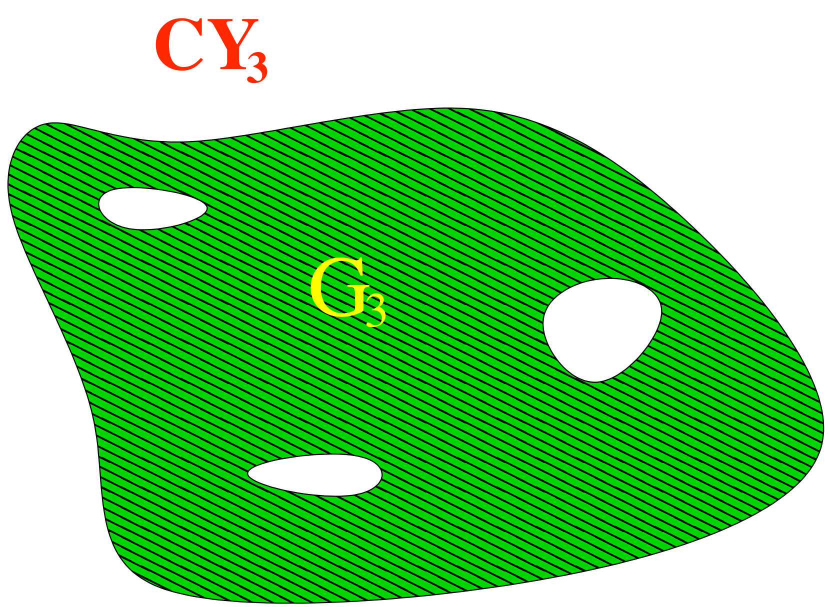


Motivation

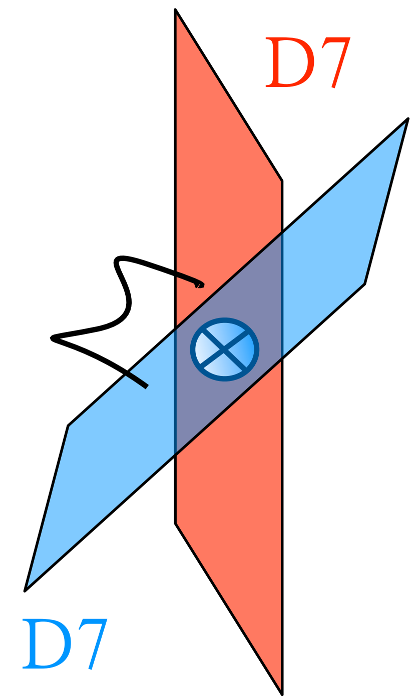
- Two popular lines of research in type II vacua are

Closed strings: Flux vacua



Moduli stabilization
de Sitter vacua
Inflation
Warping
...

Open strings: D-brane models



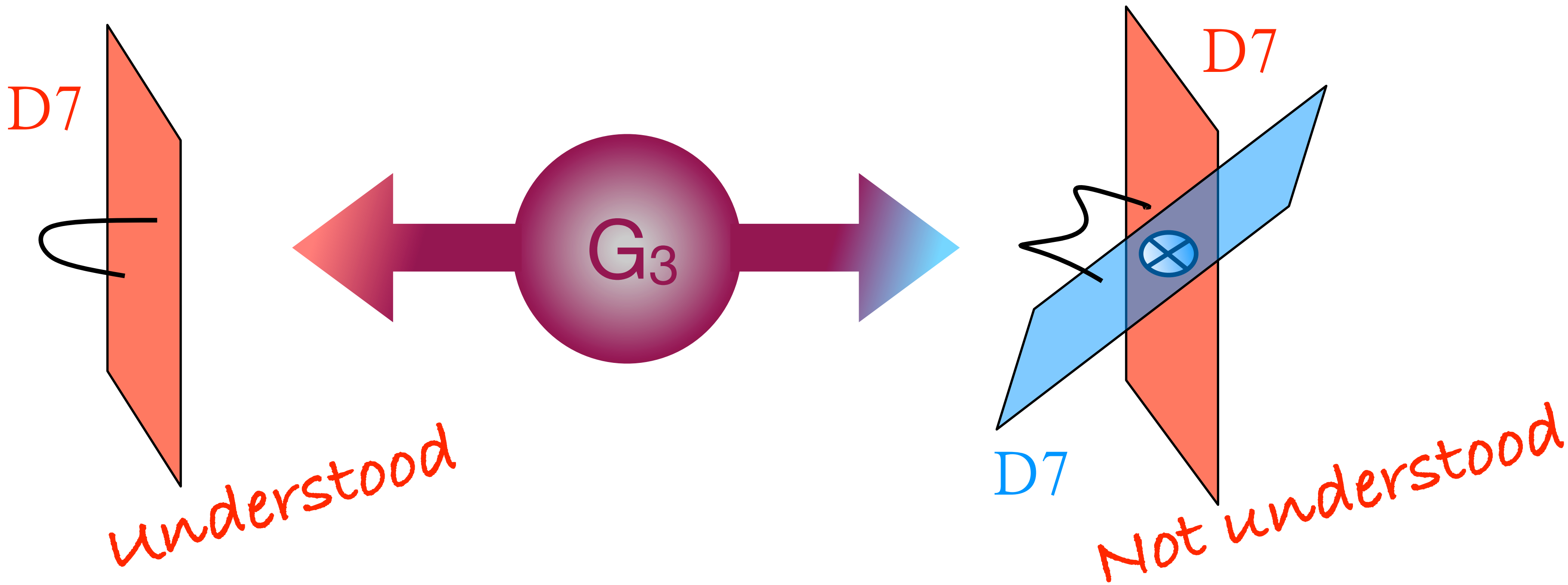
Chirality
MSSM/GUT spectra
Yukawa couplings
Instanton effects
...

Motivation

- Both subjects have greatly evolved in the past few years, but mostly independently
- Some overlapping research has shown that fluxes can have interesting effects on D-branes
  - Soft-terms/moduli stabilization — *Cámara, Ibáñez, Uranga '03; Lüst, Reffert, Stieberger '04; Gomis, F.M., Mateos '05; ...*
  - D-terms and superpotentials — *Martucci '06*
  - Instanton zero mode lifting — *Tripathy, Trivedi '05; Saulina '05; Kallosh, Kashani-Poor, Tomasiello '05; ...*
  - Warping effects — *Shiu's Talk*

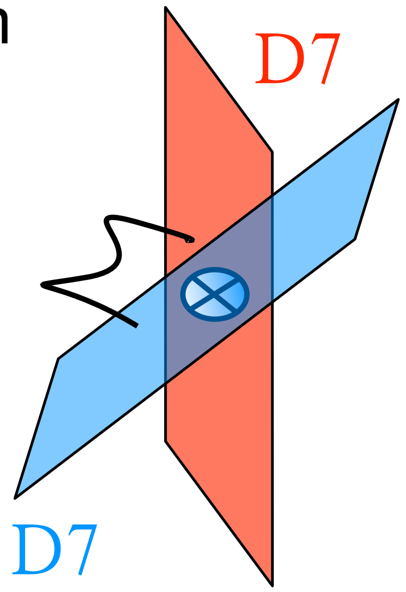
Motivation

- Both subjects have greatly evolved in the past few years, but mostly independently
- Some overlapping research has shown that fluxes can have interesting effects on D-branes
- The most interesting sector is however still missing



The problem

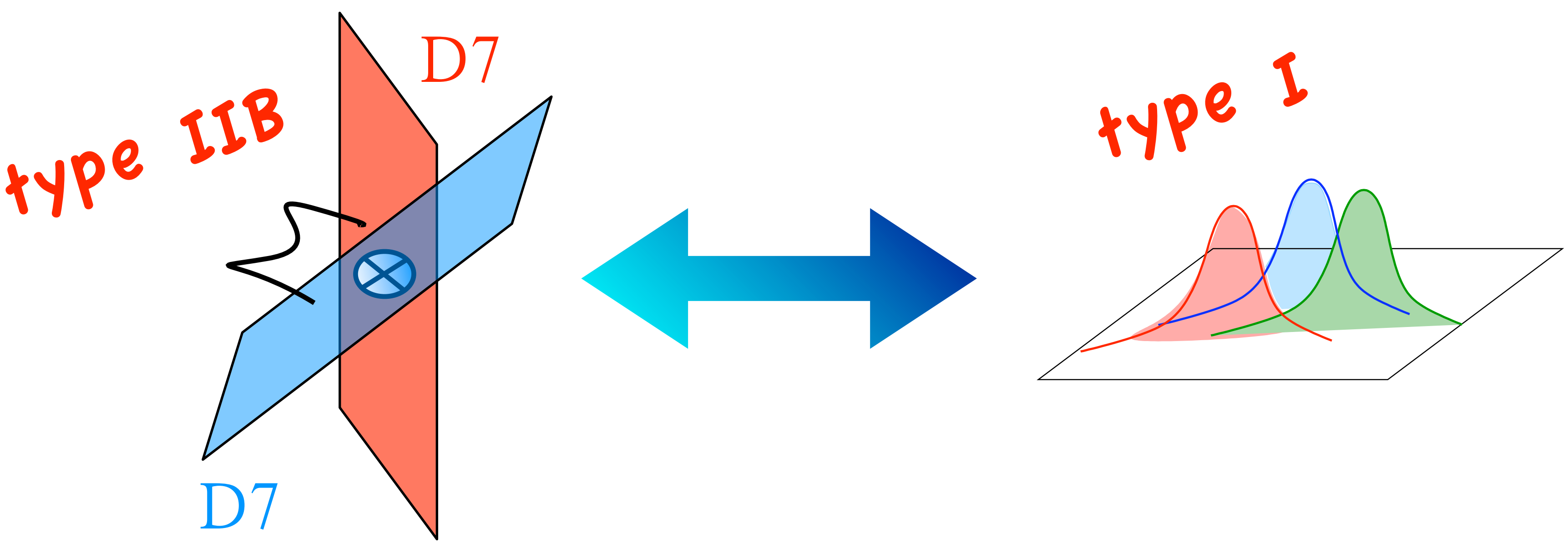
- The chiral sector of a D-brane model arises from open strings with twisted boundary conditions
- We do not know the precise effect of fluxes and warping microscopically
    - CFT tricky because of RR flux
    - Full D-brane action not available beyond U(1) gauge theories



The strategy

Idea: Consider Type I/Heterotic strings in the field theory limit

- Twisted open strings can be understood as wavefunctions
- Their coupling to fluxes can be read from the 10D action



Type I flux vacua

- The particle content of type I theory is

	bosons	fermions	
gravity	g_{MN}, C_{MN}, ϕ	ψ_M, λ	closed st.
vector	A_M^α	χ^α	open st.

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Fluxes

Torsion

$$F_3 = dC_2 + \omega_3$$

$$F_2 = dA$$

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Fluxes

Torsion

$$F_3 = dC_2 + \omega_3$$
$$F_2 = dA$$

Open string e.o.m.

$$\left(\not{D} + \frac{1}{4} e^{\phi/2} \not{F}_3\right) \chi = 0$$

$$D_K F^{KP} - \frac{e^{\phi/2}}{2} F_{MN} F^{MNP} = 0$$

Type I flux vacua

- The gravity background is of the form

$$ds^2 = Z^{-1/2} ds^2_{\mathbb{R}^{1,3}} + ds^2_{\mathcal{M}_6}$$

with $\mathcal{M}_6$ an SU(3)-structure manifold (→ forms J_{mn}, Ω_{mnp})

such that

$$\begin{aligned} Z e^{\phi} &\equiv g_s = \text{const.} \\ g_s^{1/2} e^{\phi/2} F_3 &= *_{\mathcal{M}_6} e^{-3\phi/2} d(e^{3\phi/2} J) \\ d\left(e^{\phi} J \wedge J\right) &= 0 \end{aligned}$$

Hull '86
Strominger '86

If $\mathcal{M}_6$ is complex ⇒ $\mathcal{N}$=1 SUSY vacuum

If $\mathcal{M}_6$ is not complex ⇒ $\mathcal{N}$=0 no-scale vacuum

Schulz '04
Cámara & Graña '07
Lüst, F.M., Martucci, Tsimpis '08

Twisted tori

- Ansatz for $\mathcal{M}_6$: elliptic fibration

$$ds^2_{\mathcal{M}_6} = Z^{-1/2} \sum_{a\in\Pi_2} (e^a)^2 + Z^{3/2} ds^2_{B_4}$$

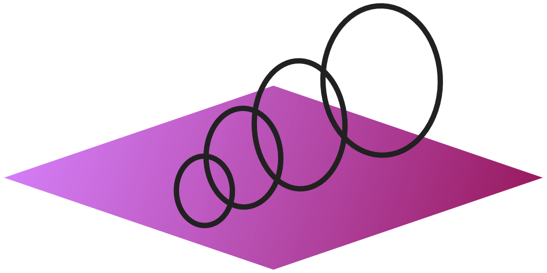
B_4 : base

Π_2 : fiber

simplest examples → (warped) twisted tori ($B_4 = T^4$)

They can be described as:

i) S^1 bundles



ii) Coset manifolds $\Gamma \setminus G$

G : nilpotent Lie group

Γ : discrete subgroup

- Parallelizable
- Explicit metric

Twisted tori

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For instance:

$$\begin{aligned} ds^2_{B_4} &= \sum_{m=1,2,4,5} (R_m dx^m)^2 \\ ds^2_{\Pi_2} &= \left[(R_3 dx^3)^2 + (R_6 \tilde{e}^6)^2\right] \\ F_3 &= -N(dx^1 \wedge dx^2 + dx^4 \wedge dx^5) \wedge \tilde{e}^6 - g_s^{-1} *_{T^4} dZ^2 \end{aligned}$$

$$\tilde{e}^6 = dx^6 + \frac{M}{2}(x^1 dx^2 - x^2 dx^1 + x^4 dx^5 - x^5 dx^4)$$

Twisted tori

<u>In our example</u>

$$d\tilde{e}^6 = M(dx^1 \wedge dx^2 + dx^4 \wedge dx^5)$$

$$de^6 = R^6 M \left(\frac{e^1 \wedge e^2}{R_1 R_2} + \frac{e^4 \wedge e^5}{R_4 R_5} \right)$$

<u>In general</u>

$$d\tilde{e}^a = \frac{1}{2} \tilde{f}^a_{bc} \tilde{e}^b \wedge \tilde{e}^c$$

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c$$

Twisted tori

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f^a_{bc} : structure constants of a 6D Lie algebra $\mathfrak{g}$

generators of $\mathfrak{g}$: $\hat{\partial}_a \equiv e_a{}^\alpha(x)\, \partial_{x^\alpha}$ $\qquad [\hat{\partial}_b, \hat{\partial}_c] = -f^a_{bc} \hat{\partial}_a$

Twisted tori

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$$\exp(\mathfrak{g}) = \mathcal{H}_5 \times \mathbb{R} \qquad G = \exp(\mathfrak{g})$$

$$\mathcal{M}_6 = \Gamma_{\mathcal{H}_5} \backslash \mathcal{H}_5 \times \mathbb{Z} \backslash \mathbb{R} \qquad \mathcal{M}_6 = \Gamma \backslash G$$

(For Z → 1)

Dimensional reduction

Following Cremades, Ibáñez, F.M. '04

- Consider a U(N) gauge group (i.e., N D9-branes)
- The bosonic d.o.f. come from the 10D gauge boson A_M

$$A_M = B_M^\alpha U_\alpha + W_M^{\alpha\beta} e_{\alpha\beta} \qquad\qquad U_\alpha : \text{ Cartan subalgebra}$$

- As usual $\langle B_m^\alpha \rangle \neq 0 \quad \Longrightarrow \quad U(N) \rightarrow \prod_\alpha U(n_\alpha) = G_{unbr}$

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- As usual $\langle B_m^\alpha \rangle \neq 0 \implies U(N) \to \prod_\alpha U(n_\alpha) = G_{unbr}$
- We can expand the bosonic fields as

$$B(x^\mu, x^i) = b_\mu(x^\mu)\, B(x^i)\, dx^\mu + \sum_m b^m(x^\mu)\, [\langle B^m \rangle + \xi^m](x^i)\, e_m \qquad U(n_\alpha) \text{ Adj.}$$

$$W(x^\mu, x^i) = w_\mu(x^\mu)\, W(x^i)\, dx^\mu + \sum_m w^m(x^\mu)\, \Phi^m(x^i)\, e_m \qquad (\bar{n}_\alpha, n_\beta) \text{ bif.}$$

... and similarly for fermions

Laplace and Dirac eqs.

- The e.o.m for the adjoint fields read (Z→1)

$$\hat{\partial}_a \hat{\partial}^a B = -m_B^2 B$$ gauge bosons

$$\left(\Gamma^a \hat{\partial}_a + \frac{1}{2} \not{f} P_+^{B_4}\right) \chi_6 = m_\chi \mathcal{B}_6^* \chi_6^*$$ fermions

... scalars

$$P_+^{B_4} = \frac{1}{2}(1 \pm \Gamma_{B_4}) \qquad \mathcal{B}_6 = \text{6D Maj. matrix}$$

- For bifundamental fields:

$$\hat{\partial}_a \rightarrow \hat{\partial}_a - i(\langle B_m^\alpha \rangle - \langle B_m^\beta \rangle)$$

see Cámara's Talk

Recap

- We want to understand the effect of fluxes on non-Abelian gauge theories
- Nice framework: type I/heterotic flux vacua → 10D field theory
- Simplest examples in terms of twisted tori
- The effect of fluxes appears in the modified Dirac and Laplace equations. For adjoint fields and Z→1:

$$\hat{\partial}_a \hat{\partial}^a B = -m_B^2 B$$

$$\left(\Gamma^a \hat{\partial}_a + \frac{1}{2} \not{f} P_+^{B_4}\right) \chi_6 = m_\chi \mathcal{B}_6^* \chi_6^*$$

Gauge Bosons

- Laplace equation $$\hat{\partial}_a \hat{\partial}^a B \quad = \quad -m_B^2 B$$

- In our example:

$$R_1\hat{\partial}_1 = \partial_{x^1} + \frac{M}{2}x^2\partial_{x^6} \qquad R_4\hat{\partial}_4 = \partial_{x^4} + \frac{M}{2}x^5\partial_{x^6}$$

$$R_2\hat{\partial}_2 = \partial_{x^2} - \frac{M}{2}x^1\partial_{x^6} \qquad R_5\hat{\partial}_5 = \partial_{x^5} - \frac{M}{2}x^4\partial_{x^6}$$

$$R_3\hat{\partial}_3 = \partial_{x^3} \qquad R_6\hat{\partial}_6 = \partial_{x^6}$$

If B does not depend on $x^6 \Rightarrow \hat{\partial}^a = \partial_a \Rightarrow$ $B = e^{2\pi i \vec{k}\cdot\vec{x}}$ $\quad \vec{k} = (k_1, k_2, k_3, k_4, k_5)$

If B depends on x^6 like $e^{2\pi i k_6 x^6} \Rightarrow$ eq. of a W-boson in a magnetized T^4, with magnetic flux k_6M

$$F_2^{\text{cl}} = k_6 M(dx^1 \wedge dx^2 + dx^4 \wedge dx^5)$$

Gauge Bosons

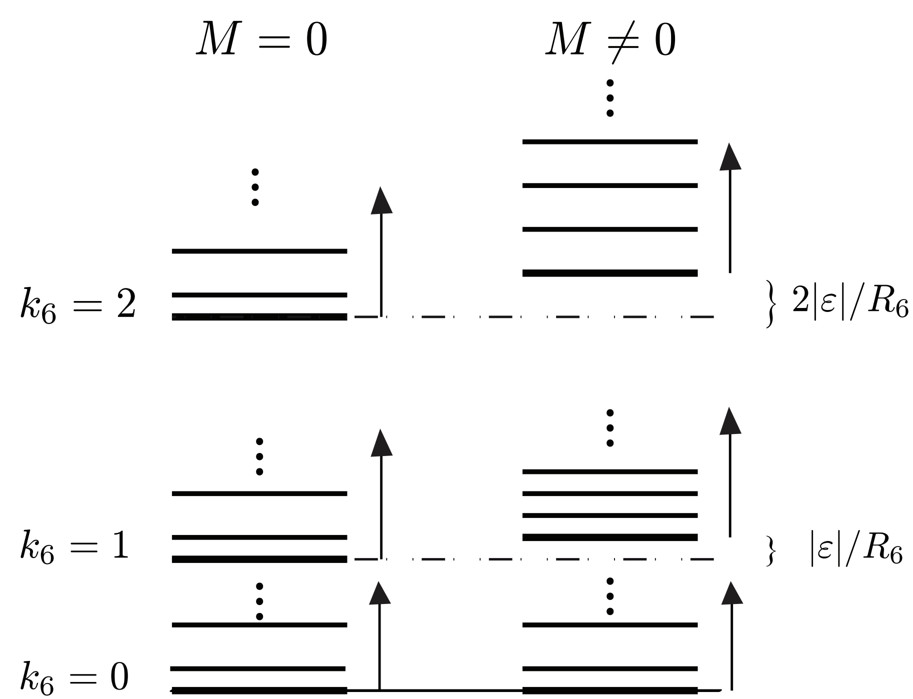
- Laplace equation

$$\hat{\partial}_a \hat{\partial}^a B \quad = \quad -m_B^2 B$$

- KK modes on the S^1 fiber are analogous to magnetized open strings ⇒ B = θ-functions & sums of Hermite functions

  - Fluxes freeze moduli ⇒ extra degeneracies

$$m_B^2 = \frac{|k_6 M|}{\pi R_1 R_2}(n+1) + \left(\frac{k_6}{R_6}\right)^2 + \left(\frac{k_3}{R_3}\right)^2$$



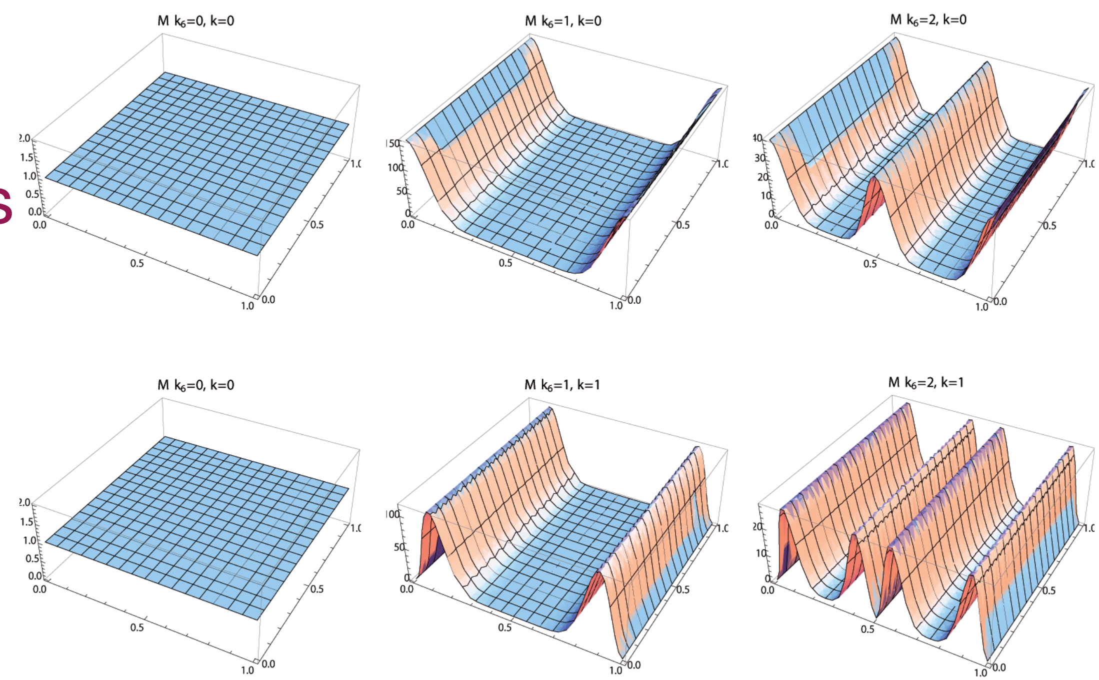
Gauge Bosons

- Laplace equation $$\hat{\partial}_a \hat{\partial}^a B \quad = \quad -m_B^2 B$$

- KK modes on the S^1 fiber are analogous to magnetized open strings ⇒ B = θ-functions & sums of Hermite functions

  - Fluxes freeze moduli ⇒ extra degeneracies

  - Wavefunctions are localized



Group Manifolds

- While the previous example was quite simple, one can solve the Laplace eq. for more general manifolds of the form Γ \ G
- A natural object to consider is the non-Abelian Fourier transform

$$\hat{f}_{\vec{\omega}}\,\varphi(\vec{s}) = \int_G B(g)\pi_{\vec{\omega}}(g)\varphi(\vec{s})dg$$

unirrep of G

auxiliary Hilbert space $\mathcal{H}$

Group Manifolds

- While the previous example was quite simple, one can solve the Laplace eq. for more general manifolds of the form Γ \ G
  - Let us consider the function

$$B_{\vec{\omega}}^{\varphi,\psi}(g) = (\pi_{\vec{\omega}}(g)\varphi, \psi)$$

scalar product in $\mathcal{H}$

  - Note that

$$\Delta\,(\pi_{\vec{\omega}}(g)\varphi, \psi) = (\pi_{\vec{\omega}}(g)\pi_{\vec{\omega}}(\Delta)\varphi, \psi)$$

  - So we can take Ψ = δ-function and φ eigenfunction
  - Finally we can impose Γ-invariance via

$$B_{\vec{\omega}}(g) = \sum_{\gamma\in\Gamma} \pi_{\vec{\omega}}(\gamma g)\varphi(\vec{s}_0)$$

Group Manifolds

- While the previous example was quite simple, one can solve the Laplace eq. for more general manifolds of the form $\Gamma \setminus G$
- By construction, we have a correspondence of unirreps of G and families of wavefunctions in $\Gamma \setminus G$
- Previous example → $\mathcal{H}_{2p+1}$ Heisenberg group ≅ $(\vec{x}, \vec{y}, z)$

$$\pi_{k'_z}\, u(\vec{s}) \quad = \quad e^{2\pi i k'_z [z + \vec{x}\cdot\vec{y}/2 + \vec{y}\cdot\vec{s}]}\, u(\vec{s} + \vec{x}) \longrightarrow \text{fiber KK modes}$$

$$\pi_{\vec{k}'_x, \vec{k}'_y} \quad = \quad e^{2\pi i (\vec{k}'_x \cdot \vec{x} \,+\, \vec{k}'_y \cdot \vec{y})} \longrightarrow \text{base KK modes}$$

Fermions

- Dirac equation

$$i(\mathbf{D}+\mathbf{F})\Psi \;=\; m_\chi \Psi^*$$

$$\mathbf{D} \;\leftarrow\; \Gamma^a \hat{\partial}_a$$

$$\mathbf{F} \;\leftarrow\; \frac{1}{2} \not{f} P_+^{B_4}$$

Moduli lifting info.

- Squared Dirac eq.

$$(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F})\Psi \;=\; |m_\chi|^2 \Psi$$

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$$(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F})\Psi \;=\; |m_\chi|^2 \Psi$$

- Previous example: **F** = 0

$$-\mathbf{D}^*\mathbf{D} \;=\; \begin{pmatrix} \hat{\partial}_m \hat{\partial}^m & 0 & 0 & 0 \\ 0 & \hat{\partial}_m \hat{\partial}^m & -\varepsilon \hat{\partial}_6 & 0 \\ 0 & \varepsilon \hat{\partial}_6 & \hat{\partial}_m \hat{\partial}^m & 0 \\ 0 & 0 & 0 & \hat{\partial}_m \hat{\partial}^m \end{pmatrix} \qquad \varepsilon = \text{flux density}$$

All entries of the matrix commute ⇒ standard diagonalization

Fermions

- Dirac equation

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Moduli lifting info.

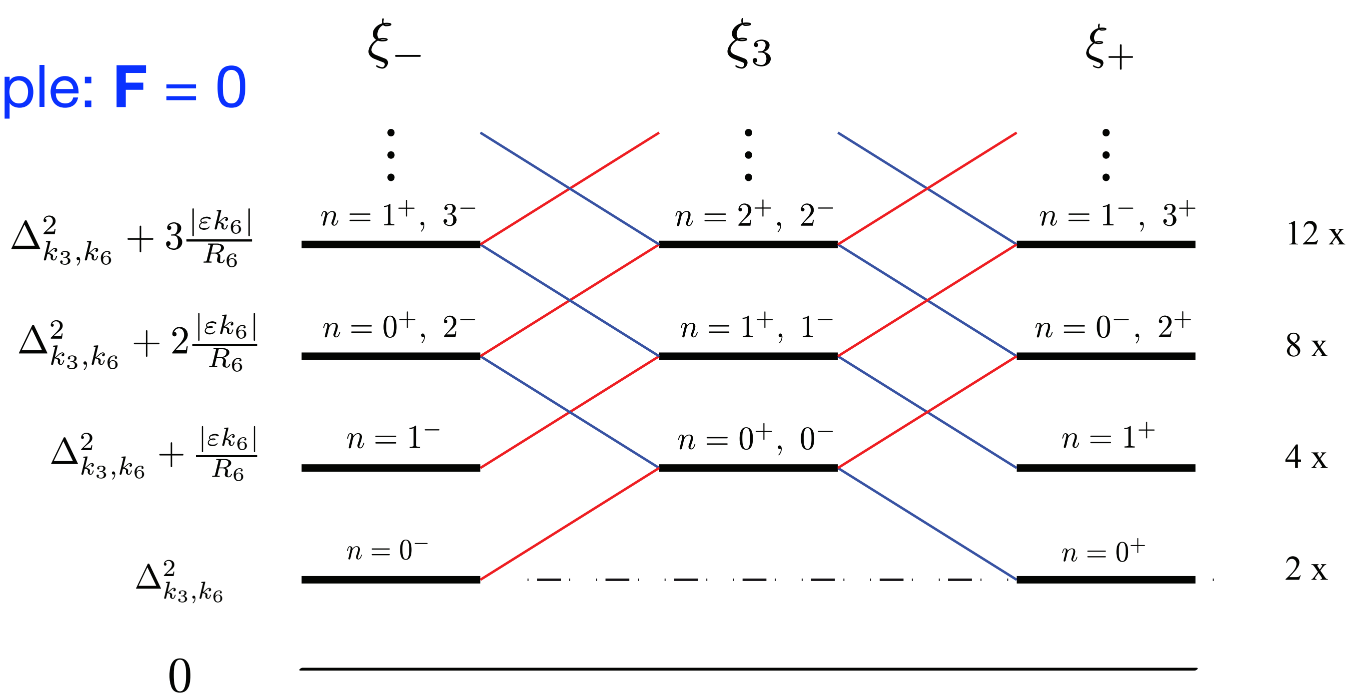
- Squared Dirac eq.

$$(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F})\Psi \;=\; |m_\chi|^2 \Psi$$

- Previous example: **F** = 0

$$\xi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} B$$

$$\xi_\pm = \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix} B$$



Fermions

- Squared Dirac eq. $(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F})\Psi \;=\; |m_\chi|^2\Psi$

  - More involved example: **F** $\neq 0$

$$-(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F}) \quad = \quad \begin{pmatrix} \hat{\partial}_m\hat{\partial}^m & 0 & 0 & 0 \\ 0 & \hat{\partial}_m\hat{\partial}^m & -\varepsilon\hat{\partial}_{z^3} & -\varepsilon\hat{\partial}_{z^2} \\ 0 & \varepsilon\hat{\partial}_{\bar{z}^3} & \hat{\partial}_m\hat{\partial}^m & \varepsilon\hat{\partial}_{z^1} \\ 0 & \varepsilon\hat{\partial}_{\bar{z}^2} & -\varepsilon\hat{\partial}_{\bar{z}^1} & \hat{\partial}_m\hat{\partial}^m - \varepsilon^2 \end{pmatrix}$$

Entries no longer commute!!

Fermions

- Squared Dirac eq. $$(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F})\Psi \;=\; |m_\chi|^2\Psi$$

  - More involved example: **F** ≠ 0

$$-(\mathbf{D}+\mathbf{F})^*(\mathbf{D}+\mathbf{F}) \quad = \begin{pmatrix} \hat\partial_m\hat\partial^m & 0 & 0 & 0 \\ 0 & \hat\partial_m\hat\partial^m & -\varepsilon\hat\partial_{z^3} & -\varepsilon\hat\partial_{z^2} \\ 0 & \varepsilon\hat\partial_{\bar z^3} & \hat\partial_m\hat\partial^m & \varepsilon\hat\partial_{z^1} \\ 0 & \varepsilon\hat\partial_{\bar z^2} & -\varepsilon\hat\partial_{\bar z^1} & \hat\partial_m\hat\partial^m - \varepsilon^2 \end{pmatrix}$$

Entries no longer commute!!

Eigenvectors:

$$\xi_3 \equiv \begin{pmatrix} \hat\partial_{\bar z^1} \\ \hat\partial_{\bar z^2} \\ \hat\partial_{\bar z^3} \end{pmatrix} B \qquad \qquad \xi_\pm \equiv \begin{pmatrix} \hat\partial_{z^3}\hat\partial_{\bar z^1} + m_{\xi_\pm}\hat\partial_{z^2} \\ \hat\partial_{z^3}\hat\partial_{\bar z^2} - m_{\xi_\pm}\hat\partial_{z^1} \\ \hat\partial_{z^3}\hat\partial_{\bar z^3} + m^2_{\xi_\pm} \end{pmatrix} B$$

$$m^2_{\xi_3} = m^2_B \qquad \qquad m^2_{\xi_\pm} = \frac{1}{4}\left(\varepsilon_\mu \pm \sqrt{\varepsilon^2_\mu + 4m^2_B}\right)^2$$

Recap II

- We have computed the spectrum of KK modes in several type I vacua based on twisted tori

- If one assumes the hierarchy $\mathrm{Vol}_{B_4}^{1/2} \gg \mathrm{Vol}_{\Pi_2}$ then one has

$$\varepsilon = m_{\text{flux}} \ll m_{\text{base}}^{\text{KK}} \ll m_{\text{fib}}^{\text{KK}}$$

Massless modes and lifted moduli	Base KK modes	Fiber KK modes
ψ = const like in T^6	ψ like in T^4	Exotic, localized ψ

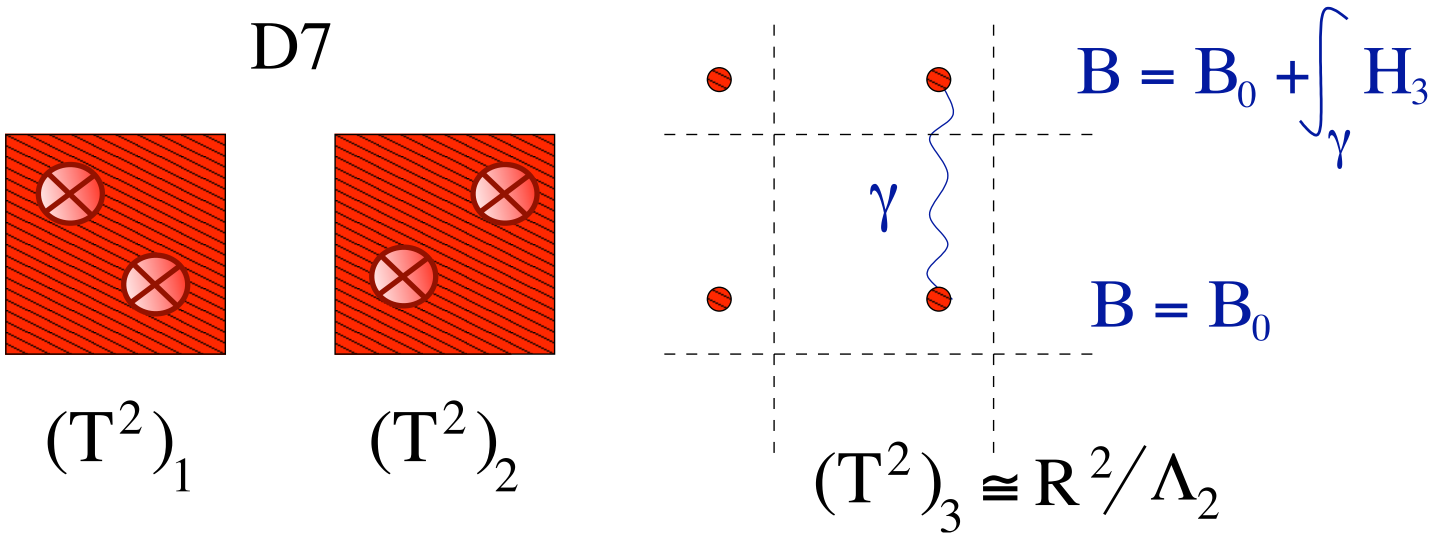
About warping

- In the above we have assumed a constant warping
- One can check that $\nabla^2_{T^4} Z^2 = -\varepsilon^2 + \ldots$
- So for $\mathrm{Vol}_{B_4}^{1/2} \gg \mathrm{Vol}_{\Pi_2}$ we have $\varepsilon \ll m_{\mathrm{base}}^{\mathrm{KK}}$ and Z = const. is a good approximation
- However, for $\mathrm{Vol}_{B_4}^{1/2} \simeq \mathrm{Vol}_{\Pi_2}$ we have
  - Warping effects
  - Fiber modes more localized ⇒ should dominate

Type IIB T-dual

- We can take our models to type IIB by T-duality on the fiber coordinates:

$$\begin{array}{c} N \text{ D9-branes} \\ \textbf{KK mode on } B_4 \simeq (T^2)_1 \times (T^2)_2 \\ \textbf{KK mode on } \Pi_2 \simeq (T^2)_3 \end{array} \longrightarrow \begin{array}{c} N \text{ D7-branes} \\ \textbf{KK mode on } (T^2)_1 \times (T^2)_2 \\ \textbf{Winding mode on } (T^2)_3 \end{array}$$



Conclusions

- We have considered type I flux vacua in order to see the effect of fluxes on open strings via field theory calculations

- Assuming constant Z, one can compute exactly the massless and massive spectrum of wavefunctions for models based on twisted tori and group quotients $\Gamma \backslash G$

- The techniques used here for adjoint fields also work for bifundamental chiral multiplets

  see Cámara's Talk

- Computing 4D couplings via wavefunctions, we can compare with the ones from 4D sugra. They indeed agree for ε small

- For ε not small, however, we expect new phenomena, in part due to warping and in part due to exotic KK modes

Outlook

- As a byproduct, we have developed a method for computing wavefunctions on group manifolds and quotients Γ \ G

- This is not only useful for type I compactifications, but also for the KK spectrum of type IIA flux vacua

  - de Sitter vacua — *Silverstein'07*; *Haque, Underwood, Shiu, van Riet'08*

  - AdS vacua — *Lüst & Tsimpis'04*; *see Villadoro's & Zagermann's Talks*

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  - de Sitter vacua — *Silverstein'07*, *Haque, Underwood, Shiu, van Riet'08*
  - AdS vacua — *Lüst & Tsimpis'04*, *see Villadoro's & Zagermann's Talks*
- We have also seen that the effect of RR fluxes is very simple once that the background eom have been applied

$$\left(\Gamma^a\hat{\partial}_a + \frac{1}{4}\left[\not{f} + e^{\phi/2}\not{F}_3\right]\right)\chi_6 \quad \rightarrow \quad \left(\Gamma^a\hat{\partial}_a + \frac{1}{2}\not{f}P_+^{B_4}\right)\chi_6$$

...hint for a CFT computation?