Flavor structure in magnetized/intersecting brane models

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outline

- Introduction and Motivation
- Magnetized extra dimension
- Yukawa coupling and Discrete symmetry
- Summary

Introduction

Motivation

Standard model from string theory

String theory is a candidate of unified theory including gravity

10 dimensional string theory

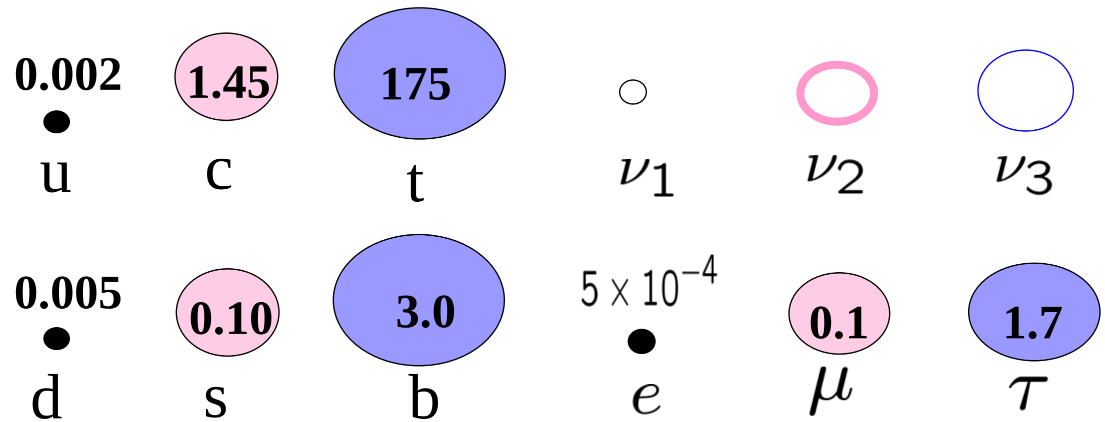
Compactification of extra 6 dimensional space

Our world (standard model) : 4-dimensional space time

We need the 4 dimensional standard model of particle including all the values of parameters

Flavor mystery

three generations for quark and lepton



Large hierarchy
between generations

$$(u : c : t) \sim (1 \times 10^{-5}, 8 \times 10^{-3}, 1)$$
$$(d : s : b) \sim (1.7 \times 10^{-3}, 3.3 \times 10^{-2}, 1)$$

small quark mixing
(except Cabbibo angle)

$$|V_{CKM}| = \begin{pmatrix} 0.97 & 0.22 & 0.0032 \\ 0.22 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix}$$

large lepton mixing

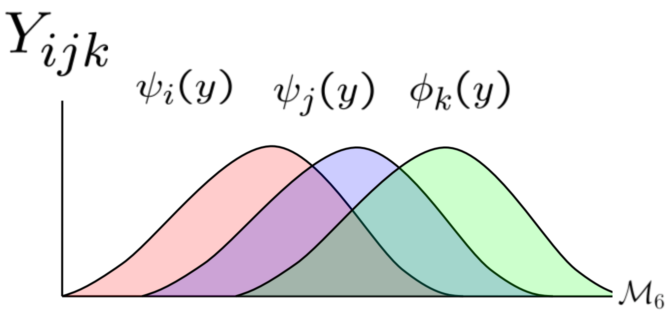
$$\sin\theta_{12}^2 = 0.3, \quad \sin\theta_{23}^2 = 0.5, \quad \sin\theta_{13}^2 \sim 0$$

Some candidate of potential phenomenological interest

localized matter fields in extra dimensions

Four dimensional Yukawa couplings from overlap integral

$$Y_{ijk} = \int dy^{D-4} \psi_L^{i,M_1}(y) \psi_R^{j,M_2}(y) (\psi_H^{k,M_3}(y))^*$$



The hierarchirally small Yukawa couplings may be obtained from overlap integral.

Discrete flavor symmetry

To explain the lepton large flavor mixing
Particle phenomenologist consider several discrete symmetries which have some interest results

e.g. $S_3, D_4, A_4, S_4, Q_6, \Delta(27), \ldots$

The origin of such discrete flavor symmetries have been investigated within the framework of extra dimensional field theory and string theory.
Geometrical interpretation is possible.
c.f. Heterotic Orbifold models

[Kobayashi, Raby, Zhang '04,
Kobayashi, Nilles, Ploger, Raby, Ratz, '06]

Extra dimensional (Yang-Mills) field theory
is essential to …

Low energy limit of string theory (Heterotic or D-brane)
-> (Super) Yang-Mills theory in extra dimensions

Yukawa and all the other couplings can be calculable in principle

GUT and SUSY are definitely included.

non-vanishing magnetic flux in extra dimensions

- Gauge symmetry and its breaking
- existence of chiral matters (super symmetry breaking)
- Possible to obtain the explicit form of wave functions

$\longrightarrow$ information about spectrum of matter (generation number) and calculation of Yukawa coupling

It may obtain phenomenological interesting results of spectrums and flavor structures (flavor symmetry etc…)

Magnetized extra dimensional models

We start from D=4+2n (super) Yang-Mills with non-vanishing magnetic flux in extra dimensions.

Four dimensional effective theory is obtained by dimensional reduction.

We analyze the flavor structure of these class of models in which we will see that non-abelian discrete flavor symmetries appear.

Magnetized extra dimenstions

Higher Dimensional SYM theory with flux

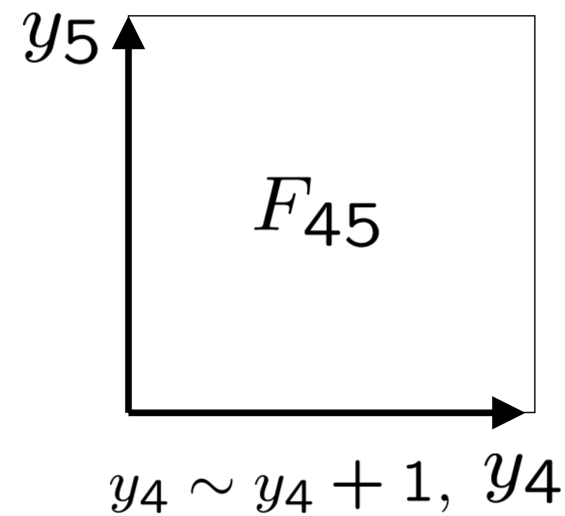
$$\boxed{\mathcal{L}_{SYM} = -\frac{1}{4g^2} Tr\{F^{MN}F_{MN}\} + \frac{i}{2g^2} Tr\{\bar{\lambda}\Gamma^M D_M \lambda\}}$$

$$\begin{aligned} \lambda(x^\mu, y^m) &= \sum_n \chi_n(x^\mu) \times \psi_n(y_m), \\ A_M(x^\mu, y^m) &= \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m) \end{aligned} \quad \Rightarrow \quad \begin{aligned} i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\ \Delta_6 \phi_{n,M}(y) &= M_{n,M}^2 \phi_{n,M} \end{aligned}$$

The wave functions $\longrightarrow$ eigenstates of corresponding internal Dirac/Laplace operator.

Higher Dimensional SYM theory with flux $U(1)$

Abelian gauge field on magnetized torus T^2



$$y_4 \sim y_4 + 1,$$
$$y_5 \sim y_5 + 1$$

Constant magnetic flux $F_{45} = 2\pi M,$

$$M \in Z$$

Dirac equation in magnetized backgroud

$$\left[\partial_i \gamma^i + A_i(z)\gamma^i\right]\psi(y) = 0$$

Dirac equation

|M| independent zero mode solutions in Dirac equation.

$$\Theta^{j}(y_4, y_5) = N_j e^{-M\pi y_4^2} \cdot \vartheta \left[\begin{array}{c} j/M \\ 0 \end{array} \right] (M(y_4 + i y_5), Mi)$$

$$(j = 0, 1, \cdots, |M| - 1)$$

Zero-mode = gaussian x theta-function

$$\vartheta \left[\begin{array}{c} a \\ b \end{array} \right] (\nu, \tau) \equiv \sum_n e^{\pi i (n+a)^2 \tau} e^{2\pi (a+n)(\nu + b)}$$ (Theta function)

Analysis for Yukawa interaction

[Cremades, Ibanez, Marchesano, '04]

$$\int dy^{D-4} Tr\left(\bar{\lambda}\Gamma^M D_M \lambda\right) \;\rightarrow\; \int dy^{D-4} Tr\left(\bar{\lambda}\Gamma^M [A_M, \lambda]\right)$$

$$F = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a\times N_a} & & 0 \\ & M_b \mathbf{1}_{N_b\times N_b} & \\ 0 & & M_c \mathbf{1}_{N_c\times N_c} \end{pmatrix},$$

$$\begin{pmatrix} & \psi_L & H \\ \psi_L^* & & \psi_R \\ H^* & \psi_R^* & \end{pmatrix},$$

$$Y_{ijk} = \int dy^{D-4} \psi_L^{i,M_1}(y)\psi_R^{j,M_2}(y)(\psi_H^{k,M_3}(y))^*$$

$$(M_1 = M_a - M_b,\ M_2 = M_b - M_c,\ M_3 = M_a - M_c)$$

Results of Yukawa couplings

by making use of **addition formula for theta functions**

$$\psi^{i,M}(z)\cdot\psi^{j,N}(z)=\sum_{m=1}^{M+N} y_{ijm}\psi^{i+j+Mm,M+N}(z)$$

orthogonal condition

$$\int d^2z\psi^{i,M}(z)\cdot(\psi^{j,N}(z))^{\dagger}=\delta ij$$

$$Y_{ijk}\equiv\int dzd\bar{z}\psi^{i,M_1}\psi^{j,M_2}(\psi^{k,M_3})^*$$

$$\propto\sum_{m\in M_3}\vartheta\begin{bmatrix}\frac{M_2 i-M_1 j+M_1M_2 m}{M_1M_2M_3}\\ 0\end{bmatrix}(a,\tau M_1M_2M_3)\times\delta_{i+j+M_1m,k}\quad \mathrm{mod}\ M_3$$

These forms are same as Heterotic orbifold and Intersecting D-brane calculations.

Coupling selection rule and Flavor symmetries

Coupling selection rule

The orthogonal condition imply

$$\boxed{i + j + k = 0 \pmod{g}}$$

$$g \equiv \mathrm{g.c.d.}(M_1, M_2, M_3)$$

g.c.d. … greatest common divisor

Allowed couplings are restricted.

There exists (I,j,k) such that satisfying this constraint.

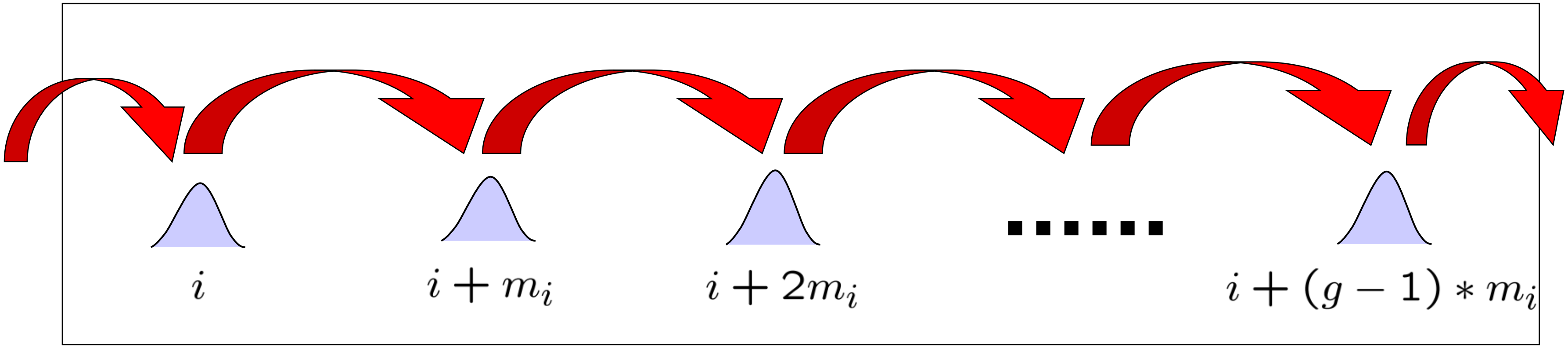
Shift Symmetry

Flux number are represented by following integer number m

$$m_1 = \frac{M_1}{g}, \quad m_2 = \frac{M_2}{g}, \quad m_3 = \frac{M_3}{g}$$

Yukawa couplings are invariant by following discrete shift

$$\left\{\begin{array}{l} i \to \tilde{i} = i + m_1 \\ j \to \tilde{j} = j + m_2 \\ k \to \tilde{k} = k + m_3 \end{array}\right. \quad \Rightarrow \quad Y_{ijk} = Y_{\tilde{i}\tilde{j}\tilde{k}}$$



Transformation property

Introducing following multiplet

$$|\psi^{M_i}\rangle = \begin{pmatrix} \psi^{i,M_i} \\ \psi^{i+m_i,M_i} \\ \psi^{i+2m_i,M_i} \\ \cdots \\ \psi^{i+(g-1)m_i,M_i} \end{pmatrix}$$

coupling selection rule $\Longleftrightarrow$ **Charge assignment of** $\mathbf{Z}_g$

$$Q = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \cdots & \\ & & & \omega^{g-1} \end{pmatrix} \qquad \omega \equiv e^{2\pi i/g}$$

discrete shift in the space $\Longleftrightarrow$ **Permutation symmetry**

$$P = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \cdots & \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix} \qquad \mathbf{Z}_g^{(P)}$$

Discrete Flavor Symmetry

Generators of transformation $\{P, Q\}$ (called by twist matrix)

Property of matrix $PQ = \omega QP$ (non-commutative)

The Closed algebra of the symmetry is $\left(\mathbf{Z}_g \times \mathbf{Z}'_g\right) \cup \mathbf{Z}_g^{(P)}$

Non-Ablein discrete flavor symmetry !

⇨ There are two types of diagonal $\mathbf{Z}_g$ matrix

$$\mathbf{Z}_g : \quad Q = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \cdots & \\ & & & \omega^{g-1} \end{pmatrix} \qquad \mathbf{Z}'_g : \quad \Omega = \begin{pmatrix} \omega & & & \\ & \omega & & \\ & & \cdots & \\ & & & \omega \end{pmatrix}$$

M=gm $\psi^{i,M}$ **corresponds to the certain multiplets under this discrete symmetry**

Some examples

- g=2 case
- g=3 case

g=2 case   M= 2, 4, 6, 8, …

$$\omega \equiv e^{2\pi i/g} = -1$$

twist matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Elements $\pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\pm\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\pm\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\Rightarrow$ **D4 flavor symmetry!**

This symmetry is same as the results of heterotic orbifold with $S^1/\mathbf{Z}_2$

But, what kind of the representation will appear ?

g=2 case   M= 2, 4, 6, 8, …

(1) m=1,   (M=gm=2)   two fields $\left(\psi^{0,2}, \quad \psi^{1,2} \right)$

⇨ **doublet** $|\psi^2\rangle = \begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix}$

(2) m=2,   (M=gm=4)   four fields $\left(\psi^{0,4}, \quad \psi^{1,4}, \quad \psi^{2,4}, \quad \psi^{3,4} \right)$

$$P : \left(\psi^{0,4}, \quad \psi^{1,4}, \quad \psi^{2,4}, \quad \psi^{3,4} \right) \to \left(\psi^{2,4}, \quad \psi^{3,4}, \quad \psi^{0,4}, \quad \psi^{1,4} \right)$$

$$Q : \left(\psi^{0,4}, \quad \psi^{1,4}, \quad \psi^{2,4}, \quad \psi^{3,4} \right) \to \left(\psi^{0,4}, \quad -\psi^{1,4}, \quad \psi^{2,4}, \quad -\psi^{3,4} \right)$$

⇨ **four singlets**

$$\mathbf{1}_{++} : \quad \left(\psi^{0,4} + \psi^{2,4}\right), \quad \mathbf{1}_{+-} : \quad \left(\psi^{0,4} - \psi^{2,4}\right)$$
$$\mathbf{1}_{-+} : \quad \left(\psi^{1,4} + \psi^{3,4}\right), \quad \mathbf{1}_{--} : \quad \left(\psi^{1,4} - \psi^{3,4}\right)$$

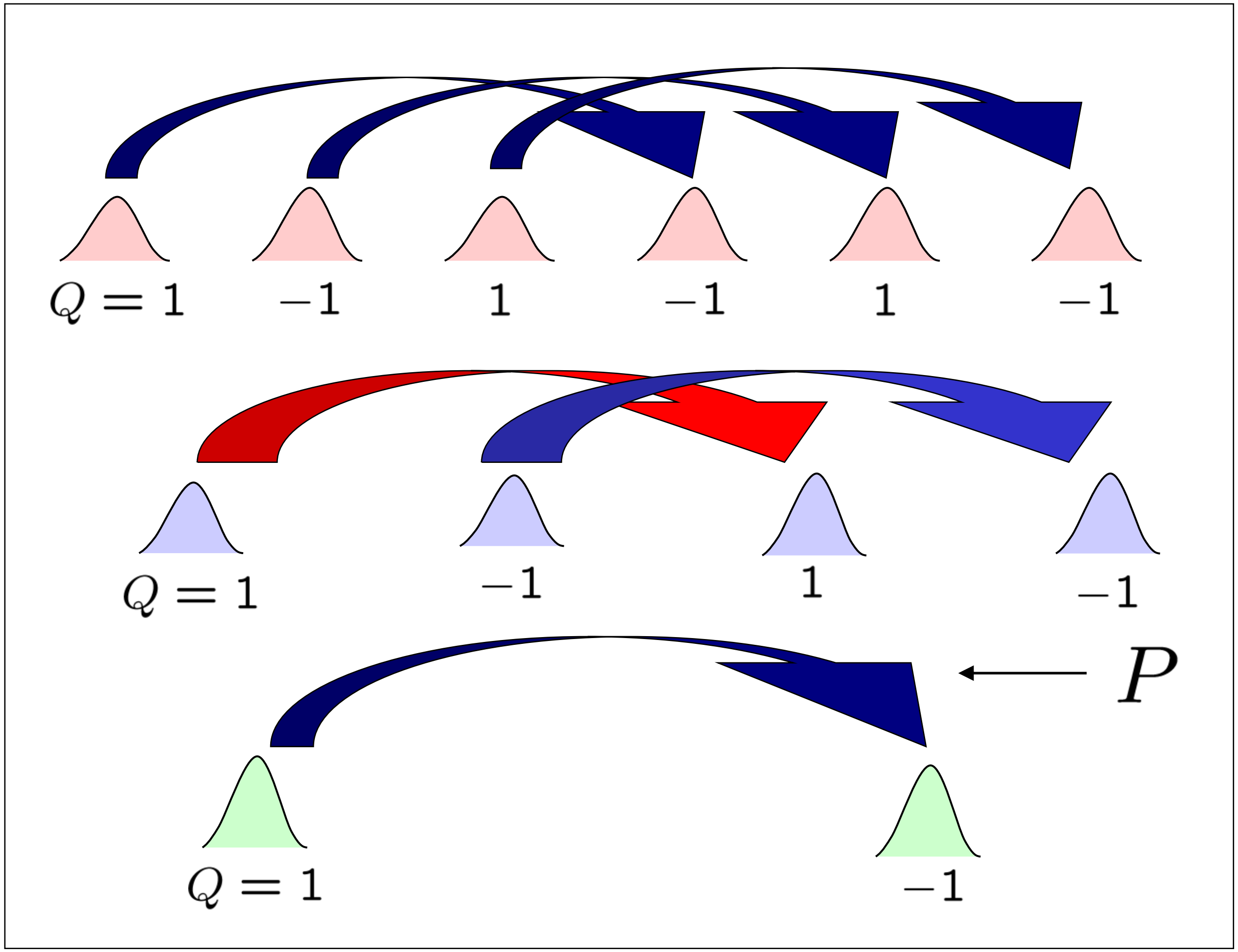
g=2

T^2

M= 6

M= 4

M= 2



Some Results of Representations

g=2 case

M	Representation of D_4
2	$\mathbf{2}$
4	$\mathbf{1}_{++},\ \mathbf{1}_{+-},\ \mathbf{1}_{-+},\ \mathbf{1}_{--}$
6	$3 \times \mathbf{2}$

g=3 case   M= 3, 6, 9, 12, …

$$(\mathbf{Z}_3 \times \mathbf{Z}_3) \cup \mathbf{Z}_3^{(P)} \Rightarrow \Delta(27)$$ **flavor symmetry!**

twist matrix $P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$ $\omega \equiv e^{2\pi i/3}$

(1) m=1, (M=gm=3) three fields $\left(\psi^{0,3}, \ \psi^{1,3}, \ \psi^{2,3} \right)$

$\Rightarrow$ $|\psi^3\rangle = \begin{pmatrix} \psi^{0,3} \\ \psi^{1,3} \\ \psi^{2,3} \end{pmatrix}$ triplet representation $\mathbf{3}$ of $\Delta(27)$

g=3 case M= 3, 6, 9, 12, … $\omega \equiv e^{2\pi i/3}$

(2) m=2, (M=gm=6)

$$\Rightarrow \quad |\psi^6\rangle_1 = \begin{pmatrix} \psi^{0,6} \\ \psi^{2,6} \\ \psi^{4,6} \end{pmatrix}, \quad |\psi^6\rangle_2 = \begin{pmatrix} \psi^{1,6} \\ \psi^{3,6} \\ \psi^{5,6} \end{pmatrix},$$

two triplet representation $\bar{\mathbf{3}}$ of $\Delta(27)$

(3) m=3, (M=gm=9)

nine singlet representations
$$\left\{ \begin{array}{lll} \mathbf{1}_{1,1}, & \mathbf{1}_{1,\omega}, & \mathbf{1}_{1,\omega^2}, \\ \mathbf{1}_{\omega,1}, & \mathbf{1}_{\omega,\omega}, & \mathbf{1}_{\omega,\omega^2}, \\ \mathbf{1}_{\omega^2,1}, & \mathbf{1}_{\omega^2,\omega}, & \mathbf{1}_{\omega^2,\omega^2}, \end{array} \right.$$

$$\Rightarrow \quad \mathbf{1}_{\omega^n,\omega^m} : \ \left(\psi^{n,9} + \omega^m \psi^{n+3m,9} + \omega^{2m} \psi^{n+6m,9} \right)$$

Some Results of Representations

g=3 case

M	Representation of $\Delta(27)$
3	$\mathbf{3}$
6	$2 \times \bar{\mathbf{3}}$
9	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{1}_5,\ \mathbf{1}_6,\ \mathbf{1}_7,\ \mathbf{1}_8,\ \mathbf{1}_9$
12	$4 \times \mathbf{3}$
15	$5 \times \bar{\mathbf{3}}$
18	$2 \times \{\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{1}_5,\ \mathbf{1}_6,\ \mathbf{1}_7,\ \mathbf{1}_8,\ \mathbf{1}_9\}$

Summary and future prospects

Summary

•We have studied the non-ablian discrete flavor symmetries , which can apper in D-dimensional N=1 super Yang-Mills theory with non-vanishing magnetic flux.

•We have found D4 or Delta(27) in Magnetized/intersecting D-brane model.

•We have shown rather simple model building in more generic class of extra dimensional models with flux.

•Future Prospects

•Extension to general non-Abelian magnetic flux is possible.

•In other compactifications, we are going to analyse the general N-point couplings and structure of flavor symmetries.

•We also consider anomaly of this discrete flavor symmetry

Thank you

Symmetry enhancement by vanishing Wilson line

Background without Wilson line case

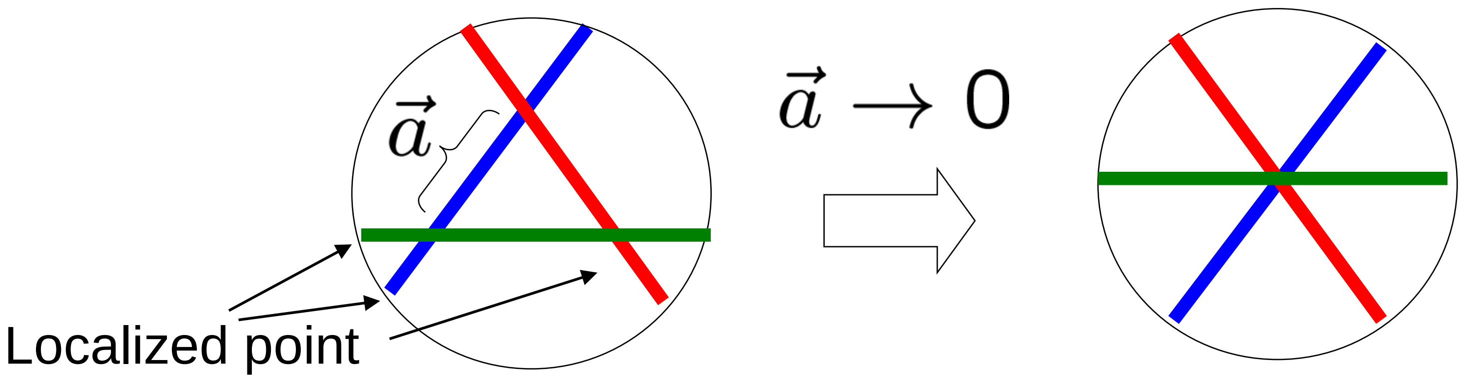
No Wilson line : $\begin{cases} a_4 = 0 \\ a_5 = 0 \end{cases}$

One can find the following property of wave functions (No Wilson line case) :

$$\boxed{\psi^{j,M}(-y_4, -y_5) = \psi^{M-j,M}(y_4, y_5)}$$

These effective action have the Z2 rotation symmetry as following

$$Z : \psi^{i,M} \to \psi^{M-i,M}$$



Representation of additional Z2 symmetry

We have g-plets in general.

$\mathbf{Z}_2$ acts on $\begin{pmatrix} \psi^{0,g} \\ \psi^{1,g} \\ \psi^{2,g} \\ \cdots \\ \psi^{(g-1),g} \end{pmatrix}$ g-plets as $Z = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ & & \cdots & & & \\ 0 & 1 & \cdots & 0 & 0 & 0 \end{pmatrix}$

We can reconsider about the discrete symmetry

which generated by these closed algebra $\{P, Q, Z\}$

$\Rightarrow$

- g=2 case
- g=3 case

g=2 case M= 2, 4, 6, 8, …

(1) m=1, (M=gm=2) Z2 act on the doublets as $Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Flavor symmetry is $D_4 \times \mathbf{Z}_2$ (direct product)

(2) m=2, (M=gm=4) four fields

$$Z : \left(\psi^{0,4}, \quad \psi^{1,4}, \quad \psi^{2,4}, \quad \psi^{3,4} \right) \rightarrow \left(\psi^{0,4}, \quad \psi^{3,4}, \quad \psi^{2,4}, \quad \psi^{1,4} \right)$$

$$\mathbf{1}_{+++} : \quad \left(\psi^{0,4} + \psi^{2,4}\right), \qquad \mathbf{1}_{+-+} : \quad \left(\psi^{0,4} - \psi^{2,4}\right)$$
$$\mathbf{1}_{-+\underline{+}} : \quad \left(\psi^{1,4} + \psi^{3,4}\right), \qquad \mathbf{1}_{---} : \quad \left(\psi^{1,4} - \psi^{3,4}\right)$$

Z2 charge

g=3 case   M= 3, 6, 9, 12, …

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\Rightarrow$ $\Delta(54)$ **flavor symmetry!**

(1) m=1, (M=gm=3) $|\psi^3\rangle = \begin{pmatrix} \psi^{0,3} \\ \psi^{1,3} \\ \psi^{2,3} \end{pmatrix}$

$\Rightarrow$ triplet representation $\mathbf{3}$ of $\Delta(54)$

(2) m=2, (M=gm=6) $|\psi^6\rangle_1 = \begin{pmatrix} \psi^{0,6} \\ \psi^{2,6} \\ \psi^{4,6} \end{pmatrix}, \quad |\psi^6\rangle_2 = \begin{pmatrix} \psi^{1,6} \\ \psi^{3,6} \\ \psi^{5,6} \end{pmatrix},$

$\Rightarrow$ two triplet representation $\bar{\mathbf{3}}$ of $\Delta(54)$

g=3 case

(3) m=3,   (M=gm=9)

$$\mathbf{1}_1 : \left(\psi^{0,9} + \psi^{3,9} + \psi^{6,9} \right)$$

$$\Rightarrow \quad \mathbf{2}_1 : \begin{pmatrix} \psi^{0,9} + \omega\psi^{3,9} + \omega^2\psi^{6,9} \\ \psi^{0,9} + \omega^2\psi^{3,9} + \omega\psi^{6,9} \end{pmatrix} \qquad \mathbf{2}_2 : \begin{pmatrix} \psi^{1,9} + \psi^{4,9} + \psi^{7,9} \\ \psi^{2,9} + \psi^{5,9} + \psi^{8,9} \end{pmatrix}$$

$$\mathbf{2}_3 : \begin{pmatrix} \psi^{1,9} + \omega\psi^{4,9} + \omega^2\psi^{7,9} \\ \psi^{8,9} + \omega^2\psi^{5,9} + \omega^2\psi^{2,9} \end{pmatrix} \qquad \mathbf{2}_4 : \begin{pmatrix} \psi^{1,9} + \omega^2\psi^{4,9} + \omega\psi^{7,9} \\ \psi^{8,9} + \omega^2\psi^{5,9} + \omega\psi^{2,9} \end{pmatrix}$$

In general g case,

$$\{Q, Z\} \Rightarrow D_g \qquad \left(PZ = Z^{-1}P\right)$$

Non-Abelian flavor symmetry

$$D_g \cup (Z_g \times Z_g)$$

Some Results of Representations

without Wilson line

g=2 case

M	Representation of $D_4 \times Z_2$
2	$\mathbf{2}_+$
4	$\mathbf{1}_{+++},\ \mathbf{1}_{+-+},\ \mathbf{1}_{-++},\ \mathbf{1}_{---}$
6	$2 \times \mathbf{2}_+,\ \mathbf{2}_-$
8	$\mathbf{1}_{+++},\ \mathbf{1}_{+-+},\ \mathbf{1}_{+++},\ \mathbf{1}_{+--},\ \mathbf{1}_{-++},\ \mathbf{1}_{-+-},\ \mathbf{1}_{---},\ \mathbf{1}_{--+}$
10	$3 \times \mathbf{2}_+,\ 2 \times \mathbf{2}_-$

g=3 case

M	Representation of $\Delta(54)$
3	$\mathbf{3}_1$
6	$2 \times \bar{\mathbf{3}}_1$
9	$\mathbf{1}_1,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4$
12	$3 \times \mathbf{3}_1,\ \mathbf{3}_2$
15	$3 \times \bar{\mathbf{3}}_1,\ 2 \times \bar{\mathbf{3}}_2$
18	$2 \times \{\mathbf{1}_1,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4\}$

Non-Abelian Discrete flavor symmetry

Magnetic flux without Wilson line non-vanishing Wilson line

$$D_g \cup \left(\mathbf{Z}_g \times \mathbf{Z}'_g\right) \xrightarrow{\langle A \rangle \neq 0} Z_g \cup \left(\mathbf{Z}_g \times \mathbf{Z}'_g\right)$$

$$(g \neq 1, 2)$$

In especially

g=2 case $$D_4 \times Z_2 \xrightarrow{\langle A \rangle \neq 0} D_4$$

g=3 case $$\Delta(54) \longrightarrow \Delta(27)$$

These results can also apply the intersecting D-brane models

Higher order couplings

4 point coupling

$$y_{i_1 i_2 i_3 \bar{i}_4} = \int d^2z \; \psi^{i_1,M_1}(z)\psi^{i_2,M_2}(z)\psi^{i_3,M_3}(z) \left(\psi^{i_4,M_4}(z)\right)^*$$

$$\Downarrow$$

Can be decompose into products of 3 point couplings by using the property of completeness Dirac operator.

$$y_{i_1 i_2 i_3 \bar{i}_4} = \sum_{s \in Z_M} y_{i_1 i_2 \bar{s}} \; y_{s i_3 \bar{i}_4}$$

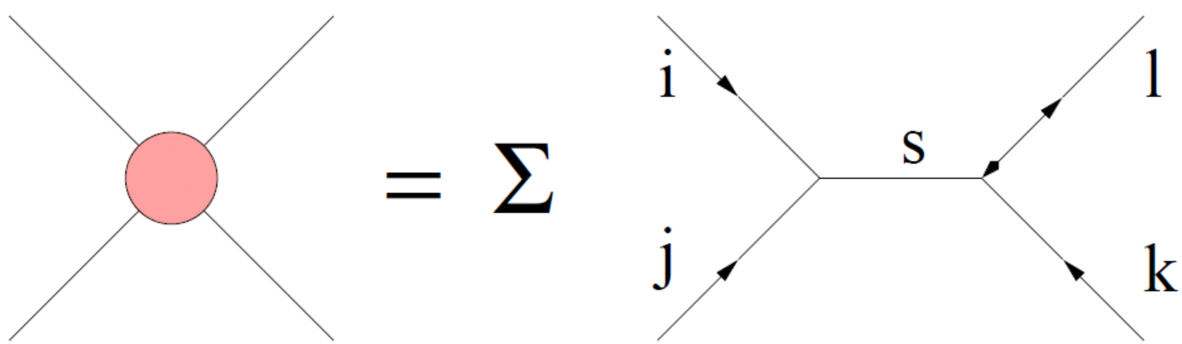
$$\begin{aligned} y_{i_1 i_2 \bar{s}} &= \int d^2z \; \psi^{i_1,M_1}(z)\psi^{i_2,M_2}(z) \left(\psi^{s,M}(z)\right)^* \\ y_{s i_3 \bar{i}_4} &= \int d^2z \; \psi^{s,M}(z)\psi^{i_3,M_3}(z) \left(\psi^{i_4,M_4}(z)\right)^* \\ M &= M_1 + M_2 = M_4 - M_3. \end{aligned}$$

4 point coupling

Coupling Selection rule

$$\boxed{i_1 + i_2 + i_3 + i_4 = 0 \pmod{g}}$$

Controlled by Zg symmetry



General N point couplings and other wave functions

These analysis is generalized to the generic N-point couplings

Then flavor symmetries arise in g.c.d(M1,M2,…M_N)=g

$$\left(\mathbf{Z}_g \times \mathbf{Z}'_g\right) \cup \mathbf{Z}_g^{(P)}$$

These mechanism is independent of geometry (backgroud).

We can extent the other compactification (s.t. S^2 or Warped backgroud) [Conlon et.al. 08, Marchesano et.al. 08]

for N-point couplings an flavor symmetries

⇨ In progress