Open String Wavefunctions in Flux Compactifications

Fernando Marchesano
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In collaboration with Pablo G. Cámara
Two popular lines of research in type II vacua are

Closed strings: Flux vacua

Open strings: D-brane models

- Moduli stabilization
- de Sitter vacua
- Inflation
- Warping

- Chirality
- MSSM/GUT spectra
- Yukawa couplings
- Instanton effects

...
Motivation

- Both subjects have greatly evolved in the past few years, but mostly independently.

- Some overlapping research has shown that fluxes can have interesting effects on D-branes:
  - Soft-terms/moduli stabilization
  - D-terms and superpotentials
  - Instanton zero mode lifting
  - Warping effects

Cámara, Ibáñez, Uranga'03
Lüst, Reffert, Stieberger'04
Gomis, F.M., Mateos'05
Martucci'06
Tripathy, Trivedi'05
Saulina'05
Kallosh, Kashani-Poor, Tomasiello'05
Shiu's Talk
Motivation

- Both subjects have greatly *evolved* in the past few years, but *mostly independently*

- Some overlapping research has shown that fluxes can have interesting effects on D-branes

- The most interesting sector is however still missing

\[ G_3 \]
The problem

- The chiral sector of a D-brane model arises from open strings with twisted boundary conditions.
- We do not know the precise effect of fluxes and warping microscopically.
  - CFT tricky because of RR flux.
  - Full D-brane action not available beyond U(1) gauge theories.
The strategy

Idea: Consider Type I/Heterotic strings in the field theory limit

- Twisted open strings can be understood as wavefunctions.
- Their coupling to fluxes can be read from the 10D action.
The particle content of *type I theory* is

<table>
<thead>
<tr>
<th>gravity</th>
<th>bosons</th>
<th>fermions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{MN}, C_{MN}, \phi$</td>
<td>$A_\alpha^M$</td>
<td>$\psi_M, \lambda$ closed st.</td>
</tr>
<tr>
<td>vector</td>
<td>$A_\alpha^M$</td>
<td>$\chi^\alpha$ open st.</td>
</tr>
</tbody>
</table>
Type I flux vacua

- The particle content of type I theory is

  \begin{align*}
  \text{gravity} & : \ g_{MN}, \ C_{MN}, \ \phi \\
  \text{vector} & : \ A_\alpha^M \\
  \text{bosons} & : \ \psi_M, \ \lambda \\
  \text{fermions} & : \ \chi^\alpha \\
  \text{fluxes} & : \ F_3 = dC_2 + \omega_3 \\
  & \quad F_2 = dA
  \end{align*}
The particle content of type I theory is

- **Bosons:** $g_{MN}, C_{MN}, \phi$
- **Fermions:** $\psi_M, \lambda$

**Closed string e.o.m.:**

\[
\left( \mathcal{D} + \frac{1}{4} e^{\phi/2} F_3 \right) \chi = 0
\]

**Open string e.o.m.:**

\[
D_K F^{KP} - \frac{e^{\phi/2}}{2} F_{MN} F^{MNP} = 0
\]
The gravity background is of the form

\[ ds^2 = Z^{-1/2} ds^2_{\mathbb{R}^{1,3}} + ds^2_{\mathcal{M}_6} \]

with \( \mathcal{M}_6 \) an SU(3)-structure manifold (→ forms \( J_{mn}, \Omega_{mnp} \)) such that

\[ Ze^\phi \equiv g_s = \text{const.} \]
\[ g_s^{1/2} e^{\phi/2} F_3 = * \mathcal{M}_6 e^{-3\phi/2} d(e^{3\phi/2} J) \]
\[ d(e^\phi J \wedge J) = 0 \]

If \( \mathcal{M}_6 \) is complex \( \Rightarrow \mathcal{N}=1 \) SUSY vacuum
If \( \mathcal{M}_6 \) is not complex \( \Rightarrow \mathcal{N}=0 \) no-scale vacuum
Twisted tori

- Ansatz for $\mathcal{M}_6$: elliptic fibration

$$\text{ds}^2_{\mathcal{M}_6} = Z^{-1/2} \sum_{a \in \Pi_2} (e^a)^2 + Z^{3/2} \text{ds}^2_{B_4}$$

simplest examples $\rightarrow$ (warped) twisted tori ($B_4 = T^4$)

They can be described as:

i) $S^1$ bundles

ii) Coset manifolds $\Gamma \backslash G$

- Parallelizable
- Explicit metric

$A_4 : \text{base}$

$\Pi_2 : \text{fiber}$

$G : \text{nilpotent Lie group}$

$\Gamma : \text{discrete subgroup}$
Twisted tori

• Ansatz for $\mathcal{M}_6$: elliptic fibration

$$ds^2_{\mathcal{M}_6} = Z^{-1/2} \sum_{a \in \Pi_2} (e^a)^2 + Z^{3/2} ds^2_{B_4}$$

simplest examples $\rightarrow$ (warped) twisted tori ($B_4 = T^4$)

For instance:

$$ds^2_{B_4} = \sum_{m=1,2,4,5} (R_m dx^m)^2$$

$$ds^2_{\Pi_2} = \left[ (R_3 dx^3)^2 + (R_6 \tilde{e}^6)^2 \right]$$

$$F_3 = -N(dx^1 \wedge dx^2 + dx^4 \wedge dx^5) \wedge \tilde{e}^6 - g_s^{-1} \ast_{T^4} dZ^2$$

$$\tilde{e}^6 = dx^6 + \frac{M}{2} (x^1 dx^2 - x^2 dx^1 + x^4 dx^5 - x^5 dx^4)$$

$\Pi_2$ : fiber

$B_4$ : base
Twisted tori

In our example

\[ d\tilde{e}^6 = M(dx^1 \wedge dx^2 + dx^4 \wedge dx^5) \]
\[ de^6 = R^6 M \left( \frac{e^1 \wedge e^2}{R_1 R_2} + \frac{e^4 \wedge e^5}{R_4 R_5} \right) \]

In general

\[ d\tilde{e}^a = \frac{1}{2} \tilde{f}^a_{bc} \tilde{e}^b \wedge \tilde{e}^c \]
\[ de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c \]
Twisted tori

In our example

\[ d\tilde{e}^6 = M (dx^1 \wedge dx^2 + dx^4 \wedge dx^5) \]
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\( f^a_{bc} \) : structure constants of a 6D Lie algebra \( \mathfrak{g} \)

Generators of \( \mathfrak{g} \) : \( \hat{\partial}_a \equiv e_a^\alpha (x) \partial_{x^\alpha} \)

\[ [\hat{\partial}_b, \hat{\partial}_c] = -f^a_{bc} \hat{\partial}_a \]
Twisted tori

In our example
\[ d\tilde{e}^6 = M(dx^1 \wedge dx^2 + dx^4 \wedge dx^5) \]
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\( f_{bc}^a \): structure constants of a 6D Lie algebra \( \mathfrak{g} \)

generators of \( \mathfrak{g} \): \( \hat{\partial}_a \equiv e_a^\alpha(x) \partial_{x^\alpha} \) \quad \[ [\hat{\partial}_b, \hat{\partial}_c] = -f_{bc}^a \hat{\partial}_a \]

\[ \exp(\mathfrak{g}) = \mathcal{H}_5 \times \mathbb{R} \]
\[ \mathcal{M}_6 = \Gamma_{\mathcal{H}_5} \backslash \mathcal{H}_5 \times \mathbb{Z} \backslash \mathbb{R} \]

In general
\[ d\tilde{e}^a = \frac{1}{2} f_{bc}^a \tilde{e}^b \wedge \tilde{e}^c \]
\[ de^a = \frac{1}{2} f_{bc}^a e^b \wedge e^c \]

(For \( Z \to 1 \))
Consider a $U(N)$ gauge group (i.e., $N$ D9-branes)

The bosonic d.o.f. come from the 10D gauge boson $A_M$

$$A_M = B_M^\alpha U_\alpha + W_M^{\alpha\beta} e_{\alpha\beta}$$

$U_\alpha$ : Cartan subalgebra

As usual $\langle B_m^\alpha \rangle \neq 0 \implies U(N) \to \prod_\alpha U(n_\alpha) = G_{unbr}$
Dimensional reduction

Following Cremades, Ibáñez, F.M. '04

- Consider a $U(N)$ gauge group (i.e., N D9-branes)

- The bosonic d.o.f. come from the 10D gauge boson $A_M$

  \[ A_M = B_\alpha^M U_\alpha + W_\alpha^\beta e_{\alpha\beta} \]

  \[ U_\alpha : \text{Cartan subalgebra} \]

- As usual  \[ \langle B_\alpha^m \rangle \neq 0 \implies U(N) \to \prod_\alpha U(n_\alpha) = G_{unbr} \]

- We can expand the bosonic fields as

  \[ B(x^\mu, x^i) = b_\mu(x^\mu) B(x^i) \, dx^\mu + \sum_m b_\mu(x^\mu) [\langle B^m \rangle + \xi^m(x^i) e_m \]

  \[ U(n_\alpha) \text{ Adj.} \]

  \[ W(x^\mu, x^i) = w_\mu(x^\mu) W(x^i) \, dx^\mu + \sum_m w_\mu(x^\mu) \Phi^m(x^i) e_m \]

  \[ (\bar{n}_\alpha, n_\beta) \text{ bif.} \]

  ... and similarly for fermions
Laplace and Dirac eqs.

The e.o.m for the adjoint fields read (Z→1)

\[ \hat{\partial}_a \hat{\partial}^a B = -m_B^2 B \]

\[ \left( \Gamma^a \hat{\partial}_a + \frac{1}{2} f P^B_+ \right) \chi_6 = m_\chi B_6^* \chi_6^* \]

... 

\[ P^B_+ = \frac{1}{2} (1 \pm \Gamma_B) \]

\[ B_6 = 6D \text{ Maj. matrix} \]

For bifundamental fields:

\[ \hat{\partial}_a \rightarrow \hat{\partial}_a - i(\langle B_\alpha^\alpha \rangle - \langle B_\beta^\beta \rangle) \]

see Cámaras Talk
Recap

- We want to understand the effect of fluxes on non-Abelian gauge theories
- Nice framework: type I/heterotic flux vacua $\rightarrow$ 10D field theory
- Simplest examples in terms of twisted tori
- The effect of fluxes appears in the modified Dirac and Laplace equations. For adjoint fields and $\mathbb{Z}\rightarrow 1$:

$$\hat{\partial}_a \hat{\partial}^a B = -m_B^2 B$$

$$\left( \Gamma^a \hat{\partial}_a + \frac{1}{2} f P_+^{B_4} \right) \chi_6 = m_\chi B_6^* \chi_6^*$$
Gauge Bosons

- Laplace equation

\[ \hat{\partial}_a \hat{\partial}^a B = -m_B^2 B \]

- In our example:

\[
\begin{align*}
R_1 \hat{\partial}_1 &= \partial_{x^1} + \frac{M}{2} x^2 \partial_{x^6} \\
R_2 \hat{\partial}_2 &= \partial_{x^2} - \frac{M}{2} x^1 \partial_{x^6} \\
R_3 \hat{\partial}_3 &= \partial_{x^3} \\
R_4 \hat{\partial}_4 &= \partial_{x^4} + \frac{M}{2} x^5 \partial_{x^6} \\
R_5 \hat{\partial}_5 &= \partial_{x^5} - \frac{M}{2} x^4 \partial_{x^6} \\
R_6 \hat{\partial}_6 &= \partial_{x^6}
\end{align*}
\]

If B does not depend on \( x^6 \) \( \Rightarrow \) \( \hat{\partial}^a = \partial_a \Rightarrow \) \( B = e^{2\pi i \vec{k} \cdot \vec{x}} \quad \vec{k} = (k_1, k_2, k_3, k_4, k_5) \)

If B depends on \( x^6 \) like \( e^{2\pi i k_6 x^6} \) \( \Rightarrow \) eq. of a W-boson in a magnetized \( T^4 \), with magnetic flux \( k_6 M \)

\[
F_2^{\text{cl}} = k_6 M (dx^1 \wedge dx^2 + dx^4 \wedge dx^5)
\]
**Gauge Bosons**

- Laplace equation
  \[ \hat{\partial}_a \hat{\partial}^a B = -m_B^2 B \]

- KK modes on the $S^1$ fiber are analogous to magnetized open strings $\Rightarrow B = \theta$-functions & sums of Hermite functions

- Fluxes freeze moduli
  $\Rightarrow$ extra degeneracies

\[
m_B^2 = \frac{|k_6 M|}{\pi R_1 R_2} (n + 1) + \left( \frac{k_6}{R_6} \right)^2 + \left( \frac{k_3}{R_3} \right)^2
\]

\[
M = 0 \quad \quad \quad M \neq 0
\]
\[
\begin{array}{c}
\vdots \\
k_6 = 2 \\
2|\varepsilon|/R_6
\end{array}
\]
\[
\begin{array}{c}
\vdots \\
k_6 = 1 \\
|\varepsilon|/R_6
\end{array}
\]
\[
\begin{array}{c}
\vdots \\
k_6 = 0
\end{array}
\]
Gauge Bosons

✶ Laplace equation

\[ \hat{\partial}_a \hat{\partial}^a B = -m_B^2 B \]

✶ KK modes on the $S^1$ fiber are analogous to magnetized open strings $\Rightarrow B = \theta$-functions & sums of Hermite functions

✧ Fluxes freeze moduli
$\Rightarrow$ extra degeneracies

✧ Wavefunctions are localized
While the previous example was quite simple, one can solve the Laplace eq. for more general manifolds of the form $\Gamma \backslash G$.

A natural object to consider is the non-Abelian Fourier transform

$$\hat{f}_{\omega} \varphi(\vec{s}) = \int_G B(g)\pi_{\omega}(g)\varphi(\vec{s}) dg$$

unirrep of G

auxiliary Hilbert space $\mathcal{H}$
While the previous example was quite simple, one can solve the Laplace eq. for more general manifolds of the form $\Gamma \backslash G$

Let us consider the function

$$B_{\omega}^{\varphi, \psi}(g) = (\pi_{\omega}(g)\varphi, \psi)$$

Note that

$$\Delta (\pi_{\omega}(g)\varphi, \psi) = (\pi_{\omega}(g)\pi_{\omega}(\Delta)\varphi, \psi)$$

So we can take $\Psi = \delta$-function and $\varphi$ eigenfunction

Finally we can impose $\Gamma$-invariance via

$$B_{\omega}(g) = \sum_{\gamma \in \Gamma} \pi_{\omega}(\gamma g)\varphi(\tilde{s}_0)$$
Group Manifolds

- While the previous example was quite simple, one can solve the Laplace eq. for more general manifolds of the form $\Gamma \backslash G$.
- By construction, we have a correspondence of unirreps of $G$ and families of wavefunctions in $\Gamma \backslash G$.
- Previous example $\rightarrow \mathcal{H}_{2p+1}$ Heisenberg group $\cong (\vec{x}, \vec{y}, z)$.

\[
\pi_{k'_{z}} u(\vec{s}) = e^{2\pi ik'_{z}[z+\vec{x}\cdot\vec{y}/2+\vec{y}\cdot\vec{s}]} u(\vec{s} + \vec{x}) \quad \text{fiber KK modes}
\]

\[
\pi_{k'_{x}, k'_{y}} = e^{2\pi i(k'_{x}\cdot\vec{x} + k'_{y}\cdot\vec{y})} \quad \text{base KK modes}
\]
Fermions

- Dirac equation
  \[ i(D + F)\Psi = m_\chi \Psi^* \]

- Squared Dirac eq.
  \[ (D + F)^*(D + F)\Psi = |m_\chi|^2\Psi \]

\[ D \leftarrow \Gamma^a \hat{\partial}_a \]
\[ F \leftarrow \frac{1}{2} f P^B_+ \]

Moduli lifting info.
Fermions

- Dirac equation
  \[ i(D + F)\Psi = m_\chi \Psi^* \]

- Squared Dirac eq.
  \[ (D + F)^*(D + F)\Psi = |m_\chi|^2\Psi \]

- Previous example: \( F = 0 \)

\[ -D^*D = \begin{pmatrix} \hat{\partial}_m \hat{\partial}^m & 0 & 0 & 0 \\ 0 & \hat{\partial}_m \hat{\partial}^m & -\varepsilon \hat{\partial}_6 & 0 \\ 0 & \varepsilon \hat{\partial}_6 & \hat{\partial}_m \hat{\partial}^m & 0 \\ 0 & 0 & 0 & \hat{\partial}_m \hat{\partial}^m \end{pmatrix} \]

\[ \varepsilon = \text{flux density} \]

All entries of the matrix commute \( \Rightarrow \) standard diagonalization
Fermions

- Dirac equation
  \[ i(D + F)\Psi = m\chi \Psi^* \]

- Squared Dirac eq.
  \[ (D + F)^*(D + F)\Psi = |m\chi|^2\Psi \]

- Previous example: \( F = 0 \)

\[ \begin{align*}
  \xi_3 &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} B \\
  \xi_{\pm} &= \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix} B
\]
Fermions

Squared Dirac eq. \( (D + F)^*(D + F)\Psi = |m_\chi|^2\Psi \)

More involved example: \( F \neq 0 \)

\[
- (D + F)^*(D + F) = \begin{pmatrix}
\hat{\partial}_m \hat{\partial}^m & 0 & 0 & 0 \\
0 & \hat{\partial}_m \hat{\partial}^m & -\epsilon \hat{\partial}_{z^3} & -\epsilon \hat{\partial}_{z^2} \\
0 & \epsilon \hat{\partial}_{\bar{z}^3} & \hat{\partial}_m \hat{\partial}^m & \epsilon \hat{\partial}_{z^1} \\
0 & \epsilon \hat{\partial}_{\bar{z}^2} & -\epsilon \hat{\partial}_{\bar{z}^1} & \hat{\partial}_m \hat{\partial}^m - \epsilon^2
\end{pmatrix}
\]

Entries no longer commute!!
Fermions

Squared Dirac eq. \((D + F)^*(D + F)\Psi = |m_\chi|^2\Psi\)

More involved example: \(F \neq 0\)

\[-(D + F)^*(D + F) = \]

\[
\left(\begin{array}{cccc}
\hat{\partial}_m \hat{\partial}_m & 0 & 0 & 0 \\
0 & \hat{\partial}_m \hat{\partial}_m & -\varepsilon \hat{\partial}_{\bar{z}3} & -\varepsilon \hat{\partial}_{\bar{z}2} \\
0 & \varepsilon \hat{\partial}_{\bar{z}3} & \hat{\partial}_m \hat{\partial}_m & \varepsilon \hat{\partial}_{\bar{z}1} \\
0 & \varepsilon \hat{\partial}_{\bar{z}2} & -\varepsilon \hat{\partial}_{\bar{z}1} & \hat{\partial}_m \hat{\partial}_m - \varepsilon^2 \\
\end{array}\right)
\]

Entries no longer commute!!

Eigenvectors:

\[
\xi_3 \equiv \begin{pmatrix} \hat{\partial}_{\bar{z}1} \\ \hat{\partial}_{\bar{z}2} \\ \hat{\partial}_{\bar{z}3} \end{pmatrix} B
\]

\[
m^2_{\xi_3} = m^2_B
\]

\[
\xi_\pm \equiv \begin{pmatrix} \hat{\partial}_{\bar{z}3} \hat{\partial}_{\bar{z}1} + m_{\xi_\pm} \hat{\partial}_{\bar{z}2} \\ \hat{\partial}_{\bar{z}3} \hat{\partial}_{\bar{z}2} - m_{\xi_\pm} \hat{\partial}_{\bar{z}1} \\ \hat{\partial}_{\bar{z}3} \hat{\partial}_{\bar{z}3} + m^2_{\xi_\pm} \end{pmatrix} B
\]

\[
m^2_{\xi_\pm} = \frac{1}{4} \left(\varepsilon_\mu \pm \sqrt{\varepsilon^2_\mu + 4m^2_B}\right)^2
\]
Recap II

- We have computed the spectrum of KK modes in several type I vacua based on twisted tori.

- If one assumes the hierarchy $\text{Vol}_{B_4}^{1/2} \gg \text{Vol}_{\Pi_2}$ then one has:

  $\varepsilon = m_{\text{flux}} \ll m_{\text{KK base}} \ll m_{\text{KK fib}}$

  - **Massless** modes and lifted moduli: $\psi = \text{const}$ like in $T^6$
  - **Base KK** modes: $\psi$ like in $T^4$
  - **Fiber KK** modes: Exotic, localized $\psi$
About warping

• In the above we have **assumed a constant warping**

• One can check that \( \nabla_{T^4}^2 Z^2 = -\epsilon^2 + \ldots \)

• So for \( \text{Vol}^{1/2}_{B_4} \gg \text{Vol}_{\Pi_2} \) we have \( \epsilon \ll m_{\text{base}}^{KK} \) and \( Z = \text{const.} \) is a **good approximation**

• However, for \( \text{Vol}^{1/2}_{B_4} \sim \text{Vol}_{\Pi_2} \) we have
  
  ✦ **Warping effects**
  
  ✦ **Fiber modes more localized** ⇒ should dominate
We can take our models to type IIB by T-duality on the fiber coordinates:

\[ N \text{ D9-branes} \]

KK mode on \( B_4 \simeq (T^2)_1 \times (T^2)_2 \) \[ \rightarrow \]

KK mode on \( \Pi_2 \simeq (T^2)_3 \)

\[ N \text{ D7-branes} \]

KK mode on \((T^2)_1 \times (T^2)_2\)

Winding mode on \((T^2)_3\)

\[ \gamma \]

\[\begin{align*}
B &= B_0 + \int_{\gamma} H_3 \\
B &= B_0 \\
(T^2)_3 &\equiv \mathbb{R}^2/\Lambda_2
\end{align*}\]
Conclusions

- We have considered type I flux vacua in order to see the effect of fluxes on open strings via field theory calculations.

- Assuming constant $Z$, one can compute exactly the massless and massive spectrum of wavefunctions for models based on twisted tori and group quotients $\Gamma \backslash G$.

- The techniques used here for adjoint fields also work for bifundamental chiral multiplets. See Cámara's talk.

- Computing 4D couplings via wavefunctions, we can compare with the ones from 4D sugra. They indeed agree for $\epsilon$ small.

- For $\epsilon$ not small, however, we expect new phenomena, in part due to warping and in part due to exotic KK modes.
As a byproduct, we have developed a method for computing wavefunctions on group manifolds and quotients $\Gamma \backslash G$

This is not only useful for type I compactifications, but also for the KK spectrum of type IIA flux vacua

- de Sitter vacua
- AdS vacua

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Silverstein ’07
Haque, Underwood, Shiu, van Riet ’08
Lüst & Tsimpis ’04

see Villadoro’s & Zagermann’s Talks
Outlook

✦ As a byproduct, we have developed a method for computing wavefunctions on group manifolds and quotients $\Gamma \backslash G$

✦ This is not only useful for type I compactifications, but also for the KK spectrum of type IIA flux vacua

✧ de Sitter vacua

✧ AdS vacua

✦ We have also seen that the effect of RR fluxes is very simple once that the background eom have been applied

\[
\left( \Gamma^a \hat{\partial}_a + \frac{1}{4} \left[ f + e^{\phi/2} F_3 \right] \right) \chi_6 \rightarrow \left( \Gamma^a \hat{\partial}_a + \frac{1}{2} f P_+ B_4 \right) \chi_6
\]

...hint for a CFT computation?

Silverstein'07
Haque, Underwood, Shiue, van Riet'08
Lüst & Tsimpis'04
see Villadoro’s & Zagermann’s Talks